

Nonrandomly Missing Data in Multiple Regression Analysis: An Empirical Comparison of Ten Missing Data Treatments

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This study investigated the effectiveness of ten missing data treatments within the context of a two-predictor multiple regression analysis with nonrandomly missing data. Five distinct types of missing data treatments were examined: deletion (both listwise and pairwise methods), deterministic imputation (with imputations based on the sample mean, simple regression and multiple regression), stochastic imputation (mean, simple regression and multiple regression), maximum likelihood estimation (ML) and multiple imputation (MI). Design factors included in the study were sample size, total proportion of missing data, and the proportion of missing data occurring in the upper stratum of each predictor. The success of each method was evaluated based on the sample estimate of R^2 and each standardized regression coefficient. Results suggest that the stochastic multiple regression imputation procedure evidenced the best performance in providing unbiased estimates of the parameters of interest. Deterministic imputation approaches and the stochastic mean imputation approach resulted in large amounts of bias in the estimates.

It is not uncommon for missing data to occur on one or more variables within an empirical investigation. Missing data may adversely affect data analyses, interpretations and conclusions. Collins, Schafer, and Kam (2001) indicate that missing data may potentially bias parameter estimates, inflate Type I and Type II error rates, and influence the performance of confidence bands. Further, because a loss of data is almost always associated with a loss of information, concerns arise with regard to reductions of statistical power. Unfortunately, researchers' recommendations for managing missing data are not in complete agreement resulting in conceptual difficulties and computational challenges (Guertin, 1968; Beale & Little, 1975; Gleason & Staelin, 1975; Frane, 1976; Kim & Curry, 1977; Santos 1981; Basilevsky, Sabourin, Hum, & Anderson, 1985; Raymond & Roberts, 1987; Schafer & Graham, 2002). Many studies that have examined missing data treatments are not comparable due to the various methods used, the stratification categories (number of variables, sample size, proportion of missing data, and degree of multicollinearity), and the criteria that measure effectiveness of the missing data treatment (Anderson, Basilevsky, & Hum, 1983). Further, Schafer and Graham (2002) argue that the treatment of missing values cannot be properly evaluated apart from the modeling, estimation, or testing procedure in which it is rooted. Before proceeding with an examination of the extant literature, a consideration of some missing data terminology is warranted.

Missing Data Terminology

Contemporary discussion of missing data and their treatment can often be confusing and at times may appear somewhat counterintuitive. For example, the term *ignorable*, introduced by Little and Rubin (1987) was not intended to convey a message that a particular aspect of missing data could be ignored, but rather under what circumstances the missing data mechanism is ignorable. Additionally, when one speaks of data missing at random, these words should not convey the notion that the missingness is derived from a random process external or unrelated to other variables under study (Collins et al., 2001).

According to Heitjan & Rubin (1991) missing data can take many forms, and missing values are part of a more general concept of *coarsened data*. This general category of missing values results when data are grouped, aggregated, rounded, censored, or truncated, resulting in a partial loss of information. The major classifications of missing data mechanisms can be best explained by the relationship among the variables under investigation. Rubin (1987) identified three general processes that can produce missing data. First, data that are missing purely due to chance are considered to represent data that are missing completely at random (MCAR). Specifically, data are missing completely at random if the probability of a missing response is completely independent of all other measured or unmeasured characteristics under examination. Accordingly, analyses of data of this nature will result in unbiased estimates of the population parameters under investigation. Second, data that are classified as missing at random (MAR), do not depend on the missing value itself, but may depend on other variables that are measured for all participants under study. Lastly, and most problematic statistically, are data missing not at random (MNAR). This type of missingness, also referred to as *nonignorable* missing data, is directly related to

the value that would have been observed for a particular variable. A commonly encountered situation, in which data would be classified as MNAR, arises when respondents in a certain income or age strata fail to provide responses to questions of this nature.

In much of the research that has been previously conducted, the key assumption has been that data are missing at random. If data are randomly missing and the percentage of missing data is not too large, researchers are advised that any missing data treatment is effective. However, this assumption of randomly missing data is tenuous in many instances. Although a few procedures have been presented to test the assumption of randomly missing data (e.g., Cohen & Cohen, 1975, 1983; Tabachnick & Fidell, 1983), Kromrey and Hines (1994) assert that this assumption is rarely tested and that the applied researcher is hard pressed to find guidance if data are missing nonrandomly.

Missing Data Treatments

Applied researchers often employ deletion or deterministic imputation procedures to manage missing data rather than choosing from among other available missing data treatments. The former procedures employ a deletion process, utilizing cases with complete data (Glasser, 1964; Haitovsky, 1968). Listwise deletion discards all cases with incomplete information, whereas pairwise deletion constructs a correlation matrix utilizing all pairs of complete data. Deterministic imputation procedures (e.g., mean substitution, simple regression, or multiple regression) provide estimates of the missing values (Santos, 1981; Kalton & Kasprzyk, 1982). In all deterministic approaches to imputation, the residual (error) term is set to zero in the estimation equation. In contrast, stochastic imputation includes a random value for the residual in the estimation equation. Some empirical evidence suggests that the stochastic imputation procedures are superior to the deterministic approaches (Santos, 1981; Kalton & Kasprzyk, 1982; Jinn & Sedransk, 1989; Keawkungal & Benson, 1989; Brockmeier, Hines, & Kromrey, 1993).

Kromrey and Hines (1994) examined the effectiveness of the deletion and deterministic imputation procedures in regression analysis with nonrandomly missing data in the context of missing data on one of two predictor variables. Systematically missing data were produced by generating 60% of the missingness above the mean value of the variable in each simulated sample. These researchers concluded that with moderate amounts of missing data, the deletion procedures yielded results similar to those obtained without missing data. Further, the deterministic imputation procedures evidenced poor performance when compared to the deletion procedures.

Within the context of a two-predictor multiple regression analysis with nonrandomly missing data, Brockmeier, Kromrey, and Hines (2000) investigated the effectiveness of eight missing data treatments on the sample estimate of R^2 and each standardized regression coefficient. These researchers varied the overall proportion of missing data in each sample, as well as the proportion of the missingness that occurred in values greater than the sample mean. The results suggested that the stochastic multiple regression imputation procedure provided the best treatment of missing data.

New Approaches to Missing Data

Traditionally, researchers have not utilized maximum likelihood estimation (ML), multiple imputation (MI), or the aforementioned stochastic imputation procedures. Kromrey (1989) and Brockmeier (1992) indicated that these methods are not typically found in the journals of applied researchers, with much of the scholarly work on maximum likelihood estimation and multiple imputation appearing in technical statistical journals. Little (1992) stated that maximum likelihood estimation is infrequently utilized due to the lack of software and the mathematical complexity of the computations. More recently, however, maximum likelihood estimation has been included in some structural equation modeling software and statistical packages. Additionally, Gregorich (1999) has created a maximum likelihood estimation program using SAS IML that is currently available and freely distributed to SAS users for noncommercial purposes. This program employs the Expectation-Maximization (EM) algorithm to estimate the maximum likelihood covariance matrix and mean vector in the presence of missing data. This maximum likelihood approach assumes that data are missing completely at random or missing at random. While a decade ago few stand-alone programs existed for employing multiple imputation as a missing data treatment, ML and MI are now becoming more popular with the implementation of these procedures in free and commercial software (Schafer & Graham, 2002). For example, a recent release of SAS (version 8.2) introduced a multiple imputation procedure.

Rubin (1996) indicated that the ultimate goal of multiple imputation is to provide statistically valid inferences in applied contexts where researchers employ different analyses and models and when there is no one accepted reason for the missing data. The multiple imputation procedure replaces each missing value with a set of plausible values that represents the degree of uncertainty about the correct value to impute. One might view this approach as an enhancement over simple imputation methods that fail to reflect the uncertainty about the predictions of the missing values, often resulting in point estimates of a variety of parameters that are not statistically valid in any generality.

Multiple imputation inference involves three distinct phases (Schafer, 1997). First, missing data are filled in m times to generate complete data sets. Second, the m complete data sets are analyzed using standard statistical analyses. Finally, the results from the m complete data sets are combined to produce inferential information. For a recent review of MI procedures, see Sinharay, Stern, and Russell (2001).

Evidence of the effectiveness of maximum likelihood estimation and multiple imputation, as missing data treatments has been somewhat limited in the past (for a recent exception, see Collins et al., 2001). However, with the innovations in the software described above, interest and enthusiasm for these alternative methods appears to be growing. A recent issue of *Psychological Methods* (2001) devoted a special section to issues surrounding missing data, with a specific focus on multiple imputation and maximum likelihood estimation.

Purpose

The purpose of this study was to investigate the effectiveness of ten missing data treatments within the context of a two-predictor multiple regression analysis with nonrandomly missing data. Further, the study investigated whether sample size, proportion of missing data occurring in the upper stratum on each predictor, and the total percentage of missing data affected the effectiveness of the ten missing data treatments. The success of each method was evaluated based on the sample estimate of R^2 and each standardized regression coefficient. Five distinct types of missing data treatments were examined: deletion (both listwise and pairwise methods), deterministic imputation (with imputations based on the sample mean, simple regression and multiple regression), stochastic imputation (mean, simple regression and multiple regression), maximum likelihood estimation and multiple imputation.

Method

This research was a Monte Carlo study designed to simulate multiple regression analyses in the presence of missing data. The use of simulation methods allows the control and manipulation of research design factors and the incorporation of sampling error into the analyses. Observations for each sample were generated under known population conditions and missing data were created in each sample.

