Application of a Structure Coefficient Rule of Thumb For Two-Group Descriptive Discriminant Analysis

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This simulation study considered the rule of thumb as noted in Pedhazur (1997) for judging the usefulness of continuous (in a previous MANOVA, dependent) variables at determining group separation in descriptive discriminant analysis; namely, that a structure coefficient value equal to or greater than .3 identifies a useful continuous variable. No research to date has tested this rule. Results indicate that the rule is generally useful for identifying variables with medium to large effects but not small effects.

In an effort to easily interpret results of complex statistical analyses, practitioners often consult the literature for guidelines, or "rules of thumb," to aid in understanding their results. Although statisticians themselves might hesitate to distill complex results into a few general criteria, they also wish to offer some means of helping researchers from other fields utilize complex results (e.g., Cohen's work with effect size guidelines, 1992). Thus, they offer "rules of thumb." In Cohen's work, the rules of thumb for identifying small, medium, and large effect sizes is based upon extensive research. However, some rules of thumb are not so obviously supported via prior research. Such is the case of interpreting structure coefficient (SC) values in discriminant analysis (DA), specifically, descriptive discriminant analysis (DDA). Before continuing about the issues surrounding SC interpretation, a more detailed discussion of DA might prove helpful.

Cooley and Lohnes (1971) describe DA as the search for the best reduced-rank linear model to account for differences among groups as such differences have been measured on a vector of p continuous variables. Though mathematically it makes no difference whether the continuous variables are viewed as independent and the grouping variable, dependent, often in DA, the grouping variable is considered the outcome variable, with orthogonal linear discriminant functions derived such that the resulting coefficients associated with the vector of p continuous, independent variables maximize group differences. The number of possible linear functions is the lesser of p and the number of groups, k, minus one. In the case of two-group MANOVA, only one linear function is possible. With slight modification in notation, this function, Z, may be written as:

$$Z = X_1v_1 + X_2v_2 + \ldots + X_pv_p = \mathbf{Xv} \quad (1)$$

as noted in Schneider, 2002 (see Tatsuoka, 1988a, for further explanation). In Equation 1, X_p is the pth continuous variable and v_p, the raw weight associated with the pth variable. The raw weights are not readily interpretable and must be converted into other coefficients. One vector of coefficients commonly used for lending meaning to the linear function is the vector of structure coefficients (SCs). In the case of two groups, only one linear function is possible. Therefore, the SSCP matrix for the total sample is reduced to a scalar, T. Using notation from Tatsuoka (1988b), if **D(.)** represents the diagonal elements of a given matrix, then the matrix of SCs based on total variance, **A**, can be written as:

$$\mathbf{A} = [\mathbf{D}(\mathrm{T})]^{-1/2}\,(\mathrm{T}\mathbf{V})\,[\mathbf{D}(\mathbf{V}'\mathrm{T}\mathbf{V})]^{-1/2}. \quad (2)$$

The elements of **A** are the SCs , a_1 thru a_p, associated with the single linear function, Z. Using the above algorithm, in order to calculate SCs based on pooled within-group variance as opposed to total variance, one need only substitute the scalar T with the scalar for pooled within-group variance, W. In the case of Equation 2, SCs are normalized because calculation of **A** includes multiplying the square root of the inverse matrix, $[\mathbf{D}(\mathbf{V}'\mathrm{T}\mathbf{V})]^{-1}$. Thus, any given vector of SCs is restricted to a length of one. However, in popular statistical computer packages (SAS and SPSS), SCs are not normalized. Furthermore, DA output in SAS includes SCs based on both total variance and pooled within-group variance; concordant output in SPSS includes only SCs based on within-group variance. Because SCs are not normalized in contemporary statistical computer software, as it applies to non-normalized SCs, Equation 3 may be rewritten as a modification of Tatsuoka's algorithm from Equation 2:

$$\mathbf{A}_{\mathbf{non}} = [\mathbf{D}(\mathrm{T})]^{-1/2}\,(\mathrm{T}\mathbf{V})\,, \quad (3)$$

where the resulting matrix on SCs, $\mathbf{A}_{\mathbf{non}}$, is not normalized.

