Demystifying Parametric Analyses: Illustrating Canonical Correlation Analysis as the Multivariate General Linear Model

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A review of the research literature suggests that teachers need to provide students with engaging problems, facilitate their discovery of analysis methods, and encourage classroom discussion and presentation of their approaches to solving problems. The present article illustrates how canonical correlation analysis can be employed to implement all the parametric tests that canonical methods subsume as special cases, including multiple regression. The point is heuristic: *all* analyses are correlational, all apply weights to measured variables to create synthetic variables, and all yield effect sizes analogous to r^2 . Knowledge of such relationships helps inform researcher judgement of analysis selection and use.

In one of his seminal contributions, the late Jacob "Jack" Cohen (1968) demonstrated that multiple regression subsumes all the univariate parametric methods as special cases, and thus provides a univariate general linear model (GLM) that can be employed in all univariate analyses. At about the same time, researchers increasingly also came to realize that ANOVA was being overused, and in many cases used when other methods would have been more useful. One source of ANOVA overuse was that too many researchers erroneously associated ANOVA as an *analysis* with the ability to make causal statements when using experimental research *designs*; however, it is the design, and not the analysis that leads to the ability to make definitive causal statements!

As Humphreys (1978) explained this phenomonon:

The basic fact is that a measure of individual differences is not an independent variable [in an experimental design], and it *does not become one* by categorizing the scores and treating the categories as if they defined a variable under experimental control in a factorially designed analysis of variance. (*p.* 873, emphasis added)

Similarly, Humphreys and Fleishman (1974) noted that categorizing variables in a nonexperimental design using an ANOVA analysis "not infrequently produces in both the investigator and his audience the illusion that he has experimental control over the independent variable. Nothing could be more wrong" (p. 468).

Furthermore, as Cliff (1987) noted, the practice of discarding variance on intervally-scaled predictor variables in order to perform ANOVA-type analyses creates problems in almost all cases:

Such divisions are not infallible; think of the persons near the borders. Some who should be highs are actually classified as lows, and vice versa. In addition, the "barely highs" are classified the same as the "very highs," even though they are different. Therefore, reducing a reliable variable to a dichotomy makes the variable *more unreliable*, not less. (*p*. 130, emphasis added)

These various realizations have led to less frequent use of ANOVA methods, and to more frequent use of general linear model approaches such as regression (cf. Edgington, 1974; Elmore & Woehlke, 1988; Goodwin & Goodwin, 1985; Willson, 1980).

Since *all* analyses are correlational, and it is the design and not the analysis that yields the capacity to make causal inferences, the practice of converting intervally-scaled predictor variables to nominal scale so that ANOVA and other OVAs (i.e., ANCOVA, MANOVA, MANCOVA) can be conducted is inexcusable in many cases.

However, canonical correlation analysis, and not regression analysis, is the most general case of the general linear model (Baggaley, 1981; Fornell, 1978; Thompson, 1991, 1998). [Structural equation modeling (SEM) represents an even broader general linear model, but SEM is somewhat different in that this analysis usually also incorporates measurement error estimation as part of the analysis (cf. Bagozzi, Fornell, & Larcker, 1981; Fan, 1996, 1997).] In an important article, Knapp (1978) demonstrated this in some detail and concluded that "virtually all of the commonly encountered parametric tests of significance can be treated as special cases of canonical correlation analysis" (p. 410).

The present article will illustrate how canonical correlation analysis can be employed to implement all the parametric tests that canonical methods subsume as special cases. The point is not that all research ought to be conducted with canonical analyses, rather the point is heuristic: all analyses are correlational, all analyses apply weights to measured variables to create synthetic variables that become the analytic focus, and all yield effect sizes analogous to r^2 that are important to interpret. For example the R^2 obtained in a multiple regression, the eta² obtained from an ANOVA, and the squared canonical correlation coefficient obtained from a canonical correlation

analysis all describe the variance-accounted-for between two variables and/or sets of variables. Ultimately, these statistics are directly analogous to the squared Pearson correlation.