Data Source

Data for this investigation were simulated to model the correlational structure observed in a sample of responses to an instrument designed to measure teachers' reported perceptions of computers and integration of technology in their classrooms (Hogarty, Lang, & Kromrey, in press). These field data were composite scores based on selected subscales from the instrument. Each subscale contained items measured on either a 5-point Likert scale ranging from *strongly disagree* to *strongly agree* or a 5-point frequency of use scale ranging from *not at all* to *every day*. Two correlation matrices from these data were selected for use as population templates in this Monte Carlo study (Table 1). The first matrix had correlations between variables that ranged from 0.33 to 0.61 with an R^2 of 0.50. In contrast, the second matrix presented lower correlations (ranging from 0.12 to 0.49) and an R^2 value of 0.25.

Table 1. *Correlation Matrices Used for Simulation*

Higher Correlated Data Set			Lower Correlated Data Set		
	Y	X ₁		Y	X ₁
X ₁	0.53		X ₁	0.49	
X ₂	0.61	0.33	X ₂	0.16	0.12

Experimental Design

The study employed a 2 X 3 X 4 X 6 experimental design. The factorial design included two between-subjects factors (population correlation matrix and sample size, $N = 50, 100,$ and 200) and two within-subjects factors (proportion of missing data in the upper stratum of each variable and percentage of missing data). The four proportions of systematically missing data in the upper stratum were 0.60, 0.70, 0.80, and 0.90. The six conditions of missing data generated were 10%, 20%, 30%, 40%, 50%, and 60%. In addition, the complete samples with no missing data were analyzed. Missing data were distributed equally across both predictor variables (i.e., with 20% missing data, 10% of the observations presented missing data on each of the regressors).

The pseudopopulations were not manipulated within the experiment, but were selected to obtain the desired correlational patterns in each data set. The sample sizes, missing data structures and missing data treatments were chosen to replicate and extend the earlier work of Kromrey and Hines (1994) and Brockmeier et al. (2000).

Conduct of the Monte Carlo Study

This research was conducted using SAS/IML version 8.2. Conditions for the study were run under Windows 98. Normally distributed random variables were generated using the RANNOR random number generator in SAS. A different seed value for the random number generator was used in each execution of the program and the program code was verified by hand-checking results from benchmark data sets. For each condition examined in the Monte Carlo study, 5000 samples were simulated. The use of 5000 samples provides adequate precision for this investigation. For example, the use of 5000 samples provides a maximum 95% confidence interval width around an observed proportion that is ± 0.014 (Robey & Barcikowski, 1992).

Within each sample, missing data were created by setting to missing the generated values of one of the two regressors variables, under the constraint that no observation may have both regressors missing. The target percentage of missing data was evenly divided between the two regressors (i.e., half of the missingness occurred on each regressor). In addition, the percentage of the missingness that was imposed on observations in the upper stratum was controlled by dividing each sample into two strata for each regressor, then randomly selecting the correct number of observations from each stratum. The proportion of data missing in the upper stratum was altered to create increasing degrees of distortion in the observed data. The probability of a missing value was established to be proportional to the value of the variable. For the majority of conditions, the upper stratum was defined as observations above the sample median value of the predictor variable. If an insufficient number of observations were available above the median (e.g., with 90% missing and 60% of these occurring above the median) the upper stratum was defined as the top 60% of the distribution.

In each generated sample, the 24 missing data conditions were independently imposed (six total percentages of missing data crossed with four proportions of missing data in the upper stratum), allowing the two missing data factors to be treated as within-subjects factors in the research design. In addition, each complete sample with no missing data was analyzed to provide a reference for the evaluation of the missing data treatments. Each sample was analyzed by computing the regression equation (obtaining the sample estimates of each standardized regression weight and the sample value of R^2) after applying each of the ten missing data treatments.

Missing Data Treatments

The missing data treatments examined in this study included deletion, deterministic imputation, stochastic imputation and model-based approaches. The treatment of missing data was considered with respect to two predictor variables, X_1 and X_2 (with missing data on one but not both predictors), and a single criterion variable, Y_1 (with no missing data). The first method, listwise deletion, necessitated the deletion of any observations in the sample with missing values for either of the predictor variables. The resulting 'complete' data set for each sample was used in the calculation of parameter estimates (R^2 and the standardized regression weights). For the second method, pairwise deletion, a correlation matrix was constructed based on all of the available data for each pair of variables. The resulting correlation matrix was subsequently analyzed to obtain the regression equation for each sample.

Three imputation techniques using deterministic methods were employed, a mean imputation approach and both simple and multiple regression imputation. For the mean imputation procedure, missing values were imputed using the sample mean value for each regressor variable. In contrast, deterministic multiple regression imputation for each sample was initiated by calculating a prediction equation based on the data available and deriving a predicted value for the variable with missing data based on the values of the other two variables. For example, to predict missing values for X_1 , the sample regression equation for predicting X_1 based on both X_2 and Y was estimated. The resulting equation was then used to compute predicted values for X_1 and those values were imputed to replace the missing data. The same process was followed for simple regression imputation, but only a single predictor variable was used (i.e., either X_2 or Y was used to predict the missing values for X_2). In each sample, the variable evidencing the stronger correlation was used as the predictor.

The three stochastic techniques examined mirrored the deterministic methods described above. However, as stated earlier, stochastic techniques differ from deterministic methods with regard to the incorporation of a random error term. For stochastic mean imputation, the imputation begins with the mean value in the sample for the particular variable with missing data, but each imputation requires the addition of a randomly selected normal deviate drawn from a distribution with $\mu = 0$ and σ^2 equal to the sample variance of the variable. The same steps were followed when employing the stochastic simple and multiple regression imputation methods as those described above for the parallel deterministic approaches with the exception of the addition of a random residual term. This residual was randomly drawn from a distribution with $\mu = 0$ and σ^2 equal to the mean squared residual from the regression that was used to derive the prediction equation.

The multiple imputation approach is an extension of the multiple regression imputation procedure. This approach replaces each missing value with a set of plausible values that represent the degree of uncertainty about the correct values to be imputed. For this study, 10 imputations were employed for the multiple imputation procedure. Rubin (1987) indicated that the efficiency of an estimate based on m imputations is approximately:

$$\left(1 + \frac{\gamma}{m}\right)^{-1};$$

where γ is the fraction of missing information for the variable being estimated. For example, with 50% missing information, $m = 5$ imputations provides an estimate of efficiency of approximately 91%, whereas $m = 10$ imputations increases the estimated efficiency to 95%.

As described earlier, first, missing data were filled in ten times to generate ten complete data sets. This phase was accomplished by first conducting a multiple regression analysis on the sample data to derive an initial set of parameter estimates. These estimates, along with the obtained covariance matrix, were then used to generate a sample of ten sets of parameter estimates. These parameter estimates were applied individually to the sample data and the predicted values imputed, resulting in ten distinct samples with complete data. The ten complete data sets were then analyzed sequentially using multiple regression analysis. Finally, the obtained R^2 values and standardized regression weights from the ten regression analyses were averaged and the resultant values were used to assess the performance of this procedure.

The final procedure examined was maximum likelihood estimation via the EM algorithm. This is an iterative approach that seeks parameter values that maximize the likelihood of the observed data. The well-known EM algorithm (Dempster, Laird, & Rubin, 1977) consists of an estimation step (or E-step) that predicts the missing values based upon estimates of the parameters, and a maximization step (or M-step) that revises the estimates of the parameters. The E and M steps are repeated until the parameter values do not change appreciably from one cycle to the next. For an excellent overview of the theory underlying this approach, both with and without missing data see Rubin (1987).

Statistical Analysis

The relative effectiveness of the missing data treatments was evaluated in terms of statistical bias and standard errors for the estimates of the standardized regression weights and the sample value of R^2 . The results were analyzed by computing the effect sizes obtained from the missing data treatment conditions relative to the complete sample condition (i.e., 0% missing data). That is,

$$\delta_{ijk} = \frac{\bar{\theta}_{ijk} - \bar{\theta}_{0k}}{\hat{\sigma}_{\theta_{0k}}}$$

where δ_{ijk} = the effect size for missing data treatment i , under missing data condition j and sample size k ,

$\bar{\theta}_{ijk}$ = the mean value of the parameter estimate over the 5000 samples,

$\bar{\theta}_{0k}$ = the mean value of the parameter estimate over the 5000 samples of size k with no missing data, and

$\hat{\sigma}_{\theta_{0k}}$ = the standard deviation of the parameter estimates across the 5000 samples of size k with no missing data.

In addition, standard deviation ratios were calculated to assess the sampling variability of each missing data treatment for the sample estimate of R^2 and standardized regression coefficients relative to the complete sample condition:

$$SD_{Ratio_{ijk}} = \frac{\hat{\sigma}_{\theta_{ijk}}}{\hat{\sigma}_{\theta_{0k}}}$$

where $SD_{Ratio_{ijk}}$ = the ratio for missing data treatment i , under missing data condition j and sample size k ,

$\hat{\sigma}_{\theta_{ijk}}$ = the standard deviation of the parameter estimates obtained across the 5000 samples with missing data treatment i , under missing data condition j and sample size k ,

$\hat{\sigma}_{\theta_{0k}}$ = the standard deviation of the parameter estimates across the 5000 samples of size k with no missing data.

Results

The results of this study are presented in terms of the effect sizes and the inflation of sampling error associated with the 10 missing data treatments. To save space, only summary statistics are reported, but complete tables are available from the first author.

