MULTIPLE LINEAR REGRESSION VIEWPOINTS
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Relationship Between Multiple Regression, Path, Factor, and LISREL Analyses

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Abstract

A basic knowledge of multiple regression concepts permits further understanding of path, factor, and lisrel analyses. Specifically, standardized partial regression coefficients (beta weights) as applied in path, factor, and lisrel analyses are presented. The multivariable methods have in common the general linear model and are the same in several respects. First, they identify, partition, and control variance. Second, they are based upon a linear combination of variables. And third, the linear weights can be computed based on standardized partial regression coefficients.

Multiple regression or the general linear model approach to the analysis of experimental data in educational research has become increasingly popular since 1967 (Bashaw and Findley, 1968). In fact today, it has become recognized as an approach that bridges the gap between correlational and analysis of variance thought in answering research hypotheses (McNeil, Kelly, & McNeil, 1975). Statistical textbooks in psychology and education often present the relationship between data analysis with multiple regression and analysis of variance (Draper & Smith, 1966; Williams, 1974a; Roscoe, 1975; Edwards, 1979). Graduate students taking an advanced statistics course are therefore provided with the multiple linear regression framework for data analysis. Given their knowledge of multiple linear regression techniques applied to univariate analysis (one dependent variable), their understanding can be extended to the relationship of multiple linear regression to various multivariate statistical techniques (Kelly, Beggs, McNeil, with Eichelberger & Lyon, 1969, pps 228-248; Newman, 1988). The article therefore expands upon this understanding and indicates the importance of the standardized partial regression coefficient (beta weight) in multiple linear regression as it is applied in path, factor, and lisrel analyses.

MULTIPLE REGRESSION

Multiple regression techniques require a basic understanding of sample statistics (n, mean, and variance), standardized variables, correlation (Pedhazur, 1982, pp 53-57), and partial correlation (Cohen & Cohen, 1975; Houston & Bolding, 1974). In standard score form the multiple regression equation is:

$$\hat{z}_y = b z_x$$

The relationship between the correlation coefficient, the unstandardized regression coefficient and the standardized regression coefficient is:

$$b = \frac{S z_x z_y}{S z_x^2} = b \frac{s_x}{s_y} = r_{xy}$$

For two independent variables, the regression equation with standard scores is:

$$\hat{z}_y = b_1 z_1 + b_2 z_2$$

And the standardized partial regression coefficients are computed by:

$$b_1 = \frac{r_{y1} - r_{y2} r_{12}}{1 - r_{12}^2} \qquad b_2 = \frac{r_{y2} - r_{y1} r_{12}}{1 - r_{12}^2}$$

The correlation between the original and predicted scores is given the special name Multiple Correlation Coefficient. It is indicated as:

$$R_{y\hat{y}} = R_{y.12}$$

And the Squared Multiple Correlation Coefficient is related as follows:

$$R^2_{y\hat{y}} = R^2_{y.12} = b_1 r_{y1} + b_2 r_{y2}$$

MULTIPLE REGRESSION EXAMPLE

A multiple linear regression example using a correlation matrix as input (SPSSX User's Guide, 3rd Edition, 1988, Chapter 13) is in the appendix. The results are:

$$R^2_{y.123} = b_1 r_{y1} + b_2 r_{y2} + b_3 r_{y3}$$

$$= (.423)\ .507 + (.363)\ .481 + (.040)\ .276$$

$$R^2_{y.123} = .40$$

A systematic determination of the most important set of variables can be accomplished by setting the partial regression weight of each variable to zero. This approach and other alternative methods are presented by Kelly, Beggs, & McNeil et al (1969) and Darlington (1968).