Understanding general linear model principles aids in realizing that parametric analyses are all fundamentally related. Individual methods, such as ANOVA or *t*-tests, can then be viewed from a global perspective which will, hopefully, facilitate thoughtful researcher judgment in selecting analyses as opposed to employing "lock-step" decision strategies that limit the utility of analyses.

The Basics of Canonical Correlation Analysis

While a comprehensive discussion of CCA is beyond a scope of the present article, the reader is referred to Thompson (1991) for an accessible and userfriendly treatment of CCA. Furthermore, neither the analytic derivations of CCA nor the equivalent derivations of the linear models for the various analyses will be visited here. Since the purpose of this article is to demonstrate equivalence of models through obtained results, the reader is referred to Knapp (1978) for mathematical demonstration of the linear models.

The theory of canonical correlation analysis (CCA) has been with us for considerable time (Hotelling, 1935), but did not come into practical use until the onset of computerization (Krus, Reynolds, & Krus, 1976). In canonical analysis, the variables are considered to be members of two or more (in practice, almost always two) variable sets (e.g., pretest and posttest scores, aptitude and achievement scores) – otherwise we would analyze the data with factor analysis so as to consider simultaneously all the relationships, but without considering the existence of variable sets. Each set will include more than one variable, otherwise we generally would use a Pearson \mathbf{r} or regression analysis. As will be shown later, these analyses are essentially the same thing anyway!

A CCA will yield many useful statistics, the most recognized of which is the canonical correlation (R_c) . The canonical correlation describes the relationship between two synthetic variables that have been modeled from their respective variable sets by applying weights to the measured variables. А canonical correlation will be produced for each function (i.e., for each set of standardized canonical function coefficients and respective measured variables). The number of functions, each of which will be perfectly uncorrelated with the others, equals the number of variables in the smaller of the variable sets. The canonical correlation can be squared to yield a varianceaccounted-for effect size (R_c^2) , or the percentage of variance explainable in the criterion variable set predictable with knowledge of the variance in the predictor set.

One advantage of CCA, and other multivariate methods, lies in its *simultaneous* examination of the variables of interest, thus reducing risk of experimentwise Type I error (Fish, 1988; Henson, in press; Thompson, in press). A second, and perhaps often overlooked, advantage is the flexibility of the analysis in looking at various research problems. One example of this versatility can be found in a measurement study involving multivariate criterion-related score validity (Sexton, McLean, Boyd, Thompson, & McCormick, 1988). Thus, CCA can be used in either substantive or measurement inquiries.

Canonical Correlation Analysis as the General Linear Model

An heuristic data set for 12 elementary, middle, and high school students will be used to illustrate that CCA can conduct the other parametric methods that it subsumes, both univariate and multivariate alike. CCA will be used to perform a t-test, Pearson correlation, multiple regression, ANOVA, MANOVA, and descriptive discriminant analysis. Table 1 lists heuristic data on four intervally scaled variables related to motivational and personality issues: attributions of effort (EFFORT), attributions of ability (ABILIT), locus of control (LOCUS), and degree of extroversion (EXTROV). Also included are grouping data indicating some experimental treatment (TREAT) and whether students are in elementary, middle, or high school (GRADE). The reader will also notice five planned contrast variables which will be described later.

Analyses will be run using the SPSS (v9.0) statistics package. The canonical correlation macro (CANCORR) is a new addition to this version of SPSS but it limits analyses to two sets of variables of equal size. Since several examples used here include analyses on variable sets of differing size, the canonical macro was not used. There is also a General Linear Model menu which can be used to run a variety of analyses. However, for the sake of consistency and clarity, a uniform command syntax will be used in the present article to illustrate the relationship between canonical correlation the other parametric analyses. This command syntax is included in the Appendix. Note that CCA is conducted here using the MANOVA command (again, suggesting that these analyses are be related). Using Table 1 variable names, the SPSS commands for CCA are:

MANOVA

LOCUS EXTROV WITH EFFORT ABILIT /PRINT=SIGNIF (MULTIV EIGEN DIMENR) /DISCRIM=(STAN ESTIM COR ALPHA(.99)).

The SAS statistical software has a more direct command for CCA: PROC CANCORR. An example of SAS syntax used to perform a similar heuristic illustration can be found in Campbell and Taylor (1996).