Researchers utilize DA for two primary purposes: prediction or description (Hubery, 1994). The first purpose involves predicting group membership based on the vector of *p* continuous variables. In the second use of DA, instead of predicting group membership, the researcher is interested utilizing DA as a *post hoc* procedure following a significant MANOVA. The focus of this paper is on the latter use of DA, commonly known as descriptive discriminant analysis (DDA). In DDA, the researcher interprets the vector of *p* coefficients to understand, for example, which of the *p* variables contributed to separation on the grouping variable and which did not. As previously mentioned, the vector of raw weights must be converted into interpretable coefficients. This paper investigates two types of SCs: Those based on total correlation matrix, $\mathbf{s_T}$, and those based upon the pooled within-group variance, $\mathbf{s_W}$ (Dalgleish, 1994). As previously noted in reference to Equations 2 and 3, these two types of SCs are available in SAS, and only $\mathbf{s_W}$ is available in SPSS. Both $\mathbf{s_T}$ and $\mathbf{s_W}$ will be examined in this study. If the researcher's goal is to identify which among the *p* continuous variables are contributing to group separation by consulting SC values, then a rule of thumb would certainly be helpful. Regarding interpretation of SC values, Pedhazur (1997) notes that SC values $\geq .3$ "are treated as meaningful" (p. 910). Pedhazur also notes that rules of thumb might be problematic and refers the reader to Dalgleish regarding testing SC significance. However, tests of the significance of SCs are not readily available for researcher use, which may be why the researcher seeks a rule of thumb in the first place. Because tests of SC significance are not readily available, testing the rule of thumb might yield useful information regarding its application to SC interpretation.

Purpose of the Study

To date, no simulation study has examined the usefulness of the rule of thumb that SCs with values of .3 or greater might meaningfully identify continuous variables influential upon group separation in DA. The primary goal of the current study was to investigate conditions under which the rule might identify "meaningful" (formerly MANOVA dependent) variables when DA is used as a *post hoc* test following a significant MANOVA, known as *descriptive discriminant analysis* (DDA) (Huberty, 1994). In this work, operationalizing of "meaningful" variables is addressed in the Procedures section. In the presence of significant differences among group means, $\mathbf{s_W}$ might be preferred to $\mathbf{s_T}$ (Dalgleish, 1994; Huberty, 1975). However, because DDA research is inconclusive regarding the utility of SCs based on both the total matrix versus those based upon the within matrix (Schneider, 2004), both types of SCs will be compared in this study.

Procedures

SAS PROC IML was employed for the current Monte Carlo, two-group simulation, with two *p*-dimensional, multivariate population matrices generated, each being $\mathcal{N}(\mu, \Sigma)$ (SAS Institute, 1999). The general procedure was employed in Schneider, 2004: In all cells, μ_1 was a *p* x 1 null vector, and μ_2, a *p* x 1 vector of effects of some combination such that $\mu_1 \neq \mu_2$. A sample of dimension *n* x *p* was then drawn from each population ($n_1 = n_2$; $p_1 = p_2$) and analyzed as a two-way MANOVA using Wilks' Λ and a special case of Bartlett's V as a test of significance:

$$V = -[N - 1 - (p + 2) / 2] \ln \Lambda \quad (3)$$

where Λ is also calculated using a modified formula

$$\Lambda = 1 / (1 + \lambda) \quad (4)$$

Based on Tatsuoka (1988a, 1988b), V is approximately a χ^2 distribution with *p* degrees of freedom. In the current work, the variables and corresponding levels manipulated were as follows:

1. Continuous variable levels $p = 2, 3, 4$, and 5.
2. Group sample size $n = 50, 100, 150$ and 200.
3. The population correlation matrices, $\mathbf{P}_1$ and $\mathbf{P}_2$. Five levels were used, reflecting five possible ranges of *p* intercorrelation (hereby denoted as **P** for population and **R**, for sample): 0 - .20; .21 - .40; .41 - .60; .61 - .80, and .81 – 1.00. For a given experiment, the exact correlation for the two groups between continuous variables *p* and *p'* (where $p \neq p'$) was randomly generated within any one of these five ranges. The two most highly correlated ranges were included to consider potential effects of collinearity upon the rule of thumb.

4. Population mean vector, μ_2. As previously mentioned, μ_1 was held constant as a null vector. Thus, μ_2 was manipulated as the vector of effects. The *p* elements of a given μ_2 were some combination of effects, with three possible levels of effect size: .2, .5 and .8. These levels were based upon Cohen's (1992) determination of small, medium, and large effects, respectively, for two independent means. For the purposes of this study, all three levels of effect were considered "meaningful," as all three could contribute to MANOVA significance. All combinations of .2, .5, and .8 were investigated, for a total of 45 *p* x 1 mean vector pairs: 6 for *p* = 2; 10 for *p* = 3; 13 for *p* = 4, and 16 for *p* = 5.