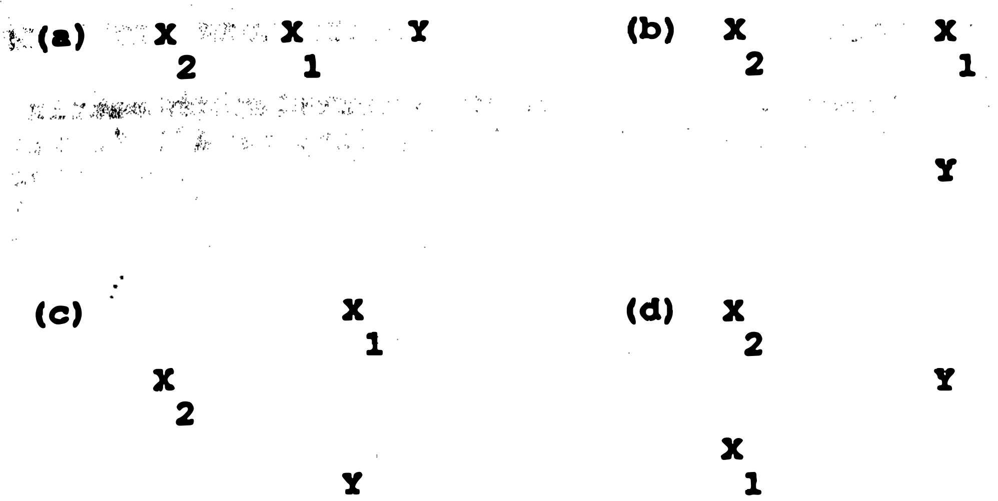
In summary, regression techniques have been shown to be robust (Bohrnstedt & Carter, 1971); applicable to contrast coding (Lewis & Mouw, 1978); dichotomous coding (McNeil, Kelly, & McNeil, 1975); and ordinal coding (Lyons, 1971) research situations. Multiple regression can also be viewed as a special case of path analysis.

PATH ANALYSIS

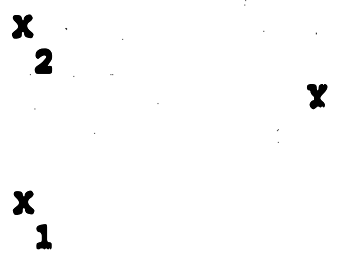
Sewall Wright is credited with the development of path analysis as a method for studying the direct and indirect effects of variables (Wright, 1921, 1934, 1960). Path analysis is not a method for discovering causes, rather it tests theoretical relationships called "causal modeling". The specified model establishes causal relationships among the variables when:

a. temporal ordering exists
b. covariation (correlation) is present
c. controlled for other causes

Model specification is necessary in examining multiple variable relationships. In the absence of a model, many different relationships among variables can be postulated with many different path coefficients being selected. For example, in a three variable model the following four relationships could be postulated:



The four different models have been considered without reversing the order of the variables. How can one decide which model is correct? Path analysis doesn't provide a way to specify the model, but rather estimates the effects once the model has been specified "a priori". Path coefficients in path analysis take on the values of a product-moment correlation and/or standardized regression coefficients in a model (Wolfle, 1977). For example given model (d):



THEN:

$b_1 = p_{y1}$ $\qquad$ $b_2 = p_{y2}$ $\qquad$ $r_{12} = p_{12}$

A path model is specified by the researcher based on theory or prior research. Variable relationships once specified, in standard score form, become standardized regression coefficients. In multiple regression, a dependent variable is regressed in a single analysis on all the independent variables. In path analysis one or more multiple regression analyses are performed. Path coefficients are computed based upon only the particular set of independent variables that lead to the dependent variable under consideration. As in regression analysis, path analysis can use dichotomous and ordinal data in the causal model (Boyle, 1970; Lyons, 1971).

MODEL SPECIFICATION

Path models permit diagramming how a particular set of independent variables lead to a dependent variable under consideration. How the paths are drawn determine whether the independent variables are correlated causes (unanalyzed), mediated causes (indirect), or independent causes (direct). The model can be tested for the significance of path coefficients (Pedhazur, 1982, pp 58-62) and a goodness-of-fit criteria (Marascuilo & Levin, 1983, pp 169-172; Tatsuoka & Lohnes, 1988, pp 98-100) which reflects the significance between the original and reproduced correlation matrix. This process is commonly called decomposing the correlation matrix (Asher, 1976, pp 32-34) according to certain rules (Wright, 1934).