ID	EFFORT	ABILIT	LOCUS	EXTROV	GRADE	TREAT	CGR1	CGR2	CTREAT	CTGR1	CTGR2
1	10	12	18	15	1	1	-1	-1	-1	1	1
2	15	14	19	16	1	1	-1	-1	-1	1	1
3	17	18	18	13	1	2	-1	-1	1	-1	-1
4	14	13	15	10	1	2	-1	-1	1	-1	-1
5	09	15	14	04	2	1	0	2	-1	0	-2
6	06	19	16	04	2	1	0	2	-1	0	-2
7	06	20	12	07	2	2	0	2	1	0	2
8	07	19	16	03	2	2	0	2	1	0	2
9	18	11	06	18	3	1	1	-1	-1	-1	1
10	17	10	04	13	3	1	1	-1	-1	-1	1
11	12	09	10	12	3	2	1	-1	1	1	-1
12	14	13	09	14	3	2	1	-1	1	1	-1

 Table 1. Heuristic Data (n=12) for Canonical Correlation Illustration

Conducting Pearson Correlation with Canonical Correlation

When examining relationships between two variables, a Pearson correlation (r) is often invoked. The reader should immediately note conceptual similarities between a Pearson r and canonical analysis, even before examining the results from the SPSS analysis. Both investigate relationships between variables, only in the canonical case the measured variables of interest occur within multivariate sets.

A Pearson r was computed for EFFORT and ABILIT. Table 2 reports the obtained results, r = -.6150, p = .033. Table 2 also reports the CCA results, including the canonical correlation (R_c) , squared canonical correlation (R_c^2) , and Wilks lambda ($\hat{\lambda}$). Wilks lambda, like R_c^2 is a variance-accountedfor type statistic. However, Wilks lambda indicates the variance not accounted for in the canonical correlation, modeled by $(1 - R_c^2)$. It is used for testing the statistical significance of R_c . As the magnitude of _ decreases (ranging from 0 to 1), the effect size (R_c^2) increases as does the likelihood of obtaining statistical significance.

For these variables, the CCA computed a squared canonical correlation coefficient of .378. The simple square root transformation of $R_c^2 = .378$ gives us $R_c =$.6148. The Pearson r and canonical correlation values are identical, save for rounding error and the fact that a canonical correlation cannot be negative. This is because the weights that are used in CCA scale the variables in the same direction, as such R_c will always range from 0 to 1. The *p* values are identical.

Herein lies the most fundamental of general linear model principles: all analyses are correlational. The canonical correlation is nothing more than a bivariate r between the synthetic variables created in CCA after the application of weights. As Thompson (1991) noted, "This conceptualization is appealing, because most researchers feel very comfortable thinking in terms of the familiar bivariate correlation coefficient" (p. 81).

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Since the present heuristic CCA only had one variable in each set, the synthetic variables reflected the same relationship as did a Pearson r between the variables without the application of weights. This result should not be surprising, given the fact that multiplicative constants do not affect the value of r. The only effect the weights had in this case was to scale the variables in the same direction, thus yielding a positive value for $R_{\rm c}$.

Conducting Multiple Regression with **Canonical Correlation**

As Cohen (1968) indicated, multiple regression subsumes all other univariate parametric analyses as special cases. Therefore, there is a directly analogous relationship between Pearson r and multiple regression. Since CCA subsumes Pearson r, it should be apparent that it will do the same for multiple regression.

A multiple regression analysis was conducted with EFFORT predicted by LOCUS and EXTROV. SPSS results of the regression and canonical analyses are found in Table 3. Again, all parallel statistics match within rounding error, with the exception of the weights. However, the difference between the weights is arbitrary at this point. Beta (B) weights and standardized function coefficients are easily converted into each other using the following formulas (Thompson, in press):

 B / R_c = Function Coefficient Function Coefficient * R = B

For example, LOCUS had a B weight of -.171156. Using $R_c = .828$ from the CCA, we find that the standardized function coefficient matches, within rounding error, that reported in Table 3 (-.171156 / .828 = -.2067). Since we know from the obtained results that the regression multiple R equals the canonical R_c , we can use the conversion formulas to find canonical function coefficients using only a regression analysis and B weights using only CCA.