Effect Sizes for Missing Data Treatments

The distributions of the obtained effect sizes for the estimation of R^2 for all conditions examined in this study are presented in Figures 1 and 2 for the high and low correlation matrices, respectively. Similar patterns of results were seen for both population matrices, although the biases associated with some of the missing data treatments (MDTs) were more extreme in the high correlation matrix. Four particularly poor MDTs are evident in these figures: the three deterministic imputation procedures and the stochastic mean imputation approach. These methods led to large biases in sample R^2 across most of the conditions examined, with the mean imputation approaches yielding underestimates of R^2 and the deterministic regression procedures yielding overestimates. The other MDTs provided relatively unbiased estimates of R^2 across the majority of conditions examined.

Figures 3 and 4 present the distributions of effect sizes associated with the estimation of the standardized regression weight for X_1 in the conditions simulated for the high and low correlation matrices, respectively. For this regression weight, the three deterministic imputation procedures and the stochastic mean imputation procedure showed relatively extreme bias in the conditions simulated in the low correlation matrix (Figure 4). For the high correlation matrix (Figure 3), although the pattern of over and under estimation remained the same, substantially less bias was evident, especially for the deterministic mean procedure.

Figures 5 and 6 present the distributions of effect sizes associated with the estimation of the standardized regression weight for X_2 . A notably different pattern was observed for this regression weight. In the high correlation matrix (see Figure 5), the same four procedures evidenced the bias pattern

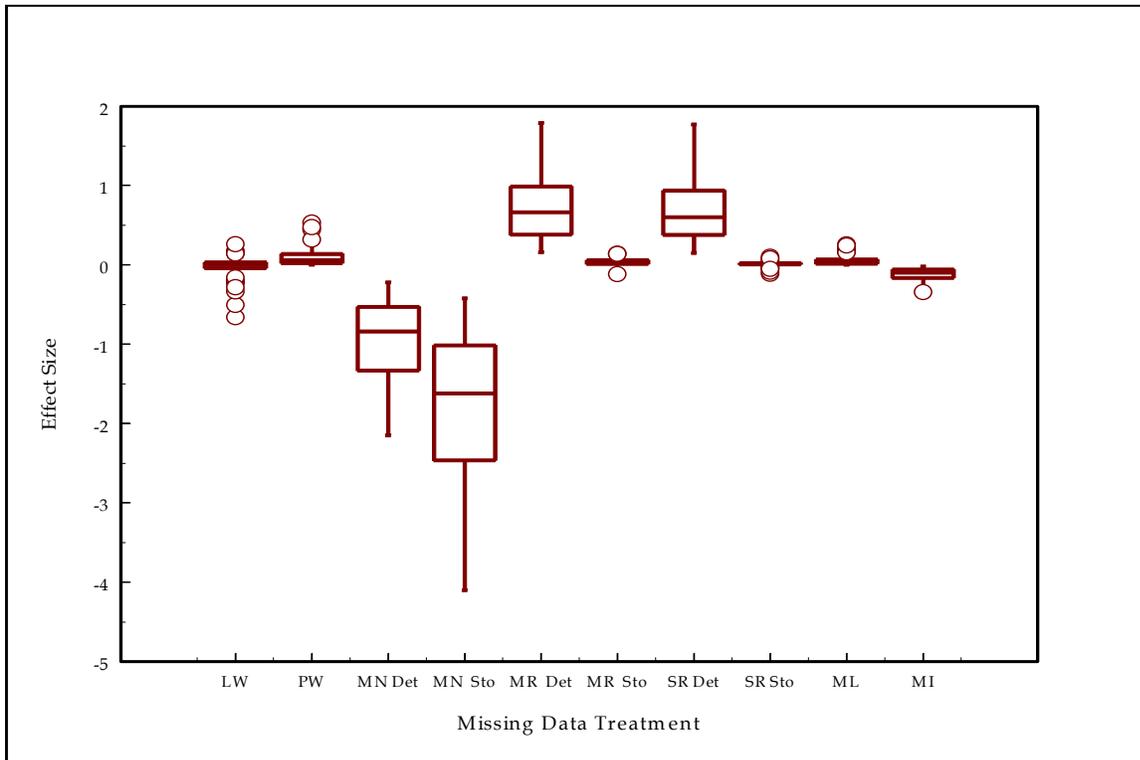


Figure 1. Distributions of Effect Sizes for Estimates of R^2 in High Correlation Matrix.

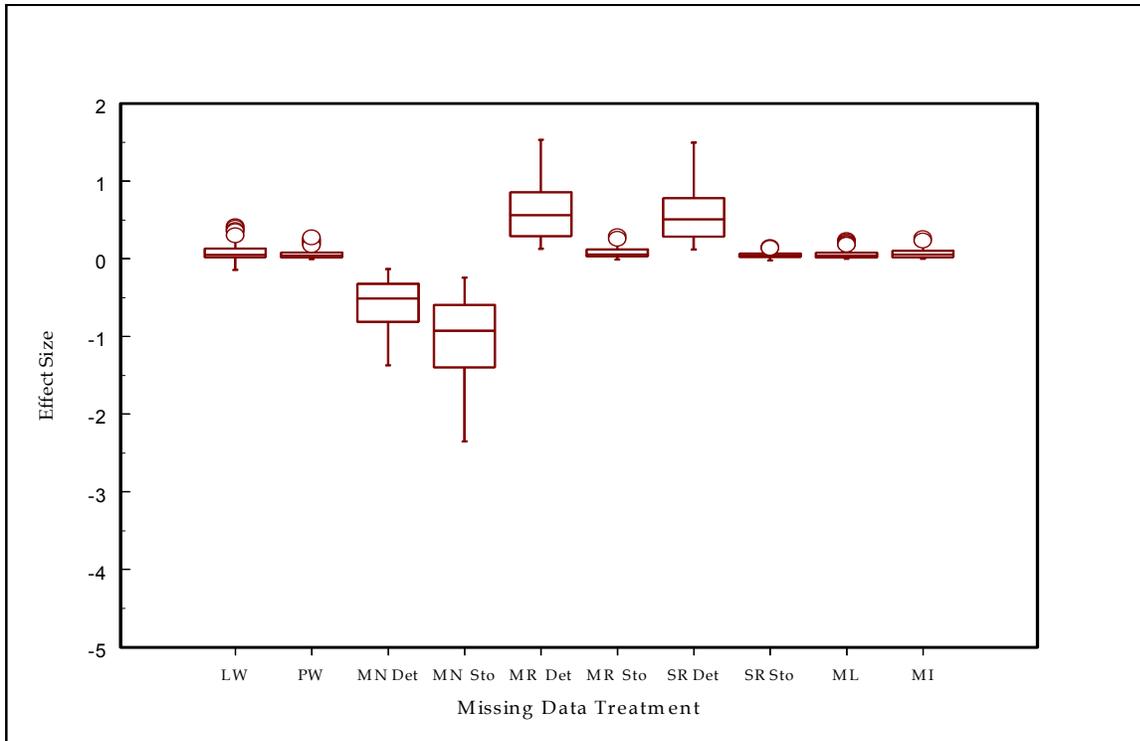


Figure 2. Distributions of Effect Sizes for Estimates of R^2 in Low Correlation Matrix.

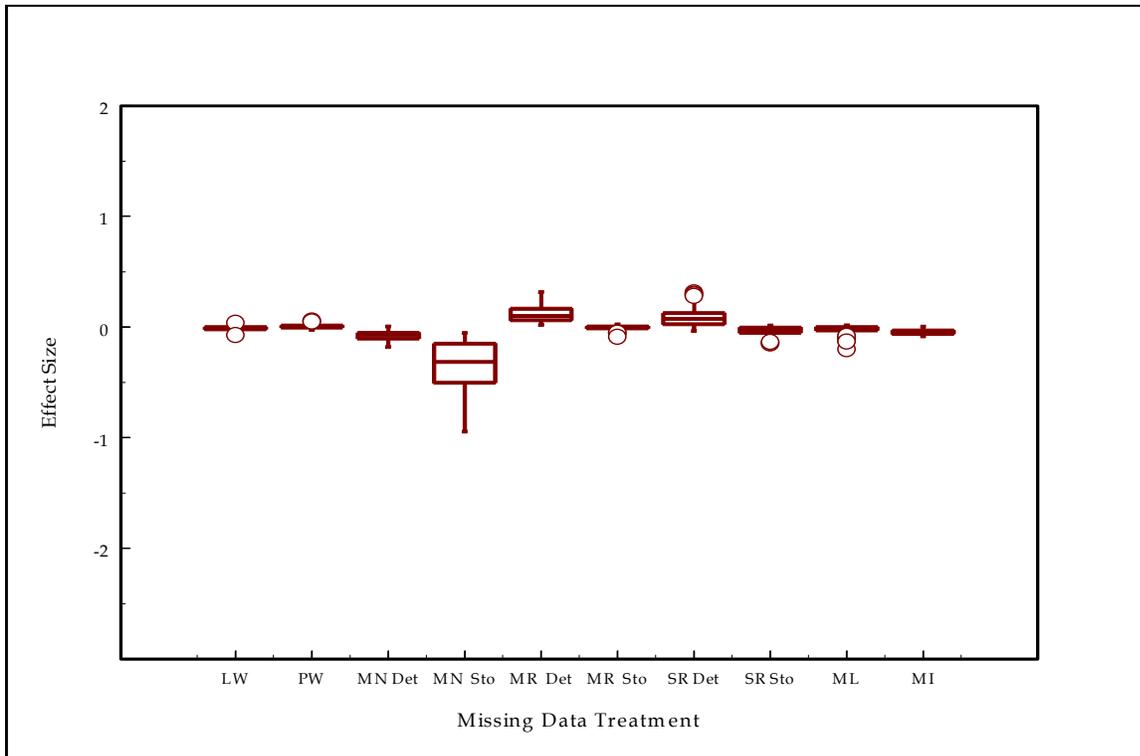


Figure 3. Distributions of Effect Sizes for Estimates of β_1 in High Correlation Matrix.