PATH ANALYSIS EXAMPLE

A four variable path analysis program is in the appendix. In order to calculate the path coefficients for the model, two regression analyses were performed. The model with the path coefficients is:

X_1 $\quad$ $p_{Y1} = .423$

$p_{21} = .224$ $\quad$ $p_{31} = -.071$ $\quad$ X_3 $\quad$ $p_{Y3} = .040$ $\quad$ Y

$p_{32} = .593$ $\quad$ $p_{Y2} = .363$

X_2

The original and reproduced correlations are presented in matrix form. The upper half represents original correlations and the lower half the reproduced correlations which include the regression of paths linking independent variables to the dependent variable.

VARIABLE	Y	X1	X2	X3
Y	1.000	.507	.481	.276
X1	.423	1.000	.224	.062
X2	.362	.224	1.000	.577
X3	.040	-.070	.593	1.000

Original Correlations (upper half)

Reproduced Correlations (lower half)

The original correlations can be completely "reproduced" if all effects: direct (DE), indirect (IE), spurious (S) and correlated (C) are included. For example:

$$r_{12} = \underset{C}{P_{12}} = .224$$

$$r_{13} = \underset{DE}{P_{31}} + \underset{IE}{P_{32}P_{21}} = .062$$

$$r_{23} = \underset{DE}{P_{32}} + \underset{S}{P_{31}P_{21}} = .577$$

$$r_{1Y} = \underset{DE}{P_{Y1}} + \underset{IE}{P_{Y2}P_{21}} + \underset{IE}{P_{Y3}P_{31}} + \underset{IE}{P_{Y3}P_{32}P_{21}} = .507$$

$$r_{2Y} = \underset{DE}{P_{Y2}} + \underset{IE}{P_{Y3}P_{32}} + \underset{S}{P_{Y1}P_{21}} + \underset{S}{P_{Y3}P_{31}P_{21}} = .481$$

$$r_{3Y} = \underset{DE}{P_{Y3}} + \underset{S}{P_{Y1}P_{31}} + \underset{S}{P_{Y2}P_{32}} + \underset{S}{P_{Y1}P_{21}P_{32}} + \underset{S}{P_{Y2}P_{21}P_{31}} = .276$$

In summary, path analysis can be carried out within the context of ordinary regression analysis and does not require the learning of any new analysis techniques (Asher, 1976, p32; Williams, 1974b). The advantage of path analysis is that it enables one to specify direct and indirect effects among independent variables. In addition, path analysis enables us to decompose the correlation between any two variables into simple and complex paths of which some are meaningful. Path coefficients and the relationship between the original and reproduced correlation matrix can also be tested for significance.

FACTOR ANALYSIS

Path models and the associated test of significance between original and reproduced correlations are used in confirmatory factor analysis. Factor analysis assumes that the observed (measured) variables are linear combinations of some underlying source variable (factor). In practice, one estimates population parameters of the measured variables from a sample (with the uncertainties of model specification and measurement error). A linear combination of weighted variables relates to multiple regression in a single factor model and to a linear causal system (path analysis - "multiple" multiple regressions) in multiple factor models. Path diagrams therefore permit representation of the causal relationships among factors and observed (measured) variables in factor analysis.

In general, the first step in factor analysis involves the study of interrelationships among variables in the correlation matrix. Factor analysis will address the question of whether these subsets can be identified by one or more factors (hypothetical constructs). Confirmatory factor analysis is used to test specific hypotheses regarding which variables correlate with which constructs (Long, 1983).

FACTOR MODELS

Factor analysis assumes that some factors, which are smaller in number than the number of observed variables, are responsible for the covariation among the observed variables. For example, given a unidimensional trait in a single factor model with four variables the diagram would be (Kim & Mueller, 1978a, p 35):