nalysis	Canonical Analysis		
615	$R_{\rm c}$.615	
.378	$R_{\rm c}^{2}$.378	
	lambda	.622	
.033	p	.033	
	615 .378	$\begin{array}{c c}615 & R_{c} \\ .378 & R_{c}^{2} \\ lambda \end{array}$	

Table 2. Conducting Pearson Correlation withCanonical (EFFORT by ABILITY)

Also of note here is the relationship between a Pearson r, the obtained multiple R from the regression, and the R_c from the canonical analysis. A regression analysis applies weights to observed (manifest) predictor variables to create a synthetic variable called *predicted* Y (or sometimes Y_{HAT}), which is a linear combination of the predictor variables. The multiple R from the regression analysis is nothing more than a Pearson correlation between predicted Y and the observed dependent measure, EFFORT in this case (R predicted Y, EFFORT). Furthermore, as shown above, the canonical correlation (R_c) also is a Pearson r between two synthetic variables. In this case, however, only the predictor set (LOCUS and EXTROV) was linearly combined via the application of weights. While technically the dependent measure (EFFORT) also was transformed by a multiplicative weight, since only one variable existed, the weight was +1 and the EFFORT variable did not change. As such, the CCA and the multiple regression yielded identical results, both of which are based on a simple Pearson r between two variables (either manifest or synthetic)!

Conducting t-test and Point-biserial Correlation with Canonical Correlation

One of the most basic of statistical analyses is the *t*-test which is used to compare means between groups. Here a *t*-test was used to evaluate if the treatment and control groups (TREAT) differed on the EFFORT variable. Results reported in Table 4 indicate that the means of the groups were not statistically significantly different, t = .310, p = .760. A canonical analysis on the same variables yielded F(1, 10) = .100, p = .760. Note that the *p* calculated values are identical between analyses. The test statistics (*t* and *F*) are different only in metric. In fact, the *F* distribution consists of squared values of the *t* distribution. Squaring t = .310 produces .096 which does match the *F* value. The slight difference in the values is arbitrary and solely due to rounding error by the statistics program.

A point-biserial correlation was also conducted to illustrate the correlational nature of even the *t*-test. In essence, a *t*-test is can be conceptualized as a correlation between one dichotomous variable (TREAT) which indicates group membership and one continuous variable (EFFORT) as the dependent

Table 3. Conducting Multiple Regression withCanonical (EFFORT by LOCUS and ABILIT)

Regression A	Analysis	Canonical Analysis		
R	.828	$R_{\rm c}$.828	
R^2	.685	$R_{\rm c}^{2}$.685	
		lambda	.315	
F(2, 9)	9.797	F(2, 9)	9.797	
р	.006	р	.006	
Beta We	ights	Function Coe	efficients	
LOCUS	171	LOCUS	207	
EXTROV	.767	EXTROV	.926	

measure. The point-biserial correlation is a generalization of the Pearson r illustrated above that allows for a dichotomy in one of the variables. Again looking at Table 4, we see that the p values are identical across the *t*-test, point-biserial, and canonical analyses. Furthermore, the point-biserial correlation matches the magnitude of the canonical correlation within rounding error. Remember that a canonical correlation cannot be negative as discussed above. The point is again made here that all analyses are correlation in nature, even those which utilize dichotomous variables.

Conducting Factorial ANOVA with Canonical Correlation

The SPSS syntax file (see Appendix) includes commands to compute the five orthogonal contrast variables reported in the Table 1 data. Planned contrasts can be used with ANOVA methods to test specific, theory-driven hypotheses as against omnibus hypotheses (Thompson, 1994). One advantage of using planned contrasts is the ease of pinpointing statistically significant effects without having to conduct post-hoc tests which include Bonferroni-type corrections for experimentwise error. It is important to note that the contrasts will yield the same overall effect [i.e., Sum of Squares (SS) explained] as the omnibus test. They are necessary here to show that CCA can conduct ANOVA.