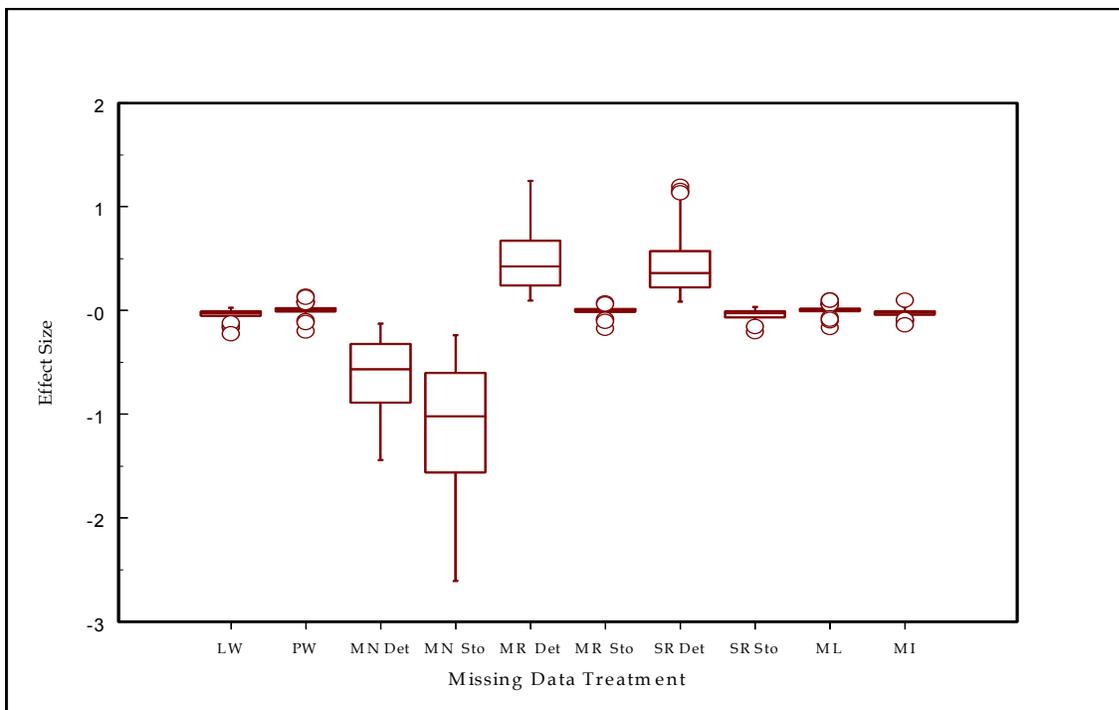


Figure 4. Distributions of Effect Sizes for Estimates of β_1 in Low Correlation Matrix.

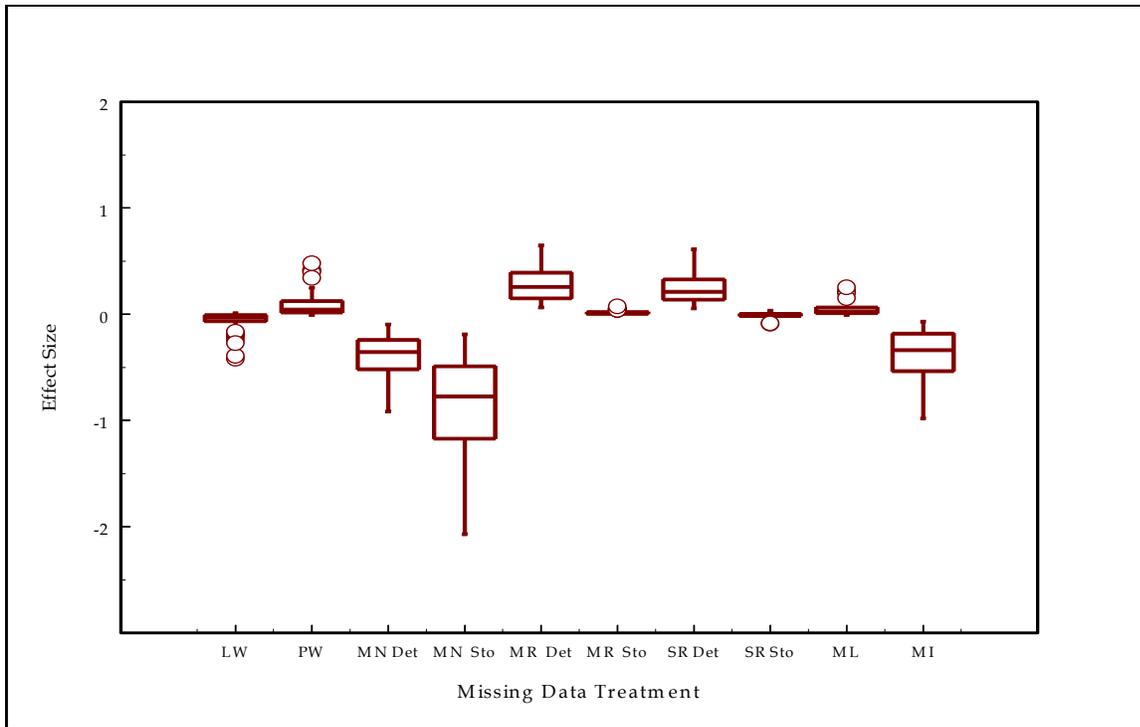


Figure 5. Distributions of Effect Sizes for Estimates of β_2 in High Correlation Matrix.

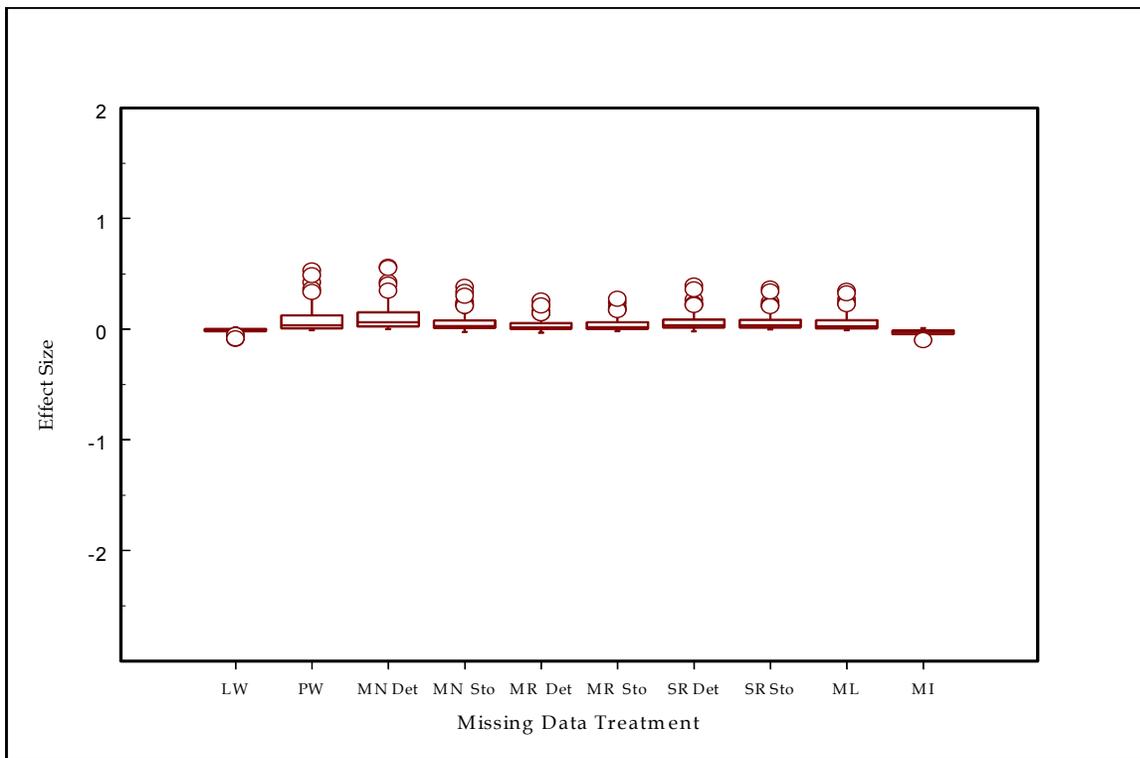


Figure 6. Distributions of Effect Sizes for Estimates of β_2 in Low Correlation Matrix.