Each *n* x *p* cell was replicated 5,000 times. For the replications where the MANOVA null hypothesis $\mathbf{H_0}$: $\mu_1 = \mu_2$ was correctly rejected within each cell, the *p* x 1 vectors of total and within SCs, $\mathbf{s_T}$ and $\mathbf{s_W}$, representing the first discriminant function, were calculated, and the proportions of $\mathbf{s_T}$ and $\mathbf{s_W}$ vectors conforming to rule of thumb (SC element values $\geq$ .3) were subsequently calculated. For all vectors, regardless of MANOVA significance, information on the proportion of individual elements conforming to the rule of thumb was also tabulated.

Results

In general, the rule of thumb that an SC value $\geq$ .3 indicates a continuous variable contributing to group separation works best for vectors involving medium and large effects across all *n* x *p* cells. The exceptions appear to be cells where *p* variable intercorrelation was highest (**R** = .81 - .99). In the case of highest intercorrelation, the proportion of sample vectors fitting the rule dropped notably, but only where the elements were some combination of medium and large effects, not where entire vectors contained either medium or large effects. As for vectors with small effects, the rule fit best where the entire vector was comprised of small effects. For remaining vectors containing at least one small effect, the rule did not fit vectors well at all. However, when the proportion of elements fitting the rule was examined (as opposed to proportions of entire vectors), it is clear that the elements with small effects are responsible for entire vector ill fit. A final notable finding is that overall, $\mathbf{s_T}$ outperformed $\mathbf{s_W}$.

In subsequent sections, discussion focuses first on vectors with either all medium or all large effects. Next are results for vectors with combined medium and large effects. Final discussion is on vectors containing 1) all small effects and 2) at least one small effect.

Vectors with All Medium or All Large Effects

Table 1 includes the proportions of both $\mathbf{s_T}$ and $\mathbf{s_W}$ conforming to the rule for *p* = 4 where all elements have medium effects (.5, as noted in Cohen, 1992). Table 2 contains the same information for *p* = 5 for all large effects (.8, as noted in Cohen). If the proportion of sample vectors conforming to the rule equaled or exceeded .8, then the result is reported in bold in all tables, for this result is deemed as indicating the rule worked well for such a cell.

For the two examples above where all effects were either medium or large, the rule of thumb worked well for almost all cells, even those with highest *p* variable intercorrelation. Note also that there appears to be little difference in the proportions of both $\mathbf{s_T}$ and $\mathbf{s_W}$ conforming to the rule, with one exception: the cell with the smallest group sample size *n* = 50 and lowest intercorrelation, **R** = .00 - .21 in Table 2.

Table 1. Proportions of SCs for *p* = 4 Vector with all medium effects

n	*R* .00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50	.6919	**.8222**	**.9049**	**.9584**	**.9790**
	.6190	*.7886*	***.8900***	***.9488***	***.9766***
100	**.9047**	**.9548**	**.9819**	**.9914**	**.9976**
	.8718	***.9403***	***.9801***	***.9903***	***.9976***
150	**.9714**	**.9886**	**.9956**	**.9988**	**1.000**
	.9526	***.9858***	***.9942***	***.9988***	***1.000***
200	**.9772**	**.9956**	**.9988**	**.9998**	**.9984**
	.9610	***.9930***	***.9986***	***.9998***	***.9978***

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

Table 2. Proportions of SCs for p = 5 Vector with all large effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		**.9208**	**.9738**	**.9910**	**.9969**	**.9976**
		.7288	***.9426***	***.9841***	***.9950***	***.9976***
100		**.9956**	**.9994**	**1.000**	**1.000**	**1.000**
		.9636	***.9990***	***1.000***	***1.000***	***1.000***
150		**.9996**	**1.000**	**1.000**	**1.000**	**1.000**
		.9978	***1.000***	***.9998***	***1.000***	***1.000***
200		**1.000**	**1.000**	**1.000**	**1.000**	**1.000**
		.9998	***1.000***	***1.000***	***1.000***	***1.000***

Note: S_T are in Roman font, and S_W are italicized. Proportions ≥ .8 are in bold.