In the present analysis, a 3 X 2 factorial ANOVA was conducted with TREAT and GRADE as independent variables and EFFORT as the dependent variable. For the CCA, the contrast variables from Table 1 were used. The total number of orthogonal contrasts that can be created equals the degrees of freedom for each main effect. The GRADE main effect has two degrees of freedom and is represented by CGR1 and CGR2. The TREAT main effect is represented by CTREAT with one degree of freedom. CTRGR1 and CTRGR2 are simply cross products of the other main effects and test the GRADE X TREAT interaction effects. Table 5 presents results for the ANOVA: GRADE, F = 19.367; TREAT, F = .510;

Table	4.	Conducting	t-test	and	Point-biserial
Correlat	ion w	vith Canonical	(EFFC	RT by	y TREAT)

<i>t</i> -test	Canonica	ıl	Point-biserial		
<i>t</i> (10)	.314	<i>F</i> (1, 10)	.100		
р	.760	р	.760	р	.760
M(TREAT1)	12.500	_		_	
SD(TREAT1)	4.848	$R_{\rm c}$.100	r	100
M(TREAT2)	11.667	$R_{\rm c}^{2}$.010	r^2	.010
SD(TREAT2)	4.320	lambda	.990		
<i>Note.</i> $R_{\rm c}$ cann	ot be neg	gative.			

GRADE X TREAT, F = 3.449. Note that the effect size (r^2) for the error term was .1323.

Obtaining comparable results with CCA requires us to take several steps. The first step involves conducting canonical analyses in four separate designs, using EFFORT as the dependent measure and the contrasts as independent variables. Design 1 included all planned contrasts, CGR1, CGR2, CTREAT, CTRGR1, and CTRGR2, to test the total effect (SOS explained). Design 2 used CTREAT, CTRGR1, and CTRGR2 to jointly test the TREAT and interaction effects. Design 3 used CGR1, CGR2, CTRGR1, and CTRGR2 to jointly test the GRADE and interaction effects. The final CCA, Design 4, used CGR1, CGR2, and CTREAT to jointly test the GRADE and TREAT effects. Table 6 displays the Wilks' lambda values for each design from the first step. Remember that λ is something of a "reverse" effect size and will equal the effect for the error term. A quick comparison of λ for the total effect (Table 6) with the error effect size (Table 5) confirms this relationship between the statistics.

After canonical lambdas have been attained, we must use them to determine the omnibus ANOVA lambdas. This was done by dividing the Design 1 total effect (lambda) by the lambdas of the other designs. For example, to find the omnibus lambda for the GRADE main effect the total lambda (.11507) was divided by the Design 2 lambda (.85793), which reflects the joint effect of the contrast variables for the TREAT main effect and the GRADE X TREAT interaction effect. This process "removes" the effects of the other hypotheses, leaving the omnibus lambda for the GRADE main effect to be .13412516 (.11507 / .85793 = .13412516 = λ). The same process was used to find the other ANOVA lambdas with results reported in Table 6.

One final step remained. ANOVA lambdas were converted into ANOVA *F* statistics using the following formula: $[(1 - \lambda)/\lambda]^*(dferror / df effect) = F$.

To illustrate, the *F* value for the GRADE main effect was modeled by [(1 - .13413) / .13413] * (6 / 2) = 19.36636. Table 6 also reports transformations for both main effects and the interaction. Note that the *F* statistics obtained by the canonical process match

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Table 5.	3 X 2 Factorial ANOVA
(EFFORT	by GRADE and TREAT)

Source	SS	df	MS	F	р	eta ²
GRADE	158.167	2	79.083	19.367	.002	.743
TREAT	2.083	1	2.083	0.510	.502	.010
G x T	28.167	2	14.083	3.449	.101	.132
Error	24.500	6				
Total	212.917	11				

those obtained by the factorial ANOVA (see Table 5), within rounding error of course.

It should also be noted that the equivalence of ANOVA and CCA can be demonstrated with dummy codes that represent group membership in the independent variable (see Fan, 1978). However, the predictors would be correlated in this case. The use of orthogonal contrast is useful here to maintain the factorial structure of the groups.

