The Use of MLR Models to Analyze Partial Interaction: An Educational Application

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Abstract

Certain research questions found in educational studies require partial interaction effects to be tested. This paper presents an application of the method of using MLR models to test a partial interaction hypothesis.

Introduction

Newman, Deitchman, Burkholder, Sanders, and Ervin (1976) addressed the issue of the importance of matching the statistical analysis with the question posed by the researcher. The use of multiple linear regression (MLR) models allows the researcher the flexibility of analysis needed to address research questions that require the testing of partial interaction (see McNeil, Kelly and McNeil; 1975). This paper presents the MLR models and the technique used to test a partial interaction research hypothesis posed in an educational study.

Research Design

A study by Drushal (1986) examined the impact of various participative decision making (PDM) techniques. The techniques examined in the study were Delphi Survey Technique (DST), Social Judgment Analysis (SJA), Nominal Group Technique (NGT), and a control group. The students in the control group were not exposed to any of the PDM techniques.

Seminary students were randomly assigned to one of the four groups. Through participation in a decision making technique, students selected the criteria to be considered in making a curriculum choice for a Sunday school. After experiencing the assigned decision making technique, participants responded to the Participative Management Survey (PMS). The PMS is a survey composed of research-based statements on leadership, trust, communication and participative decision making (see Drushal, 1986). Each student in the study received a total score on the PMS instrument. These total scores served as the values of the dependent variable for the MLR models used to test the partial interaction research question presented in the next section of this paper.

Research Hypothesis

One of the research hypotheses of interest to the researchers was:

H_1: The difference between the average of the mean PMS scores for females in the PDM groups and the mean PMS score for females in the control group will exceed the difference between the average of the mean PMS scores for males in the PDM groups and the mean PMS score for males in the control group.

To test this research hypothesis, a test of partial interaction was required. The construction and analysis of MLR models readily allowed the researchers to test this partial interaction hypothesis.

Full MLR Model

The full MLR model used to test the partial interaction hypothesis contains the interaction effect between the two independent variables--instructional techniques and gender. There were four instructional techniques and the two levels of gender. The full MLR model, which is a full interaction model, was:

$$y = au + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 x_6 + b_7 x_7 + e$$

where:

y = PMS score for each student
x_1 = 1 if student in DST group and female; 0 otherwise
x_2 = 1 if student in SJA group and female; 0 otherwise
x_3 = 1 if student in NGT group and female; 0 otherwise
x_4 = 1 if student in Control group and female; 0 otherwise
x_5 = 1 if student in DST group and male; 0 otherwise
x_6 = 1 if student in SJA group and male; 0 otherwise
x_7 = 1 if student in NGT group and male; 0 otherwise
x_8 = 1 if student in Control group and male; 0 otherwise
a = constant term
e = error term
u = unit vector

It is interesting to note that the R_2 value of this full model will equal the R_2 value generated by a oneway ANOVA of the scores of the eight groups.

Since the computer program used to compute the parameters for the full MLR model includes a unit vector, the variable x_8 was not included in the model. Thus, the value for a—the constant term—represents the mean PMS score for the males in the control group. The b_1 value represents the difference between the mean PMS score for females in the DST group and the value for the constant term a, which is the mean PMS score for males in the control group. The other b values contained in the full MLR model would be interpreted in a similar fashion.

Restriction

The restriction made on the full model to obtain the restricted MLR model required that the difference between the average of the mean PMS scores of the females in the PDM groups and the mean PMS score for females in the control group be equal to the difference between the average of the mean PMS scores of the males in the PDM groups and the mean PDM score for males in the control group. Thus, the restriction was:

$$(b_1 + b_2 + b_3)/3 - b_4 = (b_5 + b_6 + b_7)/3$$

The left-hand side of the restriction represents the difference between the PMS mean scores of the females assigned to the PDM groups and the mean score of the females in the control group. The right-hand side of the restriction represents the difference between the average of the mean PMS scores for males in the PDM groups and the mean PDM score for the males in the control group.

Again, it is interesting to note that in view of the fact that the R_2 value of the full model corresponds to the R_2 value that would be generated by an ANOVA of the scores, this restriction can be thought of as a contrast of the eight group means. The restriction specifies the contrast. Williams (1976 and 1979) discussed the use of MLR models to conduct contrasts of group means.