Table 3. Proportions of SCs for p = 5 Vector with all medium effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.5525*	.7494**	**.8612******	**.9288**	**.9744**
		.4341	*.6875*	***.8338***	***.9170***	***.9694***
100		**.8322**	**.9176**	**.9691**	**.9906**	**.9966**
		.7443	***.8961***	***.9628***	***.9886***	***.9959***
150		**.9502**	**.9832**	**.9950**	**.9990**	**.9996**
		.9076	***.9774***	***.9930***	***.9986***	***.9996***
200		**.9700**	**.9954**	**.9978**	**1.000**	**.9998**
		.9364	***.9926***	***.9972***	***1.000***	***.9998***

Note: SC_T are in Roman font, and SC_W are italicized. Proportions ≥ .8 are in bold.
*d_1= .5525 - .4341 = **.1184** **d_2= .7494 - .6875 = **.0619** ***d_3= .7494 - .6875 = **.0619**

In this case, $\mathbf{s_T}$ noticeably outperforms $\mathbf{s_W}$ in identifying correct contribution to group separation based upon the rule of thumb ($\mathbf{s_T}$ = .9208; $\mathbf{s_W}$ = .7288). For other p vectors with either all medium or all large effects, $\mathbf{s_W}$ did not fare as well as $\mathbf{s_T}$ for lower levels of n and/or **R**, with the most notable difference evident where p = 5 and all effects were medium (Table 3).

As is evident in Table 3, the difference in performance of $\mathbf{s_T}$ versus $\mathbf{s_W}$ is less noticeable as proportion of sample vectors conforming to the rule increases. Furthermore, for cells where all effects are either medium or large, an n x **R** interaction is evident. As for the difference in performance of $\mathbf{s_T}$ and $\mathbf{s_W}$, where n = 50, this difference is most noticeable for the cell with the lowest intercorrelation (d_1 = .5525 - .4341 = **.1184**) and less noticeable as intercorrelation increases (d_2 = .7494 - .6875 = **.0619**; d_3 = .8612 - .8338 = **.0274**).

Vectors with a Combination or Medium and Large Effects

As previously mentioned, the rule of thumb fit well for vectors with a combination of medium and large effects, with the exception of cells with the highest p variable intercorrelation. Tables 4 thru 6 provide representative examples of the drop in the proportion of conforming cells at highest intercorrelation for cells where p = 3, 4, and 5, respectively. As was true for vectors with all medium or all large effects, an n x **R** interaction is present, this time for all cells excepting those of the highest intercorrelation. In this interaction, fewer cells fit the rule of thumb as n and **R** decreased. Furthermore, regardless of the number of p variables, the rule worked well consistently for cells in the center of the tables, with sample sizes $n \geq 100$ and intercorrelation **R** = .21 - .80.

As the results in Table 5 show, not all cells for the highest level of intercorrelation yielded poor results. For cells with highest intercorrelation where group sample size n = 100 and 200, the rule fit well (Table 5) (n = 100: $\mathbf{s_T}$ = .9194; $\mathbf{s_W}$ = .8562, and n = 200: $\mathbf{s_T}$ = .9940; $\mathbf{s_W}$ = .9882). However, even where the rule fit erratically for cells with highest correlation (Table 5) as opposed to not fitting well at all (Tables 4 and 6), no pattern was evident except that $\mathbf{s_T}$ consistently outperformed $\mathbf{s_W}$.

Table 4. Proportions of SCs for *p* = 3 Vector with two medium and one large effect

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.7787	**.8431**	**.8707**	**.8914**	.7708
		.7019	***.8015***	***.8476***	***.8678***	*.6516*
100		**.9298**	**.9446**	**.9608**	**.9716**	.4500
		.8910	***.9196***	***.9505***	***.9590***	*.0074*
150		**.9698**	**.9848**	**.9894**	**.9896**	.6174
		.9518	***.9722***	***.9826***	***.9826***	*.0860*
200		**.9886**	**.9940**	**.9958**	**.9960**	.6844
		.9746	***.9896***	***.9928***	***.9926***	*.1196*

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

Table 5. Proportions of SCs for *p* = 4 Vector with one medium and three large effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.7666	**.8484**	**.8890**	**.9012**	.7246
		.6286	*.7878*	***.8493***	***.8732***	*.5310*
100		**.8988**	**.9330**	**.9548**	**.9652**	**.9194**
		.7958	***.8802***	***.9318***	***.9476***	***.8562***
150		**.9424**	**.9664**	**.9838**	**.9816**	.3008
		.8392	***.9360***	***.9728***	***.9702***	*0*
200		**.9562**	**.9840**	**.9920**	**.9936**	**.9940**
		.8640	***.9618***	***.9778***	***.9882***	***.9882***