$d_Y = .735$

$b_Y = .677$ Y U_Y

$d_1 = .917$

X_1 U_1

F $b_1 = .402$

$d_2 = .600$

$b_2 = .800$ X_2 U_2

$d_3 = .843$

$b_3 = .535$ X_3 U_3

WHERE:

b_i = standardized regression coefficient

The variance of each observed variable is therefore comprised of the proportion of variance determined by the common factor and the proportion determined by the unique factor, which together equal the total variance of each observed variable. Therefore:

$$S_i^2 = b_i^2 + d_i^2 = 1$$

The correlation between a common factor and a variable is:

$$r_{F,X_i} = b_i$$

The correlation between a unique factor and a variable is:

$$r_{U,X_i} = d_i$$

The correlation between observed (measured) variables sharing a common factor is:

$$r_{X_i,X_j} = b_i b_j$$

And finally, the variance attributed to the factor as a result of the linear combination of variables is:

$$h^2 = \frac{\Sigma b_i^2}{M} = R^2_{F.1234}$$

Where: M = number of variables

b_i^2 = squared factor loadings

Note: Σb_i^2 = eigenvalue

b_i^2 = communality

FACTOR ANALYSIS EXAMPLE

A single factor analysis program with four variables in a correlation matrix format is in the appendix. The path diagram is the same as above (Kim & Mueller, 1978a, p 35) with the weights as follows:

$$b_Y = .677 \quad b_1 = .402 \quad b_2 = .800 \quad b_3 = .535$$

And, factor scores computed as:

$$F = b_y Y + b_1 X_1 + b_2 X_2 + b_3 X_3$$

Multiplying the coefficients between pairs of variables gives the following correlation matrix:

VARIABLE	Y	X1	X2	X3
Y	b_1^2	.27	.54	.36
X1	.27	b_2^2	.32	.22
X2	.54	.32	b_3^2	.43
X3	.36	.22	.43	b_4^2

The common factor variance is:

$$R^2_{F.1234} = \frac{\Sigma b_i^2}{M} = \frac{.46 + .16 + .64 + .29}{4} = .39$$

The unique factor variance is:

$$1 - R^2_{F.1234} = \frac{\Sigma (1 - b_i^2)}{M} = \frac{.54 + .84 + .36 + .71}{4} = .61$$

In summary, factor loadings (variable weights) are standardized regression coefficients. As such, linear weighted combinations of variables loading on a factor are used to compute factor scores (Kim & Mueller, 1978b p 60). The weights are also the correlation between the observed (measured) variables and the factor (hypothetical construct). If the variable correlations (weights) are squared and summed, they describe the proportion of variance determined by that factor. This is traditionally known as an eigenvalue, but termed communality in factor analysis. When all variables are standardized, then the linear weights are called standardized regression coefficients (regression analysis), path coefficients (path analysis), or factor loadings (factor analysis). The factor analysis approach is distinquished from regression or path analysis in that observed variable correlation is explained by a common factor (hypothetical construct). In factor analysis therefore the correlation between observed variables is the result of sharing a common factor rather than a variable being the direct cause (path analysis) or predictor of another (regression analysis).

LISREL

Linear structural relationships (lisrel) are often diagrammed by using multiple factor path models where the factors (hypothetical contructs) are viewed as latent traits (Joreskog & Sorbom, 1986, pp I.5-I.7). The lisrel model consists of two parts: the measurement model and the structural equation model. The measurement model specifies how the latent variables or hypothetical constructs are measured in terms of the observed (measured) variables and describes their measurement properties (reliability and validity). The structural equation model specifies the causal relationship among the latent variables and is used to describe the causal effects and the amount of unexplained variance. The lisrel model includes or encompasses a wide range of models, for example: univariate or multivariate regression models, confirmatory factor analysis, and path analysis models (Joreskog & Sorbom, 1986, pp I.3, I.9-I.12). Cuttance (1983) presents an overview of several lisrel submodels with diagrams and explanations. Wolfle (1982) presents an indepth presentation of a single model to introduce and clarify lisrel analysis. The lisrel program therefore permits regression, path, and factor analysis whereby model specification and measurement error can be assessed.

MEASUREMENT ERROR

Fuller (1987) extensively covers lisrel and factor analysis models and especially extends regression analysis to the case where the variables are measured with error. Wolfe (1979, pp 48-51) presents the relationship between lisrel, regression and path analysis especially in regards to how measurement error effects the regression coefficient (path coefficient). Errors of measurement in statistics have been studied extensively (Wolfe, 1979). Cochran (1968) studied it from four different aspects: (1) types of mathematical models, (2) standard techniques of analysis which take into account measurement error, (3) effect of errors of measurement in producing bias and reduced precision and what remedial procedures are available, and (4) techniques for studying error of measurement. Cochran (1970) also studied the effects of error of measurement on the squared multiple correlation coefficient.