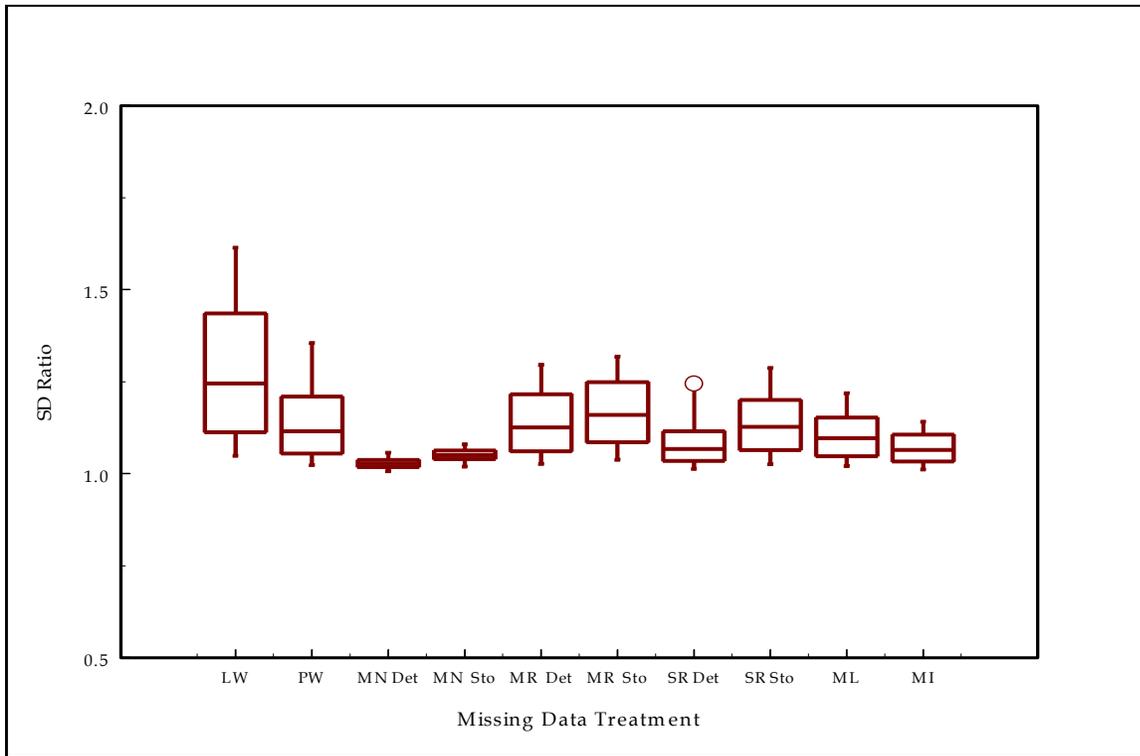


Figure 7. Distributions of Standard Deviation Ratios for Estimates of R^2 in High Correlation Matrix.

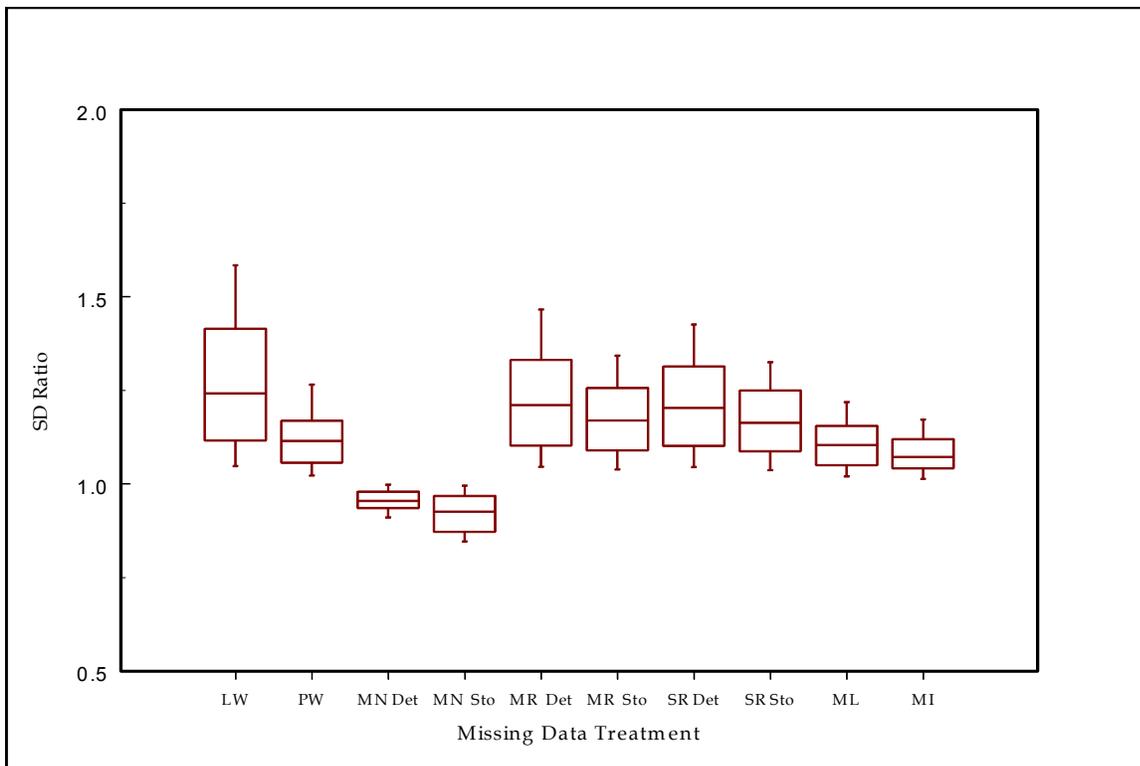


Figure 8. Distributions of Standard Deviation Ratios for Estimates of R^2 in Low Correlation Matrix.

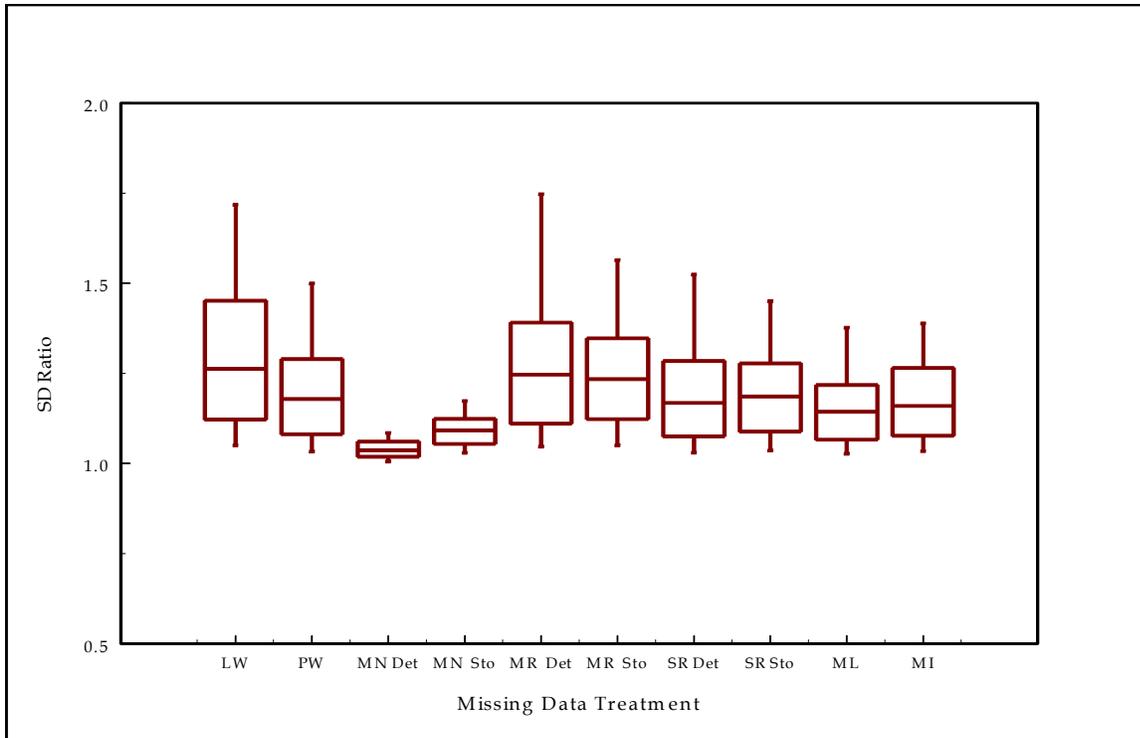


Figure 9. Distributions of Standard Deviation Ratios for Estimates of β_1 in High Correlation Matrix.

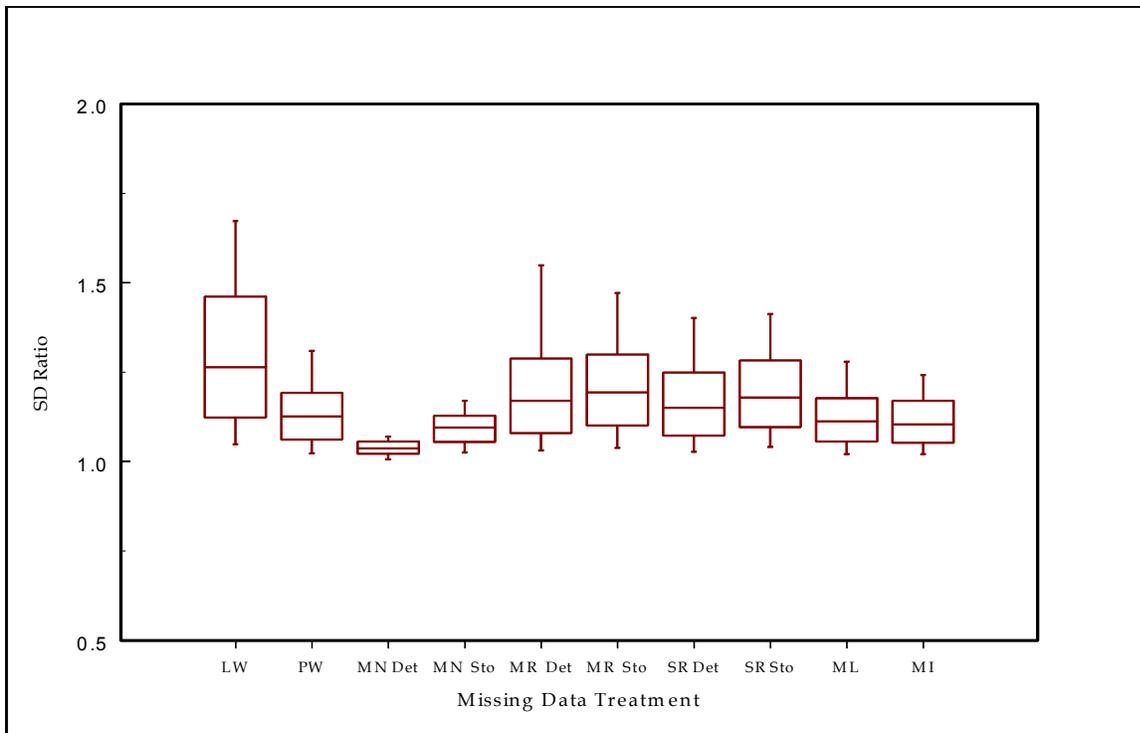


Figure 10. Distributions of Standard Deviation Ratios for Estimates of β_1 in Low Correlation Matrix.