Conducting Factorial MANOVA with Canonical Correlation

Since SPSS can use the MANOVA command to perform CCA, it would seem that the two are related. To illustrate the relationship, a 3 X 2 factorial MANOVA was computed with EFFORT and ABILIT as dependent variables and GRADE and TREAT as independent measures. Results from this analysis are found in Table 7. Since MANOVA is a multivariate method, Wilks lambdas are reported by SPSS and are used to test statistical significance of the *F* values.

The comparable canonical analysis was performed using the same process as with the ANOVA above. Four CCA designs using the contrast variables were run with canonical lambdas reported in Table 8. The subsequent conversion of these values to MANOVA lambdas is also found in Table 8. The reader will note the equivalence of the MANOVA $_$ s in Table 7 with those obtained through the canonical analysis in Table 8. The final conversion to *F* values was not necessary here since the MANOVA uses the λ value to calculate *F* statistics, unlike the SOS value used in ANOVA. However, the same full model *F*-test formula used in the ANOVA section can be used to find the *F* statistics in this case.

Conducting Discriminant Analysis with Canonical Correlation

Discriminant analysis is a multivariate method that can either be used predictively to classify persons into groups or descriptively where variables identify latent structures among groups (Huberty, 1994). The descriptive discriminant analysis (DDA) case is especially useful as the preferred substitute for a oneway MANOVA or as a post hoc analysis to multi-way MANOVA analyses.

Step One: Canonical Analyses on Four Designs					
Design	Independent Variables			lambda	
1	CGF	R1, CGR2	, CTREAT	.11507	
	C	TGR1, C	TGR2		
2	CTR	EAT, CT	GR1, CTGR2	.85793	
3	CGF	R1, CGR2	, CTGR1, CTGR2	.12485	
4	CGF	R1, CGR2	, CTREAT	.24736	
Step Two: Conversion of Canonical Lambdas to ANOVA Lambdas					
ANO	VA			ANOVA	
Effe	ct	Designs	Designs Transformation		
GRADE	Ξ	1 / 2	.11507/.85793	.13412516	
TREAT	•	1/3	.11507/.12485	.92166600	
G x T 1 / 4		1 / 4	.11507/.24736	.46519243	
Step Tl	hree:		on of ANOVA Lam	odas to	
	<i>F</i> -ratio				
Sour	ce Transformation		F-ratio		
GRADE	_	[(113413)/.13413]*(6/2)		19.36636	
TREAT	$\Gamma \qquad [(192167)/.92167]*(6/1)$		0.50992		
GxT		[(14651	9)/.46519]*(6/2)	3.44898	

 Table 6. Conduct ANOVA with Canonical Analysis (EFFORT by Contrasts)

To demonstrate the DDA and CCA relationship, a descriptive discriminant analysis was conducted with TREAT as the nominally scaled predictor variable and EFFORT and ABILIT as criterion variables. Table 9 reports a statistically non-significant result $\chi^2(2, 9) = .648$, $p = .723$. The canonical analysis was conducted using the planned contrast variable CTREAT as the predictor. Results of the CCA are also reported in Table 9. The reader will note that the analyses yield identical results. One arbitrary difference is in the reporting of a χ^2 statistic for the discriminant analysis as opposed to the CCA <i>F</i> value. As with the <i>t</i> and <i>F</i> distributions described above, the difference is arbitrary since the χ^2 and <i>F</i> statistics represent the same value expressed in a different metric. The χ^2 statistic can be calculated by multiplying the <i>F</i> value by $(j * k)$, where <i>i</i> is the number of variables in the predictor set and <i>k</i>
j is the number of variables in the predictor set and k
is the number of variables in the criterion set. In this case, $F = .33602$, so $\chi^2 = (1 * 2).33602 = .67204$.
This transformation approximates the χ^2 reported in Table 9 with the difference due to rounding.
rable 9 with the unreference due to founding.