LISREL-FACTOR ANALYSIS EXAMPLE

A LISREL factor analysis program with a correlation matrix as input is in the appendix. The factor analytic model in matrix notation is:

$$X = L_x x + q_d$$

Where:
- X = observed variables
- L = structural weights (factor loadings)
- x = latent trait (factor)
- q_d = error variance (unique variance)

The LISREL results are:

a. L = LAMBDA X (structural weights-factor loadings)

$Y = .677 \quad X_1 = .402 \quad X_2 = .800 \quad X_3 = .535$

b. q_d = THETA DELTA (unique factor variance)

$Y = .54 \quad X_1 = .84 \quad X_2 = .36 \quad X_3 = .71$

c. b^2 = LAMBDA X^2 (common factor variance)

$Y = .46 \quad X_1 = .16 \quad X_2 = .64 \quad X_3 = .29$

The concept of model specification and goodness of fit pertains to the original correlation matrix and the estimated correlation matrix. The estimated correlation matrix is:

$$\sigma = \begin{matrix} .272 & & \\ .542 & .321 & \\ .362 & .215 & .427 \end{matrix}$$

The original correlation matrix is:

$$S = \begin{matrix} .507 & & \\ .481 & .224 & \\ .276 & .062 & .577 \end{matrix}$$

The goodness of fit index (GFI) using the unweighted least squares approach (ULS) is then computed as:

$$GFI = 1 - 1/2 \text{ trace } (S - \sigma)^2$$

$$GFI = 1 - 1/2 \ (1.308 - 1.02)^2$$

$$GFI = 1 - .041$$

$$GFI = .959$$

LISREL-REGRESSION ANALYSIS EXAMPLE

A LISREL regression program with a correlation matrix as input is in the appendix. The regression model in matrix notation is:

$$Y = G X + z$$

Where: Y = dependent variable
G = gamma matrix (beta weights)
X = independent variables
z = errors of prediction (error variance)

The LISREL results are the same as in the previous regression program:

$$R^2_{y.123} = G_1 r_{y1} + G_2 r_{y2} + G_3 r_{y3}$$

$$R^2_{y.123} = (.423) .507 + (.363) .481 + (.040) .276$$

$$R^2_{y.123} = .40$$

CONCLUSION

The appropriate statistical method to use is often an issue of debate. It sometimes requires more than one approach to analyzing data. The rationale for choosing between the alternative methods of analysis is usually guided by research hypotheses or questions.

The multivariable methods discussed have in common the general linear model and are the same in several respects. First, they identify, partition, and control variance. Second, they are based on linear combinations of variables. And third, the linear weights can be computed based on standardized partial regression coefficients.

The multivariable methods however have different applications. Multiple regression seeks to identify and estimate the amount of variance in the dependent variable attributed to one or more independent variables (prediction). Path analysis seeks to identify relationships among a set of variables (explanation). Factor analysis seeks to identify subsets of variables from a much larger set (common/shared variance). Lisrel determines the degree of model specification and measurement error. The different methods were derived because of the need for prediction, explanation, common variance, model and measurement error assessment type applications.

Multiple regression techniques are robust except for model specification and measurement errors (Borhnstedt & Carter, 1971). Multiple regression techniques are also useful in understanding path, factor, and LISREL applications. LISREL permits regression, path, and factor analyses whereby model specification and measurement error can be assessed. Lisrel also permits univariate or multivariate least squares analysis in either single sample or multiple sample (across populations) research settings. An understanding of multiple regression and general linear model techniques can therefore greatly facilitate ones understanding of the testing of research questions in multivariable situations.