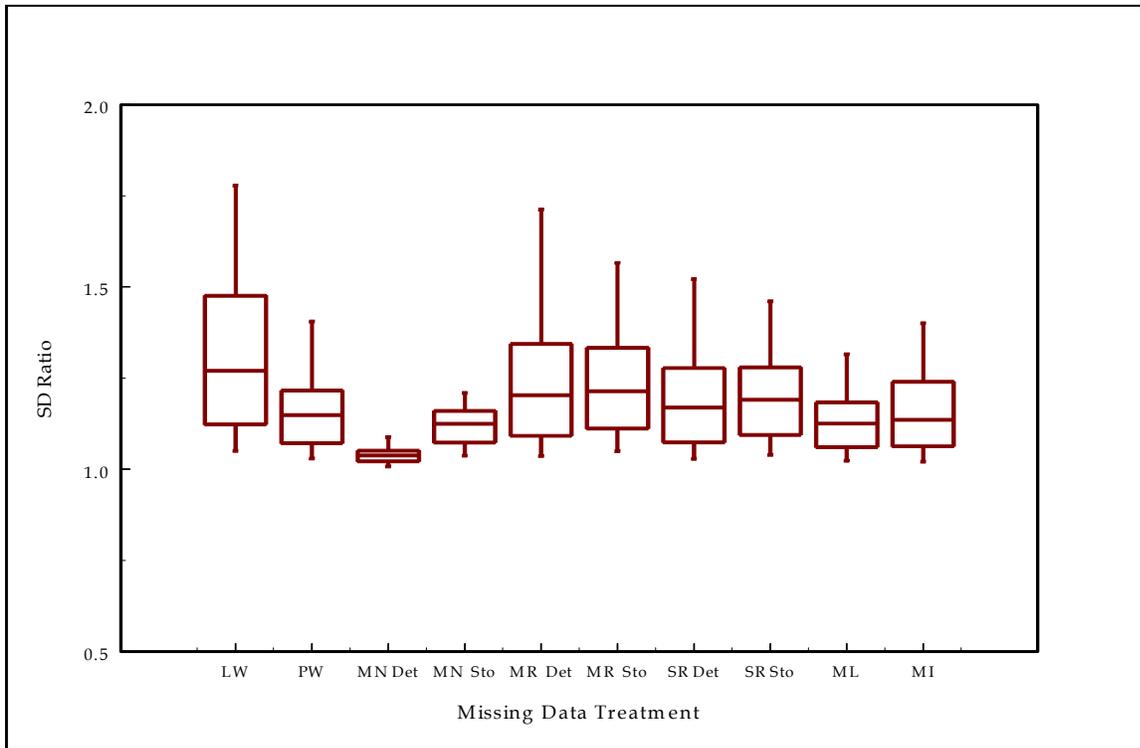


Figure 11. Distributions of Standard Deviation Ratios for Estimates of β_2 in High Correlation Matrix.

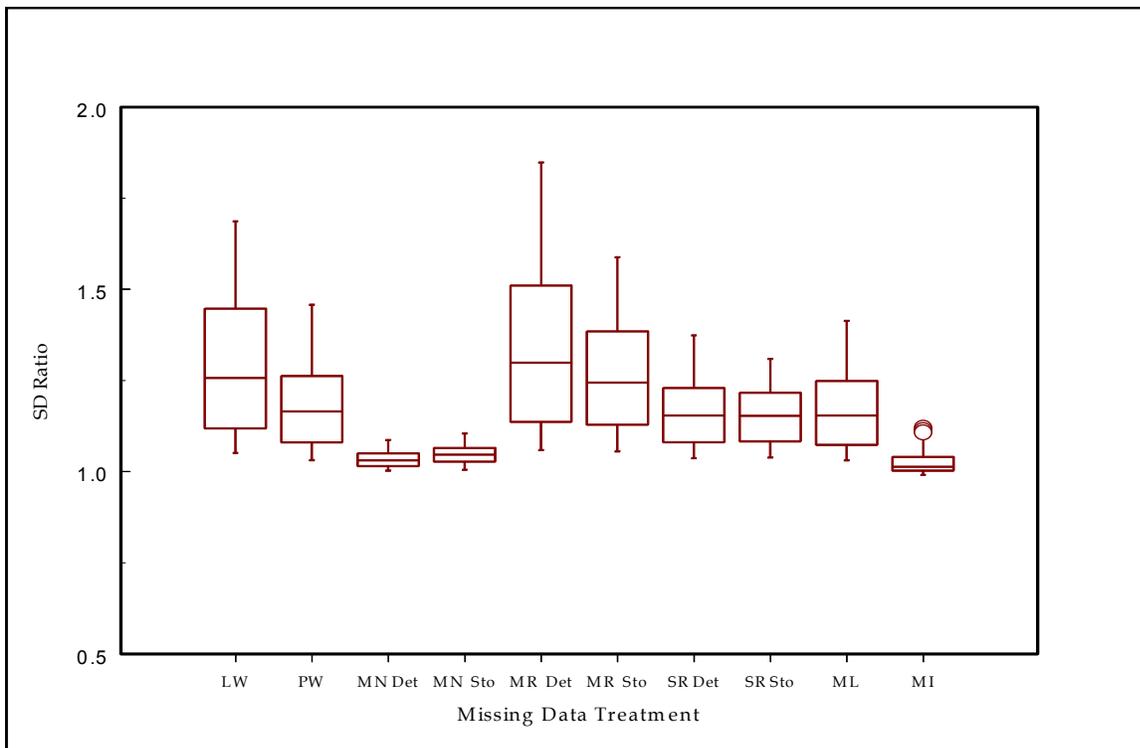


Figure 12. Distributions of Standard Deviation Ratios for Estimates of β_2 in Low Correlation Matrix.

seen before, but the MI procedure also revealed substantial negative bias. In the low correlation matrix (Figure 6), for most conditions very little bias was evident in any of the methods. When bias was present, the bias was consistently in a positive direction (overestimation of the regression weight).

To aid in the interpretation of the distributions of effect sizes observed, the differences in mean effect sizes were examined across each of the design factors in the study. Table 2 presents the mean effect sizes across conditions examined in each of the two population correlation matrices. On average, the two deletion procedures provided relatively unbiased estimates of all parameters (with the average effect size not exceeding 0.10). For the three deterministic procedures, substantial bias was evident in the estimates for both matrices, with the exception of the estimation of β_1 in the high correlation matrix and β_2 in the low correlation matrix. The simple regression, multiple regression, and stochastic imputation procedures performed well on average, with the average bias not exceeding 0.08, and the MI procedure performed well with the exception of the estimation of β_2 in the high correlation matrix, a condition that led to substantial negative bias. Finally, the maximum likelihood (EM) approach provided reasonably unbiased estimates of all parameters across the conditions.

Table 3 presents the patterns of effect sizes by sample size. Neither deletion procedure was appreciably influenced by the sample size, with the average bias ranging from -0.02 to 0.10 across the parameters and conditions examined. In contrast, the deterministic imputation procedures and the stochastic mean imputation procedure showed greater bias with larger samples, often doubling in magnitude as sample size increased from 50 to 200. The remaining stochastic imputation procedures and the EM approach showed minimal bias across sample sizes examined, with the exception of the MI procedure in the estimation of the second regression weight.

Table 4 presents the pattern of mean effect sizes by the percentage of missing data. The deletion procedures, deterministic imputation procedures and the stochastic mean imputation procedure all evidenced greater degrees of bias with larger percentages of missing data. In the most extreme condition (60% missing data), the stochastic mean imputation procedure yielded an effect size of -2.31 for the estimation of R^2 . The stochastic regression procedures (both simple and multiple) also evidenced greater bias with more missing data, but the increase was quite small (reaching only as large as 0.07 for simple regression and 0.14 for multiple regression). The bias observed with the MI procedure in the estimation of the second regression weight is also evident in this table, with increasing bias accompanying greater proportions of missing data. Finally, the EM approach showed a small increase in bias with greater amounts of missing data (with mean effect size reaching as high as 0.16 in the estimation of R^2).

Table 5 presents the pattern of mean effect sizes by the proportion of missing data occurring in the upper stratum of each regressor. Surprisingly, for most of the MDTs, the estimation bias was not strongly related to this factor. Exceptions were pairwise deletion, which showed larger bias in the estimation of R^2 and β_2 with increasing proportions of missing data in the upper stratum, and the maximum likelihood procedure, which showed greater bias in the estimation of β_2 .

As a final approach to guide interpretation of the success of the missing data treatments, the proportions of conditions in which the effect size was less than 0.30 in absolute value were computed (see Table 6). Effect sizes this small are considered to present little or no practical problem to researchers (Kromrey & Hines, 1991). The criterion of 0.30 was chosen (rather than Cohen's 0.50, a medium effect size) because the regression coefficients and the sample estimate of R^2 are subject to both substantive interpretation and tests of statistical significance. The best performance, overall, in terms of providing relatively unbiased estimates of the three parameters of interest was the stochastic multiple regression imputation procedure. With this method, none of the conditions examined yielded effect sizes greater than 0.30. Closely following the stochastic multiple regression approach were the stochastic simple regression approach and the maximum likelihood (EM) approach. Both of these methods provided effect sizes less than 0.30 for all conditions except for two, both of which involved the estimation of the second regression weight. The deletion procedures also performed relatively well, providing relatively unbiased estimates in more than 90% of the conditions examined. The deterministic imputation procedures and the stochastic mean imputation procedures produced very poor performance in this analysis. For example, none of these procedures produced effect sizes less than 0.30 for the estimates of R^2 in more than 24% of the conditions.