The restriction can be more clearly explained by referring to a graph of the interaction effect between the instructional methods and gender, which was estimated by the regression coefficients of the full MLR model. Gender was placed along the X axis of Figure 1. Recall that each of the regression coefficients of the full MLR model represents the differences between the mean

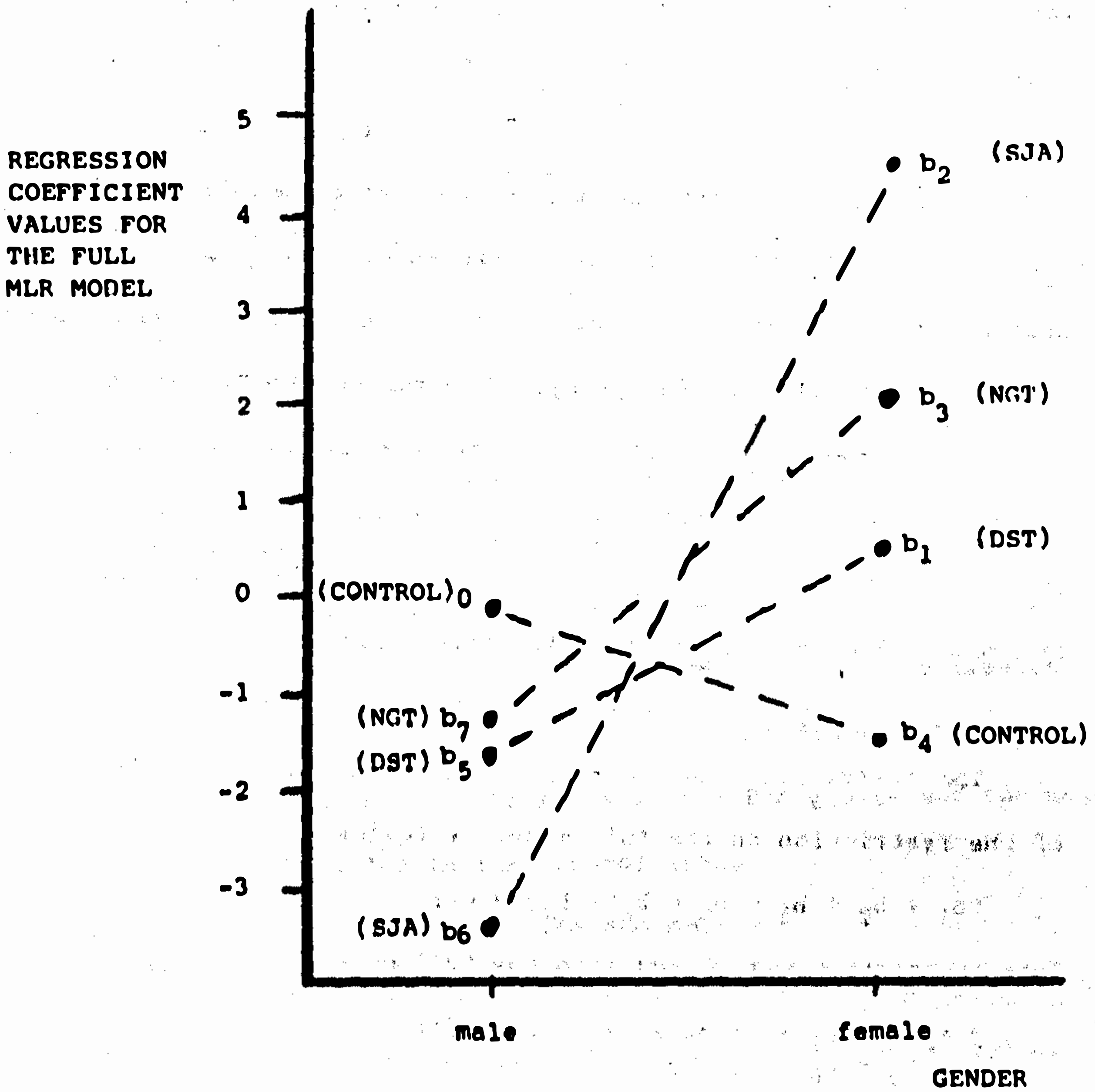

Figure 1

Interaction Effect Estimated by the Full MLR Model

PMS score for a given instructional group and gender, and the mean PMS score for males in the control group. Thus, the Y axis of Figure 1 represents the differences in the mean PMS scores of the various combinations of groups and gender, and the value for the constant term a, which is the mean PMS score of the males in the control group.

In Figure 1 the distance between average of points b_1, b_2 and b_3, and point b_4 represents the difference between the mean PMS scores for females in the three PDM groups and the mean PDM score for females in the control group. The restriction requires that this distance equal the distance between the average of points b_5, b_6 and b_7, and the 0 point, which is the difference between the average of the mean PMS scores of the males in the PDM groups and the mean PMS score for males in the control group.

