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A General Model for Estimating and Correcting the Effects of Nonindependence in Meta-Analysis

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Abstract

This paper describes a general meta-analysis model that can be used to represent the four types of meta-analysis commonly conducted. The model explicitly allows for nonindependence among study outcomes, providing exact statistical solutions when the nonindependence can be estimated. Also discussed are the directional biases that result if nonindependence is ignored.

A General Model for Estimating and Correcting the Effects of Nonindependence in Meta-Analysis

Over the past several years there has been a dramatic increase in the use of meta-analytic procedures. At the same time there has been relatively little attention given to some of the problems that are encountered when traditional statistical procedures are applied to the nontraditional data bases that meta-analysts encounter (for exceptions, see Rosenthal & Rubin, 1986; Strube, 1985a; Tracz & Elmore, 1985; Tracz, Newman, & McNeil, 1986). One of the more prevalent and serious problems encountered in a meta-analysis occurs when studies give rise to multiple outcomes. In such cases, the assumption of independence is violated with potentially serious inferential consequences. To date, there has been no clear exposition of the nature or direction of bias that exists when nonindependence is ignored. The purpose of this paper is thus twofold. First, I will present a general model of nonindependence that encompasses the four major types of meta-analysis that are conducted. This model also provides an exact solution for the correction of nonindependence. Second, I will indicate the inferential consequences of ignoring nonindependence.

A General Model for Meta-Analysis

There are four basic types of meta-analysis that are typically conducted. First, the meta-analyst may examine study outcome defined in terms of an effect size estimate (e.g., Δ *d*, g, or *r*) or in terms of an estimate of statistical significance (e.g., *p* or *Z*). Second, within these two outcome classes, the meta-analyst can perform two basic tasks (Rosenthal, 1983) by either combining study outcomes or contrasting study outcomes. The former task represents an interest in the overall outcome whereas the latter task corresponds to a search for moderators of study outcome.

What often goes unnoticed is that the various specific statistical procedures described in the literature for carrying out these four types of meta-analysis all represent special cases of a more general approach. In particular, all can be represented as special cases of the following formula:

$$Z = \frac{\Sigma \lambda_i \psi_i}{(\Sigma \lambda_i^2 \sigma_i^2 + 2\Sigma \lambda_i \lambda_j \sigma_i \sigma_j \rho_{ij})^{1/2}} \qquad (i \neq j)$$

This formula represents a weighted linear combination of elements, ψ, divided by the standard deviation of that linear combination. When the linear combination is tested against the null mean of zero, the ratio will be approximately normally distributed for modest sample sizes. There are several things to note about the formula. First, the elements to be combined or contrasted can be either effect sizes or an index of statistical significance. Second, if $\psi = Z$, and all Z are independent, then the formula provides the familiar Stouffer solution for combined probabilities (see Strube, 1985a). Third, if ψ are to be combined, then all $\lambda = 1$. Finally, if ψ are to be contrasted, then $\Sigma\lambda$ must equal zero (as in ANOVA or regression). As can be seen, all four types of meta-analysis can be represented.

What makes this approach additionally useful is that it provides a means of accounting for nonindependence. As the formula and the variance-covariance matrix in Figure 1 indicate, nonindependence serves to alter the size of the standard deviation of the linear combination. Under the assumption of independence, all covariance terms are zero, and the estimate of the standard deviation of the linear combination is based solely on the main diagonal of the variance-covariance matrix (formulae for estimating the variances of several common effect sizes can be found in Hodges & Olkin, 1985; Rosenthal, 1984). Thus it is the off-diagonal elements that are of particular interest when there is nonindependence.