Table 2. Effect Sizes for Missing Data Treatments by Population Correlation Matrix

Statistic	R Matrix	Deletion			Deterministic Imputation			Stochastic Imputation			
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	MI	ML
R^2	High	-0.02	0.10	-0.95	0.70	0.73	-1.80	0.01	0.04	-0.12	0.05
	Low	0.09	0.07	-0.59	0.57	0.61	-1.04	0.05	0.08	0.07	0.06
β_1	High	-0.01	0.01	-0.08	0.09	0.12	-0.35	-0.03	-0.01	-0.05	-0.02
	Low	-0.03	0.01	-0.62	0.42	0.48	-1.12	-0.04	0.00	-0.02	0.00
β_2	High	-0.06	0.09	-0.40	0.25	0.28	-0.87	-0.01	0.01	-0.38	0.05
	Low	-0.01	0.08	0.11	0.06	0.04	0.06	0.06	0.04	-0.03	0.06

Table 3. Effect Sizes for Missing Data Treatments by Sample Size

Statistic	N	Deletion			Deterministic Imputation			Stochastic Imputation			
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	MI	ML
R^2	50	0.10	0.10	-0.53	0.44	0.51	-0.96	0.04	0.09	0.01	0.08
	100	0.02	0.08	-0.73	0.60	0.63	-1.36	0.03	0.05	-0.02	0.05
	200	-0.02	0.07	-1.04	0.86	0.86	-1.94	0.03	0.03	-0.05	0.04
β_1	50	-0.02	0.02	-0.24	0.14	0.22	-0.50	-0.06	0.01	-0.05	0.01
	100	-0.02	0.01	-0.33	0.25	0.29	-0.70	-0.03	0.00	-0.03	-0.01
	200	-0.02	-0.01	-0.48	0.39	0.40	-1.01	-0.02	-0.01	-0.03	-0.02
β_2	50	-0.02	0.07	-0.10	0.10	0.12	-0.28	0.01	0.03	-0.14	0.04
	100	-0.04	0.08	-0.14	0.15	0.15	-0.39	0.03	0.02	-0.20	0.05
	200	-0.04	0.10	-0.19	0.22	0.20	-0.54	0.05	0.03	-0.27	0.06

Table 4. *Effect Sizes for Missing Data Treatments by Percentage of Missing Data*

Statistic	Percent	Deletion			Deterministic Imputation			Stochastic Imputation				
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	MI	ML	
R^2	10	0.01	0.01	-0.23	0.18	0.18	-0.44	0.01	0.02	-0.01	0.01	
	20	0.02	0.02	-0.44	0.34	0.35	-0.84	0.02	0.03	-0.02	0.02	
	30	0.02	0.04	-0.68	0.53	0.55	-1.26	0.03	0.05	-0.03	0.03	
	40	0.01	0.07	-0.89	0.71	0.74	-1.64	0.03	0.06	-0.03	0.05	
	50	-0.01	0.13	-1.12	0.90	0.95	-2.02	0.02	0.06	-0.04	0.08	
	60	0.16	0.23	-1.24	1.15	1.25	-2.31	0.07	0.14	-0.01	0.16	
β_1	10	0.00	0.00	-0.10	0.08	0.08	-0.21	0.00	0.00	-0.02	0.00	
	20	-0.01	0.00	-0.20	0.15	0.16	-0.40	-0.01	0.00	-0.03	0.00	
	30	-0.02	0.00	-0.31	0.22	0.24	-0.63	-0.02	-0.01	-0.04	-0.01	
	40	-0.03	-0.01	-0.41	0.29	0.32	-0.85	-0.04	-0.01	-0.04	-0.02	
	50	-0.05	0.00	-0.52	0.35	0.42	-1.09	-0.07	-0.02	-0.04	-0.03	
	60	-0.01	0.05	-0.55	0.45	0.57	-1.26	-0.06	0.02	-0.05	0.01	
β_2	10	0.00	0.01	-0.06	0.04	0.04	-0.12	0.01	0.00	-0.05	0.01	
	20	0.00	0.02	-0.11	0.09	0.08	-0.24	0.01	0.01	-0.10	0.01	
	30	-0.02	0.04	-0.15	0.13	0.13	-0.36	0.02	0.02	-0.17	0.03	
	40	-0.04	0.09	-0.17	0.18	0.18	-0.46	0.04	0.03	-0.23	0.05	
	50	-0.10	0.17	-0.17	0.23	0.23	-0.55	0.05	0.05	-0.32	0.10	
	60	-0.05	0.19	-0.20	0.26	0.28	-0.68	0.04	0.06	-0.35	0.12	

Table 5. Effect Sizes for Missing Data Treatments by Percentage of Missingness in the Upper Stratum

Statistic	Percent	Deletion			Deterministic Imputation			Stochastic Imputation			
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	MI	ML
R^2	60	0.07	0.06	-0.76	0.64	0.68	-1.42	0.03	0.06	-0.04	0.05
	70	0.07	0.07	-0.76	0.64	0.68	-1.42	0.04	0.06	-0.03	0.05
	80	0.03	0.09	-0.77	0.63	0.67	-1.42	0.03	0.06	-0.02	0.06
	90	-0.03	0.12	-0.77	0.62	0.66	-1.42	0.03	0.05	-0.01	0.07
β_1	60	-0.01	0.02	-0.34	0.27	0.31	-0.73	-0.02	0.01	-0.05	0.01
	70	-0.01	0.01	-0.34	0.27	0.31	-0.73	-0.03	0.01	-0.04	0.01
	80	-0.03	0.00	-0.35	0.25	0.30	-0.74	-0.04	-0.01	-0.04	-0.01
	90	-0.04	-0.01	-0.36	0.24	0.28	-0.75	-0.05	-0.02	-0.02	-0.04
β_2	60	-0.01	0.02	-0.19	0.13	0.14	-0.45	0.00	0.01	-0.20	0.01
	70	-0.02	0.05	-0.17	0.14	0.15	-0.43	0.01	0.02	-0.20	0.03
	80	-0.04	0.10	-0.14	0.16	0.16	-0.40	0.03	0.03	-0.20	0.06
	90	-0.07	0.18	-0.07	0.19	0.18	-0.33	0.06	0.06	-0.22	0.11

Table 6. Number and Proportions of Conditions with Effect Sizes Less than 0.30 in Absolute Value

Statistic	Listwise	Pairwise	Deletion			Deterministic Imputation			Stochastic Imputation		
			Mean	SR	MR	Mean	SR	MR	Mean	SR	MR
R^2	N	136	140	24	35	35	5	144	144	143	144
	%	94%	97%	17%	24%	24%	3%	100%	100%	99%	100%
β_1	N	144	144	88	97	92	40	144	144	144	144
	%	100%	100%	61%	67%	64%	28%	100%	100%	100%	100%
β_2	N	142	134	89	120	111	77	142	144	106	142
	%	99%	93%	62%	83%	77%	53%	99%	100%	74%	99%

Table 7. Standard Deviation Ratios for Missing Data Treatments by Population Correlation Matrix

Statistic	R Matrix	Deletion			Deterministic Imputation			Stochastic Imputation			
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	MI	ML
R^2	High	1.28	1.13	1.03	1.08	1.14	1.05	1.13	1.17	1.07	1.10
	Low	1.27	1.12	0.96	1.21	1.22	0.92	1.17	1.18	1.08	1.11
β_1	High	1.30	1.20	1.04	1.20	1.28	1.09	1.20	1.25	1.18	1.15
	Low	1.30	1.13	1.04	1.17	1.20	1.09	1.20	1.21	1.11	1.12
β_2	High	1.31	1.16	1.04	1.19	1.24	1.12	1.20	1.23	1.16	1.13
	Low	1.30	1.19	1.03	1.16	1.35	1.05	1.15	1.27	1.03	1.17

Table 8. Standard Deviation Ratios for Missing Data Treatments by Sample Size

Statistic	N	Deletion			Deterministic Imputation			Stochastic Imputation			
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	MI	ML
R^2	50	1.28	1.14	1.00	1.16	1.19	0.99	1.16	1.18	1.07	1.11
	100	1.27	1.12	0.99	1.14	1.18	0.99	1.15	1.17	1.08	1.10
	200	1.28	1.12	0.99	1.14	1.18	0.98	1.15	1.17	1.08	1.10
β_1	50	1.33	1.19	1.04	1.22	1.29	1.09	1.23	1.26	1.15	1.16
	100	1.29	1.16	1.04	1.18	1.23	1.09	1.19	1.22	1.14	1.13
	200	1.28	1.15	1.04	1.15	1.21	1.10	1.17	1.21	1.14	1.12
β_2	50	1.33	1.20	1.04	1.19	1.34	1.09	1.19	1.28	1.11	1.18
	100	1.29	1.16	1.04	1.17	1.28	1.08	1.17	1.24	1.09	1.15
	200	1.28	1.15	1.03	1.17	1.26	1.08	1.17	1.23	1.08	1.14

Table 9. Standard Deviation Ratios for Missing Data Treatments by Percentage of Missing Data