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

Table 6. Proportions of SCs for *p* = 5 Vector with four medium and one large effect

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.5041	.6701	.7679	**.8094**	.4996
		.3466	*.5824*	*.7156*	*.7616*	*.1918*
100		.7828	**.8774**	**.9260**	**.9279**	.7004
		.6382	***.8252***	***.9014***	***.9033***	*.4170*
150		.8810	**.9606**	**.9736**	**.9828**	.3928
		.7662	***.9334***	***.9622***	***.9740***	*.0022*
200		**.9554**	**.9852**	**.9914**	**.9936**	.5884
		.8982	***.9718***	***.9832***	***.9894***	*.0400*

Note: SC_T are in Roman font, and SC_W are italicized. Proportions $\geq$ .8 are in bold.

Vectors with All Small Effects

Tables 7 though 10 contain the results for the four *p* x 1 mean vector pairs where all elements had small effects (.2, as noted in Cohen, 1992). In the case of vectors containing small effects, only those with all small effects had cells that fit well with the rule of thumb that an SC value $\geq$ .3 indicates a continuous variable contributing to group separation. As was true for previously reported results, if the proportion of sample vectors conforming to the rule equaled or exceeded .8, then the result is reported in bold in all tables, for this result is deemed as indicating the rule worked well for such a cell.

It is clear that as one reads Tables 7 through 10 in sequence, the proportion of SC values fitting the rule well (i.e., cells with the proportion equaling or exceeding .8 for SC vectors conforming to the rule) narrows as the number of *p* variables increases. Specifically, in the case of vectors with all small effects, as the number of *p* variables increases, both group sample size *n* and intercorrelation **R** must increase in order for the rule of thumb to work well. The vectors with all small effects present the clearest evidence of a three-way interaction, *p* x *n* x **R**, in this entire study. The most noticeable decrease in the number of *n* x **R** cells fitting the rule occurs when the number of *p* variables increases from 2 to 3. Seventeen cells show SC proportions fitting the rule as equaling or exceeding .8 where *p* = 2. However, this number drops to ten cells where *p* = 3. The drop is no so drastic when *p* = 4 (6 cells fit the rule) or *p* = 5 (3 cells).

Table 7. Proportions of SCs for p =2 Vector with both small effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.6859	.7728	**.8261**	**.8593**	**.8811**
		.6726	*.7546*	***.8152***	***.8552***	***.8811***
100		.7957	**.8298**	**.8738**	**.9176**	**.9382**
		.7874	***.8246***	***.8690***	***.9153***	***.9373***
150		**.8479**	**.8989**	**.9176**	**.9460**	**.9701**
		.8437	***.8984***	***.9166***	***.9455***	***.9701***
200		**.8658**	**.9169**	**.9422**	**.9816**	**.9878**
		.8616	***.9162***	***.9396***	***.9803***	***.9878***

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

Table 8. Proportions of SCs for p = 3 Vector with all small effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.4670	.5347	.6466	.7469	**.8159**
		.4336	*.5197*	*.6248*	*.7362*	***.8035***
100		.5560	.6511	.7635	**.8219**	**.8523**
		.5429	*.6416*	*.7583*	***.8201***	***.8468***
150		.6638	.7673	**.8162**	**.8941**	**.8996**
		.6543	*.7595*	***.8111***	***.8911***	***.8980***
200		.7139	**.8280**	**.8720**	**.9284**	**.9733**
		.7060	***.8237***	***.8685***	***.9252***	***.9728***

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

Table 9. Proportions of SCs for p = 4 Vector with all small effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.2638	.3811	.5202	.6331	.7382
		.2257	*.3585*	*.4933*	*.6071*	*.7236*
100		.3335	.4943	.6832	.7376	**.8466**
		.3126	*.4754*	*.6741*	*.7312*	***.8433***
150		.4891	.6134	.7491	**.8327**	**.8952**
		.4707	*.6049*	*.7448*	***.8301***	***.8931***
200		.4626	.6638	**.8125**	**.9086**	**.9339**
		.4446	*.6548*	***.8070***	***.9070***	***.9294***