Table 9. Conducting Mul	tiple Regression with
Canonical (EFFORT by LO	OCUS and ABILIT)

Discriminan	t Analysis	Canonical Analysis		
$R_{\rm c}$.264	$R_{\rm c}$.264	
$R_{\rm c}^{2}$.070	$R_{\rm c}^{2}$.070	
lambda	.931	lambda	.931	
	.648	F	.336	
df	2, 9	df	2, 9	
p	.723	p	.723	

Table 7.	3 X 2 Factorial ANOVA
(EFFORT	and ABILIT by GRADE and TREAT)

	and ADILI	I UY OF	CADL and	IKLAI
Source	lambda	df	F	р
GRADE	.05061	4, 10	8.61299	.003
TREAT	.61798	2, 5	1.54541	.300
GxT	.44653	4, 10	1.24122	.354

Table 8. Conduct MANOVA with Canonical Analysis (EFFORT and ABILIT by Contrasts)						
Step One: Canonical Analyses on Four Designs						
Design	Indep	pendent Va	lambda			
1	CGR1, CGR2, CTREAT			.03184		
	CTGR1, CTGR2					
2	CTR	EAT, CT	.62924			
3	CGR	R1, CGR2	.05153			
4	CGR	R1, CGR2	.07132			
Step Ty	Step Two: Conversion of Canonical Lambdas to					
MANOVA Lambdas						
ANO	VA			ANOVA		
Effe	ct	Designs	Transformation	Lambda		
GRADE		1 / 2	.03184/.62924	.05060072		
TREAT	•	1/3	.03184/.05153	.61789249		
G x T		1 / 4	.03184/.07132	.44643859		

Conclusion

The purpose of the present article has been to illustrate that canonical correlation analysis represents the multivariate parametric general linear model. As such, CCA can be used to conduct the univariate and multivariate analyses that CCA subsumes, including multiple regression. The point is heuristic and not intended to suggest that all analyses should be conducted with CCA. In fact, it is quite clear in the ANOVA and MANOVA examples that CCA, at least as reported by SPSS, is the long way to the same results. However, CCA would be superior to ANOVA and MANOVA when the independent variables are intervally scaled, thus eliminating the need to discard variance.

Knowing that there is a general linear model and understanding that all parametric analyses are intricately related can be of great educational value to both students and teachers of quantitative methods as well as practicing researchers. Knowing these relationships facilitates understanding of commonalties and differences among all the parametric methods and serves to inform researcher judgement concerning analysis selection and use.