Statistic	Percent	Deletion			Deterministic Imputation			Stochastic Imputation				
		Listwise	Pairwise	Mean	Mean	SR	MR	Mean	SR	MR	MI	ML
R^2	10	1.06	1.03	1.00	1.04	1.04	1.04	1.01	1.04	1.05	1.02	1.03
	20	1.12	1.06	1.00	1.07	1.08	1.08	1.01	1.08	1.09	1.04	1.05
	30	1.20	1.09	1.00	1.11	1.14	1.14	1.00	1.12	1.14	1.06	1.08
	40	1.29	1.13	0.99	1.16	1.19	1.19	0.99	1.17	1.19	1.08	1.11
	50	1.43	1.19	0.98	1.22	1.28	1.28	0.96	1.23	1.25	1.11	1.15
	60	1.57	1.25	0.98	1.29	1.36	1.36	0.95	1.28	1.32	1.14	1.20
β_1	10	1.06	1.04	1.01	1.04	1.05	1.05	1.03	1.05	1.06	1.03	1.03
	20	1.12	1.07	1.02	1.08	1.10	1.10	1.06	1.09	1.11	1.06	1.06
	30	1.21	1.12	1.03	1.13	1.17	1.17	1.08	1.15	1.18	1.11	1.11
	40	1.31	1.18	1.05	1.19	1.24	1.24	1.11	1.21	1.25	1.16	1.15
	50	1.46	1.25	1.06	1.28	1.37	1.37	1.13	1.29	1.34	1.22	1.21
	60	1.63	1.33	1.07	1.38	1.51	1.51	1.14	1.38	1.44	1.28	1.27
β_2	10	1.06	1.04	1.01	1.04	1.06	1.06	1.03	1.05	1.06	1.02	1.04
	20	1.12	1.08	1.02	1.08	1.12	1.12	1.05	1.09	1.12	1.03	1.07
	30	1.21	1.13	1.03	1.13	1.20	1.20	1.08	1.14	1.20	1.06	1.12
	40	1.31	1.18	1.04	1.19	1.29	1.29	1.10	1.19	1.27	1.09	1.16
	50	1.47	1.26	1.05	1.27	1.45	1.45	1.11	1.26	1.37	1.14	1.23
	60	1.64	1.35	1.07	1.36	1.63	1.63	1.13	1.33	1.48	1.20	1.30

Table 10. Standard Deviation Ratios for Missing Data Treatments by Percentage of Missingness in the Upper Stratum

Statistic	Percent	Deletion			Deterministic Imputation			Stochastic Imputation			
		Listwise	Pairwise	Mean	SR	MR	Mean	SR	MR	ML	MI
R^2	60	1.27	1.12	0.99	1.15	1.18	0.99	1.15	1.17	1.08	1.10
	70	1.28	1.12	0.99	1.15	1.18	0.99	1.15	1.17	1.08	1.10
	80	1.28	1.12	0.99	1.15	1.18	0.99	1.15	1.17	1.08	1.10
	90	1.28	1.13	0.99	1.15	1.18	0.98	1.15	1.17	1.07	1.10
β_1	60	1.30	1.16	1.04	1.18	1.24	1.09	1.20	1.23	1.14	1.14
	70	1.30	1.16	1.04	1.18	1.24	1.09	1.20	1.23	1.14	1.14
	80	1.30	1.17	1.04	1.18	1.24	1.09	1.20	1.23	1.14	1.14
	90	1.30	1.17	1.04	1.18	1.24	1.09	1.20	1.23	1.14	1.14
β_2	60	1.30	1.17	1.04	1.18	1.29	1.08	1.18	1.25	1.09	1.15
	70	1.30	1.18	1.04	1.18	1.29	1.09	1.18	1.25	1.09	1.15
	80	1.30	1.17	1.04	1.18	1.29	1.08	1.18	1.25	1.09	1.15
	90	1.31	1.17	1.03	1.18	1.30	1.08	1.18	1.25	1.09	1.15

Sampling Error Inflation

The distributions of the standard deviation ratios for R^2 in samples from high and low correlation matrices are presented in Figures 7 and 8, respectively. The least inflation of sampling error was seen with the mean imputation procedures (both deterministic and stochastic), while the largest increase was seen with the listwise deletion approach. With the exception of some conditions observed with the listwise deletion procedure, the increase in sampling error did not exceed 50% across any of the conditions examined. Somewhat greater increases in sampling error were evident for the regression weights (see Figures 9 and 10 for the estimation of β_1 , and Figures 11 and 12 for the estimation of β_2).

The average inflation of sampling error was similar across the two population matrices (see Table 7) and across the sample sizes examined (see Table 8), with slightly less inflation observed with larger samples. In contrast, the inflation was directly related to the proportion of missing data (see Table 9). For example, all of the MDTs evidenced minimal sampling error inflation when only 10% of the data were missing (for these conditions, the greatest increase in sampling error across parameters and across methods was only 6%). However, as the proportion of missing data increased, the sampling error also increased for all of the MDTs with the exception of the mean imputation procedures in the estimation of R^2 . For both the stochastic and deterministic mean imputation approaches, the sampling error decreased as the proportion of missing data increased. Finally, sampling error inflation was not associated with the proportion of missing data that occurred in the upper stratum of each regressor (see Table 10).

Discussion and Conclusions

Researchers in many fields are often confronted with challenges regarding appropriate methods for dealing with missing data. This issue continues to be a pervasive concern for applied researchers conducting inquiries across a multitude of research contexts. If missing data are ignored or improperly handled, resulting parameter estimates are likely to be biased, inferences distorted and conclusions unsubstantiated. Our investigation was designed to inform the treatment of nonrandomly missing data across a variety of commonly encountered situations in the conduct of multiple regression analysis. In this vein, we examined the influence of various configurations of missing data across two predictor variables. We studied two distinct correlational structures (both low and high correlations among variables) and three levels of sample size (small, medium and large). Missing data were simulated by varying the proportion of missingness in the upper stratum of each variable (i.e., above the median in most instances) and the total percentage of missingness.

The influence of the degree of relationship between the variables under investigation was negligible for the deletion procedures, but rather substantial for the deterministic procedures. When the influence of sample size was examined, the deletion procedures appeared relatively unaffected, yet sample size evidenced a dramatic effect on the deterministic procedures and the stochastic mean imputation procedures, with larger sample sizes resulting in rather substantially biased estimates. For most of the conditions examined, the influence of the proportion of missing data in the upper stratum had little effect on most of the missing data treatment methods. As expected, the most problematic conditions appeared to be those in which the proportion of missing data in the upper stratum was high, coupled with a large percentage of missing data.

The influence of missing data on sample estimates of R^2 varied considerably among the missing data techniques across the conditions examined. In many instances, nonbiased parameter estimates were evidenced for the two stochastic regression approaches and the maximum likelihood estimation method. Both pairwise and listwise deletion methods and the multiple imputation approach were nearly as effective, yielding relatively few conditions presenting some degree of bias. In contrast, the stochastic mean substitution approaches and all three deterministic approaches evidenced considerable bias, suggesting that these methods should be avoided when faced with missing data of this nature. Similar patterns were observed for these four missing data treatments for both the high and low correlation matrices, however, the bias associated with these procedures was more pronounced when the variables were more highly correlated.

If we turn our attention to the estimation of the regression parameters, we find that both deletion procedures, the stochastic regression approaches and the two model-based methods (i.e., multiple imputation and maximum likelihood estimation) consistently yielded unbiased estimates of the standardized regression weight for X_1 . Once again, the three deterministic procedures and the stochastic mean imputation procedure showed a similar pattern of bias across both the high and low correlation matrices, but for these analyses, bias was considerably less pronounced for the high correlation matrix conditions.

Of interest is the notably different pattern inherent in the bias associated with the estimation of the standardized regression weight for X_2 . For the high correlation matrix conditions, the same four procedures evidenced extreme bias, but surprisingly the multiple imputation procedure exhibited substantial negative bias. In contrast, multiple imputation evidenced very little bias for the lower correlation condition across the various treatments and conditions.

When we consider the results from this examination of ten missing data treatments under various conditions of missing data on two predictors, we find no convincing evidence to recommend any of the deterministic procedures or the stochastic mean imputation approach as valid approaches to dealing with issues of missing data. On the contrary, the results reported here provide substantial evidence that these methods should be avoided. However, these results suggest that the use of stochastic regression methods, multiple imputation techniques and maximum likelihood estimation warrant careful consideration under many circumstances. Further, the simple deletion methods performed relatively well in many conditions. Although the stochastic multiple regression imputation approach performed the best over all of the conditions examined, the simpler deletion methods may be sufficient in many circumstances.

The results of this study need to be considered in the context of the limitations of this research. First, the findings are somewhat narrowly focused on a two-predictor multiple regression model. The success of these methods in more complex situations, such as factor analysis or path analysis remains uncertain, *Multiple Linear Regression Viewpoints*, 2003, Vol. 29(1)

although recent work suggests promise for the maximum likelihood approach in structural equation modeling (Enders, 2001). Clearly, additional research is called for to examine these methods, as well as others, in multiple contexts under supplementary conditions. Second, the structure of the missing data was such that the missingness was balanced across the two predictors, and the missingness was never allowed to occur across both predictors for any observation. Additionally, the behavior of the missing data techniques was explored for only two correlational structures. The nature and structure of the simulated data sets may additionally limit the ability to generalize the results beyond the conditions examined. Further, the simulated data used in this study were multivariate normal. The success of these missing data treatments with nonnormal distributions will require additional research. Finally, the focus of this work was on bias and sampling error of R^2 and standardized regression weights. The impact of missing data and their treatment on Type I error rates, statistical power, and the accuracy of confidence intervals constructed around parameter estimates requires additional investigation.

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