Restricted MLR Model

The restriction was manipulated to facilitate the placement of the restriction on the full model as follows:

$$(b_1 + b_2 + b_3 - b_5 - b_6 - b_7)/3 = b_4$$

This restriction was placed into the full model as follows:

$$y = au + b_1 x_1 + b_2 x_2 + b_3 x_3 + ((b_1 + b_2 + b_3 - b_5 - b_6 - b_7)/3) x_4 + b_5 x_5 + b_6 x_6 + b_7 x_7 + e$$

Multiplying the restriction by x_4 and collecting like regression coefficients produced the following restricted model:

$$y = au + b_1 (x_1 + \frac{x_4}{3}) + b_3 (x_2 + \frac{x_4}{3}) + b_3 (x_3 + \frac{x_4}{3}) +$$

$$b_5 (x_5 - \frac{x_4}{3}) + b_6 (x_6 - \frac{x_4}{3}) + b_7 (x_7 - \frac{x_4}{3}) + e$$

To facilitate the analysis of the restricted MLR model by the computer, the following variables were calculated:

$$x_9 = x_1 + x_4/3$$

$$x_{10} = x_2 + x_4/3$$

$$x_{11} = x_3 + x_4/3$$

$$x_{12} = x_5 - x_4/3$$

$$x_{13} = x_6 - x_4/3$$

$$x_{14} = x_7 - x_4/3$$

Thus, the restricted model took the form:

$$y = au + b_9 x_9 + b_{10} x_{10} + b_{11} x_{11} + b_{12} x_{12} + b_{13} x_{13} + b_{14} x_{14} + e$$

Due to the nature of the restriction, this restricted model requires that the difference between the average PMS scores for females in the PDM groups and the mean PMS score of the females in the control group be equal to the difference between the average of the mean PMS scores of the males in the PDM groups and the mean PMS score of the males in the control group.

Test of the MLR Models

To determine whether the data supported the researcher hypothesis , an F test of the difference between the R^2 values of the full and restricted models was required. The results of the analysis are presented in Table 1. Since the research hypothesis was directional, the critical F value of 2.75 for the alpha level of .05 corresponded to the critical value of a directional or

Table 1

F Test of the Partial Interaction Research Hypothesis

Hypothesis and Models	R^2	df	F	Critical F	Sig
H_1: The difference between the average of the mean PMS scores for females in the PDM groups and the mean PMS score for females in the control group will exceed the difference between the average of the mean PMS scores for males in the PDM groups and the mean PMS score for males in the control group.					
Full Model: $Y = au + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_5 x_5 + b_6 x_6 + b_7 x_7 + e$	.1263				
Restriction: $(b_1 + b_2 + b_3)/3 - b_4 = (b_5 + b_6 + b_7)/3$		1/329	6.02	2.75	Sig
Restricted Model: $Y = au + b_9 x_9 + b_{10} x_{10} + b_{11} x_{11} + b_{12} x_{12} + b_{13} x_{13} + b_{14} x_{14} + e$	.0855				

one-tailed test. The F test revealed that the calculated F value of 6.02 did exceed the critical F value of 2.75.

Even though the calculated F value exceeded the critical value, the researchers had to check the signs of the regression coefficients contained in the restriction before it could be determined whether the directional research hypothesis was supported by the data. That is, the difference between the average of the mean PMS scores for females in the PDM groups and the mean PMS score for the females in the control group had to exceed the difference between the average of the mean PMS scores for males in the PDM groups and the mean PDM score for the males in the control group.

The regression coefficient values for the full MLR model were as follows:

$b_1 = .78$
$b_2 = 4.92$
$b_3 = 2.07$
$b_4 = -1.47$
$b_5 = -1.59$
$b_6 = -3.22$
$b_7 = -1.07$

To support the directional statement contained in the research hypothesis, the left-hand side of the restriction had to be greater than the right-hand side of the restriction. That is:

$$(b_1 + b_2 + b_3)/3 - b_4 > (b_5 + b_6 + b_7)/3$$

The regression coefficients indicated that the value of 4.06 for the left-hand side of the restriction was indeed greater than the value of -1.96 for the right-hand side of the restriction. Therefore, the signs of the regression coefficients as well as the

F test of the difference between the R^2 values of the full and restricted MLR models supported the research hypothesis.

Summary

Researchers should not be hesitant to include partial interaction questions in research projects because of the perceived difficulty of testing such hypotheses. As indicated by the procedures presented in this paper, the use of MLR models allows researchers to analyze partial interaction questions in a versatile and straightforward manner.

References

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