APPENDIX

MULTIPLE REGRESSION PROGRAM

```
TITLE REGRESSION WITH CORRELATION MATRIX INPUT
COMMENT VARIABLE MEANS=0; VARIANCES=1; CONSTANT=0
MATRIX DATA VARIABLES=Y X1 X2 X3/N=100
BEGIN DATA
1.000
 .507   1.000
 .481    .224   1.000
 .276    .062    .577  1.000
END DATA
REGRESSION MATRIX=IN(*)/
           MISSING=LISTWISE/
           VARIABLES=Y X1 X2 X3/
           DEPENDENT=Y/
           ENTER X1 X2 X3/
FINISH
```

PATH ANALYSIS PROGRAM ONE

A. VARIABLE 3 REGRESSED ON VARIABLES 1 AND 2

```
TITLE PATH ANALYSIS EXAMPLE WITH CORRELATION MATRIX INPUT
COMMENT  VARIABLE MEANS=0; VARIANCES=1; CONSTANT=0
MATRIX DATA VARIABLES=Y X1 X2 X3/N=100
BEGIN DATA
1.000
 .507  1.000
 .481   .224  1.000
 .276   .062   .577  1.000
END DATA
REGRESSION MATRIX=IN(*)/
           MISSING=LISTWISE/
           VARIABLES=Y X1 X2 X3/
           DEPENDENT=X3/
           ENTER X1 X2/
FINISH
```

PATH ANALYSIS PROGRAM TWO

B. VARIABLE Y REGRESSED ON VARIABLES 1, 2, AND 3

```
TITLE PATH ANALYSIS EXAMPLE WITH CORRELATION MATRIX INPUT
COMMENT VARIABLE MEANS=0; VARIANCES=1; CONSTANT=0
MATRIX DATA VARIABLES=Y X1 X2 X3/N=100
BEGIN DATA
1.000
 .507  1.000
 .481   .224  1.000
 .276   .062   .577  1.000
END DATA
REGRESSION MATRIX=IN(*)/
           MISSING=LISTWISE/
           VARIABLES=Y X1 X2 X3/
           DEPENDENT=Y/
           ENTER X1 X2 X3/
FINISH
```

FACTOR ANALYSIS PROGRAM

```
TITLE FACTOR ANALYSIS EXAMPLE WITH CORRELATION MATRIX INPUT
COMMENT VARIABLE MEANS=0; VARIANCES=1; CONSTANT=0
MATRIX DATA VARIABLES=Y X1 X2 X3/N=100
BEGIN DATA
1.000
 .507  1.000
 .481   .224  1.000
 .276   .062   .577  1.000
END DATA
FACTOR VARIABLES=Y X1 X2 X3/
       MATRIX=IN(COR=*)/
       CRITERIA=FACTORS(1)/
       EXTRACTION=ULS/
       ROTATION=NOROTATE/
       PRINT CORRELATION DET INITIAL EXTRACTION ROTATION/
       FORMAT SORT/
       PLOT EIGEN/
       FINISH
```

LISREL FACTOR ANALYSIS PROGRAM

```
TITLE 'LISREL FACTOR ANALYSIS WITH CORRELATION MATRIX INPUT'
INPUT PROGRAM
NUMERIC DUMMY
END FILE
END INPUT PROGRAM
USERPROC NAME=LISREL
DATA FOR GROUP ONE
DA NG=1 NI=4 NO=100
LA
'Y' 'X1' 'X2' 'X3'
KM SY
1.000
 .507 1.000
 .481  .224 1.000
 .276  .062  .577 1.000
MO NX=4 NK=1 TD=DI,FR PH=ST
LK
'FACTOR'
PA LX
4 * 1
OU ULS SE TV PC RS VA FS SS MI
END USER
```

LISREL REGRESSION ANALYSIS PROGRAM

```
TITLE 'LISREL REGRESSION ANALYSIS WITH CORRELATION MATIRX'
INPUT PROGRAM
NUMERIC DUMMY
END FILE
END INPUT PROGRAM
USERPROC NAME=LISREL
DATA FOR GROUP ONE
DA NG=1 NI=4 NO=100
LA
'Y' 'X1' 'X2' 'X3'
KM SY
1.000
 .507 1.000
 .481  .224 1.000
 .276  .062  .577 1.000
MO NY=1 NX=3 PS=DI
OU ULS SE TV PC RS VA SS MI TO
END USER
```

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