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

Table 10. Proportions of SCs for p = 5 Vector with all small effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.1168	.2471	.4252	.5507	.6601
		.0880	*.2096*	*.3896*	*.5217*	*.6444*
100		.1825	.3750	.5706	.7444	.7868
		.1557	*.3530*	*.5517*	*.7363*	*.7797*
150		.2229	.4943	.6636	.7991	**.8804**
		.2004	*.4731*	*.6512*	*.7937*	***.8755***
200		.2938	.5708	.7326	**.8276**	**.9222**
		.2757	*.5569*	*.7231*	***.8229***	***.9210***

Note: SC_T are in Roman font, and SC_W are italicized. Proportions $\geq$ .8 are in bold.

As was true for all previous p x 1 mean vectors of effects discussed in this paper, $\mathbf{s_T}$ consistently outperformed $\mathbf{s_W}$, with the differences between $\mathbf{s_T}$ and $\mathbf{s_W}$ proportions being less pronounced as proportions of SC vectors fitting the rule increased. Too, as was true for vectors containing either all medium or all large effects, the drop was not present in proportions conforming to the rule for cells with the highest intercorrelation.

Table 11. Proportions of SCs for p = 2 Vector with one small and one large effect

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.4594	.4551	.4254	.4266	.2118
		.4154	*.4145*	*.3757*	*.3685*	*.1130*
100		.4355	.4238	.3999	.2780	.2120
		.3896	*.3767*	*.3449*	*.1986*	*.1240*
150		.4180	.4246	.4006	.3226	.0112
		.3612	*.3606*	*.3390*	*.2532*	*0*
200		.3906	.3948	.3888	.3160	.0402
		.3280	*.3278*	*.3194*	*.2352*	*.0048*

Note: S_T are in Roman font, and S_W are italicized. Proportions ≥ .8 are in bold.
MANOVA power: $1 - \beta = .9504$

Table 12. Proportions of SCs for p = 3 Vector with two small and one medium effect

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.3320	.4031	.4321	.4457	.3164
		.2978	*.3715*	*.3972*	*.4195*	*.2698*
100		.3866	.4627	.4790	.4329	.0116
		.3543	*.4377*	*.4560*	*.4063*	*0*
150		.4333	.4896	.5112	.4376	.0056
		.4063	*.4679*	*.4884*	*.4055*	*0*
200		.4544	.5199	.5235	.5529	.0148
		.4201	*.4964*	*.4950*	*.5255*	*.0006*

Note: S_T are in Roman font, and S_W are italicized. Proportions ≥ .8 are in bold.
MANOVA power: $1 - \beta = .5964$

Table 13. Proportions of SCs for p = 4 Vector with one small and three large effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.2256	.2944	.3277	.2735	.1912
		.1262	*.2086*	*.2463*	*.1879*	*.0806*
100		.1874	.2412	.2280	.1752	.0072
		.0874	*.1620*	*.1540*	*.0900*	*0*
150		.1154	.1712	.2168	.1746	.0868
		.0418	*.0886*	*.1346*	*.0900*	*.0218*
200		.0998	.1128	.1870	.0816	.0004
		.0328	*.0374*	*.1006*	*.0202*	*0*

Note: S_T are in Roman font, and S_W are italicized. Proportions ≥ .8 are in bold.
MANOVA power: $1 - \beta = 1.000$

Table 14. Proportions of SCs for p = 5 Vector with two small, one medium, two large effects

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.0953	.1329	.1281	.1200	.0338
		.0506	*.0831*	*.0750*	*.0476*	*0*
100		.0546	.0912	.1086	.1052	.0786
		.0192	*.0494*	*.0502*	*.0504*	*.0178*
150		.0240	.0824	.0836	.1162	.0016
		.0046	*.0402*	*.0350*	*.0638*	*0*
200		.0224	.0530	.0520	.0574	.0084
		.0054	*.0180*	*.0168*	*.0166*	*0*

Note: SC_T are in Roman font, and SC_W are italicized. Proportions ≥ .8 are in bold.
MANOVA power: $1 - \beta = .9928$