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APPENDIX

SPSS Command Syntax for Canonical Demonstration

TITLE ' Canonical correlation demonstration '. TITLE ' Robin K. Henson '. COMMENT Heuristic data for 12 cases COMMENT EFFORT - attributions of effort COMMENT ABILIT - attributions of ability COMMENT LOCUS - external vs internal locus of control COMMENT EXTROV - degree of extroversion scale COMMENT GRADE - elementary(1), middle(2), high(3) school COMMENT TREAT - treat(1), control(2) groups. SET BLANKS=SYSMIS UNDEFINED=WARN PRINTBACK LISTING. DATA LIST FILE='c:\ccaasglm.txt' FIXED RECORDS=1 /ID 1-2 EFFORT 4-5 ABILIT 7-8 LOCUS 10-11 EXTROV 13-14 GRADE 16 TREAT 18. EXECUTE. COMMENT Show that cca can do Pearson r. CORRELATIONS /VARIABLES=EFFORT ABILIT /PRINT=TWOTAIL NOSIG /MISSING=PAIRWISE . MANOVA EFFORT WITH ABILIT /PRINT=SIGNIF (MULTIV EIGEN DIMENR) /DISCRIM=(STAN ESTIM COR). COMMENT Show that cca can do multiple regression. REGRESSION /MISSING LISTWISE /STATISTICS COEFF OUTS R ANOVA /CRITERIA=PIN(.05) POUT(.10) /NOORIGIN /DEPENDENT EFFORT /METHOD=ENTER LOCUS EXTROV MANOVA LOCUS EXTROV WITH EFFORT /PRINT=SIGNIF (MULTIV EIGEN DIMENR) /DISCRIM=(STAN ESTIM COR). COMMENT Show that cca can do t-test and point biserial correlation. T-TEST GROUPS=TREAT(1 2) /MISSING=ANALYSIS /VARIABLES=EFFORT /CRITERIA=CIN(.95) .

```
MANOVA
 TREAT WITH EFFORT
 /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
 /DISCRIM=(STAN ESTIM COR).
COMMENT Show cca can do point-biserial
    which is a generalization of r.
CORRELATIONS
 /VARIABLES = treat effort
 /PRINT=TWOTAIL NOSIG
 /MISSING=PAIRWISE .
COMMENT Show that cca can do
      factorial ANOVA.
COMMENT Compute contrast variables
      to do cca.
IF (GRADE = 1) CGR1 = -1.
IF (GRADE = 2) CGR1 = 0.
IF (GRADE = 3) CGR1 = 1.
COMMENT Tests equality of the
   means of elementary(4) vs
    high school(4) students.
EXECUTE.
IF (CGR1 = -1) CGR2 = -1.
IF (CGR1 = 0) CGR2 = 2.
IF (CGR1 = 1) CGR2 = -1.
EXECUTE.
COMMENT Tests equality of means
    of middle(4) vs
    elementary high school(8) students.
IF (TREAT = 1) CTREAT = -1.
IF (TREAT = 2) CTREAT = 1.
EXECUTE.
COMMENT Tests equality of means of
   treatment (6) vs control groups (6).
COMPUTE CTRGR1 = CGR1 * CTREAT.
COMPUTE CTRGR2 = CGR2 * CTREAT.
EXECUTE.
COMMENT Tests treatment by grade
    interaction effects.
COMMENT Show contrast variables
     are orthogonal.
CORRELATIONS
 /VARIABLES=CGR1 CGR2 CTREAT
           CTRGR1 CTRGR2
  /PRINT=TWOTAIL SIG
  /MISSING=PAIRWISE .
COMMENT Step one: run factorial ANOVA
       and cca on constrast variables.
ANOVA
  VARIABLES=EFFORT
  BY GRADE(1 3) TREAT(1 2)
  /MAXORDERS ALL
  /METHOD UNIQUE
  /FORMAT LABELS .
```

```
MANOVA
 CGR1 CGR2 CTREAT CTRGR1 CTRGR2
                                      MANOVA
   WITH EFFORT
 /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
                                       CTREAT CTRGR1 CTRGR2
 /DISCRIM=(STAN ESTIM COR).
                                          WITH EFFORT ABILIT
MANOVA
                                        /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
 CTREAT CTRGR1 CTRGR2 WITH EFFORT
                                        /DISCRIM=(STAN ESTIM COR).
 /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
                                      MANOVA
 /DISCRIM=(STAN ESTIM COR).
                                       CGR1 CGR2 CTRGR1 CTRGR2 WITH EFFORT
MANOVA
                                       ABILIT
 CGR1 CGR2 CTRGR1 CTRGR2 WITH EFFORT
                                       /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
 /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
                                        /DISCRIM=(STAN ESTIM COR).
 /DISCRIM=(STAN ESTIM COR).
                                      MANOVA
MANOVA
                                        CGR1 CGR2 CTREAT WITH EFFORT ABILIT
 CGR1 CGR2 CTREAT WITH EFFORT
                                         /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
 /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
                                         /DISCRIM=(STAN ESTIM COR).
 /DISCRIM=(STAN ESTIM COR).
                                       COMMENT Show cca can do
COMMENT Show cca can do MANOVA.
                                             discriminant analysis.
MANOVA
                                      DISCRIMINANT
  EFFORT ABILIT BY GRADE(1 3)
                                        /GROUPS=TREAT(1 2)
                                        /VARIABLES=EFFORT ABILIT
       TREAT(1 2)
  /PRINT SIGNIF(MULT UNIV )
                                        /ANALYSIS ALL
  /NOPRINT PARAM(ESTIM)
                                        /PRIORS EQUAL
  /METHOD=UNIQUE
                                        /CLASSIFY=NONMISSING POOLED .
  /ERROR WITHIN+RESIDUAL
                                     MANOVA
  /DESIGN .
                                       EFFORT ABILIT WITH CTREAT
                                        /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
MANOVA
 CGR1 CGR2 CTREAT CTRGR1 CTRGR2
                                       /DISCRIM=(STAN ESTIM COR).
   WITH EFFORT ABILIT
 /PRINT=SIGNIF (MULTIV EIGEN DIMENR)
 /DISCRIM=(STAN ESTIM COR).
```

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