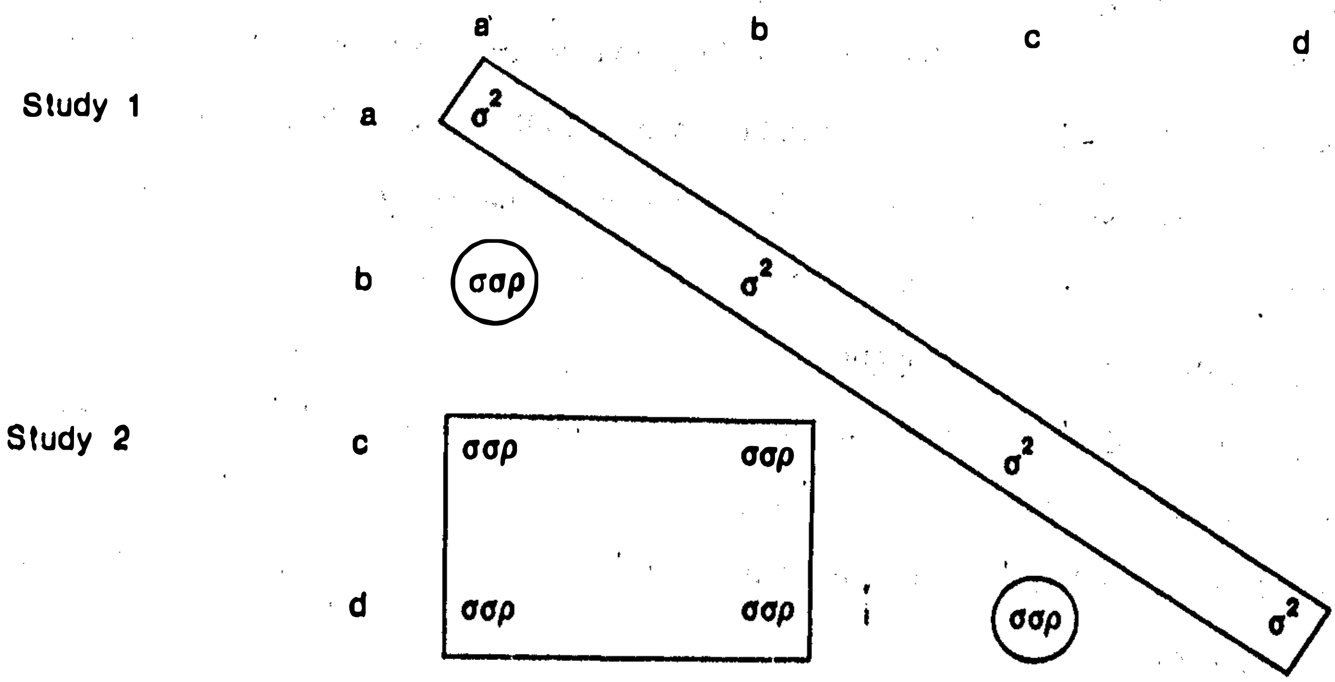

Figure 1. Variance-covariance matrix for two studies, each with two outcomes.

Nonindependence will arise in a meta-analysis whenever the same study (or subject, for N = 1 research, see Strube, 1985b) provides more than one effect size or significance level to be combined or compared. In that case, one must attempt to estimate the magnitude of the off-diagonal elements of the variance-covariance matrix (see Strube, 1985a). Actually, we need not estimate all of the off-diagonal elements. It is probably safe to assume that effect sizes and significance levels from different studies are independent, and thus the corresponding covariances are zero. Thus, in Figure 1, the covariances in the lower left box can be assumed to be zero. Only the circled covariances need to be estimated. If reasonable estimates for these covariances can be obtained, then an exact combination or contrast is possible.

Consequences of Nonindependence

Given current reporting practices, it may be difficult to estimate the needed covariances. It is still important to recognize the type of influence that nonindependence has so that, even if it cannot be adjusted statistically, it can serve to temper one's conclusions.

Figure 2 displays four basic types of questions that could be asked in a meta-analysis, as represented by the weights (λ) that would be used in our formula. We also have listed 3 studies each of which gave rise to 2 outcomes measures that we will assum are positively correlated. In the first case, all outcomes are added (a combined result i desired), that is, all λ are positive and thus the influence of nonindependence is to infla the denominator of the formula. Accordingly, failing to adjust for nonindependence wi inflate the likelihood of a Type I error. In the second case, two studies are compared. Because the comparison is across correlated units, the influence of nonindependence is inflate the denominator of the formula (i.e., cross-product of λs is positive). Again, failing to take nonindependence into account will inflate the Type I error rate. The th case represents a contrast where the two different outcomes within studies are

		Type of Contrast			
		λ_1	λ_2	λ_3	λ_4
Study 1	A	1	1	1	1
	B	1	1	-1	-1
Study 2	A	1	-1	1	-1
	B	1	-1	-1	1
Study 3	A	1	0	1	0
	B	1	0	-1	0
Type of Error Increased		Type I	Type I	Type II	Type II

Figure 2. Four common meta-analytic contrasts and their associated inferential errors when nonindependence is ignored.

compared. Because the comparison is within studies, the influence of the nonindependence is to decrease the denominator of the formula (all $\lambda_i \ \lambda_j$ are negative). In this case, failing to adjust for nonindependence will inflate the Type II error rate. The final case represents a pattern of contrasts corresponding to an interaction. Here interest is in whether the difference between the two outcome measures depends on the study. Here too, the effect of unadjusted nonindependence is inflate the Type II error rate.

Thus it can be seen that the effect of nonindependence on the outcome of a met analysis depends on the type of question being asked.

Summary

In sum, the meta-analyst must be aware of the influence of nonindependence. Where possible, the effect of nonindependence should be adjusted statistically. If this i not possible, the meta-analyst must quality conclusions, taking into account the known directional effects of nonindependence on the likelihood of making Type I and Type II errors. If nonindependence is ignored, meta-analysts may introduce stubborn and erroneous conclusions into the literature.

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