Mixed Vectors Containing at Least One Small Effect

Tables 11 through 14 contain results representative of the 23 *p* x 1 vectors involving at least one element with a small effect. What is clear from examination of Tables 11 through 14 is that the rule of thumb that an SC value ≥ .3 indicates a continuous variable contributing group separation does not fit well at all for vectors containing even only one element with a small effect. Furthermore, there is no pattern evident across these tables to indicate that increased group sample size would remediate the problem. For example, in Table 12, for all except the highest level of *p* variable intercorrelation (**R** .00 - .80), it seems that as group sample size *n* increases, so does the proportion of cells fitting the rule. However, results of Table 13 show the opposite effect: For the same range of **R** intercorrelation, the proportion of cells fitting the rule decreases as group sample size *n* increases. Finally, Tables 11 and 14 yield results neither consistently increasing nor decreasing as group size *n* increases but instead fluctuates, showing yet another pattern for all except the highest level of intercorrelation (**R** .00 - .80).

As for the highest level of *p* variable intercorrelation (**R** = .81 - .99), proportions of cells fitting the rule drop in a similar fashion to the vectors containing mixed medium and large effects (Tables 4 – 6). In the case of vectors including at least one small effect (Tables 11 – 14), cells were likely to approach or reach zero proportions fitting the rule than was true for vectors containing only medium and large effects. Too, even though general results were poor for both types of SCs, $\mathbf{s_T}$ continued to outperform $\mathbf{s_W}$ as far as rule fit was concerned.

An interesting comparison involves Tables 5 and 13. The only difference between the *p* = 4 vectors for the two tables is that the single medium effect in Table 5 is replaced with a small effect in Table 13. In both vectors, the remaining three elements are large effects. What is noteworthy is the difference the change from medium to small effect has upon the fit of the entire vector to the rule of thumb. Tables 15 through 18 contain detailed information regarding rule fit for specific elements in a vector and correspond to Tables 11 though 14, respectively, where the information is on rule fit for entire vectors. As one can see, the presence of elements with small effects in these mixed vectors would reduce the fit of the entire vector. For example, the proportion of vectors conforming to the rule where *p* = 2 with one small and one large effect where the MANOVA was correctly rejected (MANOVA power: $1 - \beta = .9504$.) was $\mathbf{s_T}$ = .4594 and $\mathbf{s_W}$ = .4154 for *n* = 50 and **R** = .00 - .20 (Table 11). However, as one examines proportions of elements conforming to the rule for these same conditions across all 5000 replications (Table 15), one sees that the proportion fitting the rule was high for the element with the large effect (both $\mathbf{s_T}$ and $\mathbf{s_W}$ = .9502) and low for the element with the small effect ($\mathbf{s_T}$ = .4368 and $\mathbf{s_W}$ = .3950). Another example involves the proportion of vectors conforming to the rule where *p* = 5 with two small, one medium, and two large effect where the MANOVA was correctly rejected (MANOVA power: $1 - \beta = .9928$.) was $\mathbf{s_T}$ = .1052 and $\mathbf{s_W}$ = .0504 for *n* = 100 and **R** = .61 - .80 (Table 14). As one examines proportions of elements conforming to the rule for these same conditions across all 5000 replications (Table 18), one sees that the proportion fitting the rule was high for the elements with the one medium ($\mathbf{s_T}$ = .9182 and $\mathbf{s_W}$ = .8466) and two large effects (both elements $\mathbf{s_T}$ = .9998 and $\mathbf{s_W}$ = .9996) and low for the two elements with small effects (first small element: $\mathbf{s_T}$ = .2110 and $\mathbf{s_W}$ = .1186; second small element: $\mathbf{s_T}$ = .2104 and $\mathbf{s_W}$ = .1218).

Discussion

Research practitioners often search the literature for guidelines regarding interpretation of statistical analysis results. The researcher interested in interpreting structure coefficients (SCs) in discriminant analysis (DA) might use the rule of thumb as noted in Pedhazur (1997) that an SC value ≥ .3 indicates a continuous variable useful for contributing to separation on the grouping variable. However, this rule has apparently not been tested before this study. In the case of two-group MANOVA, results indicate that the rule of thumb works well for vectors with medium or large effects (.5 and .8, respectively, as noted in Cohen, 1992) but not well for small effects (.2, as noted in Cohen). The exception appears to be *p* = 2 continuous variables where both effects are small (Table 7). Because the most common effect size in many fields is the medium effect size (Cohen), the rule of thumb could prove useful for practitioners despite the apparent poor results for vectors involving small effects.

Table 15. Proportions of Elements Fitting the Rule for the *p* = 2 Vector in Table 11Population Effects: One Small and One Large Effect, Respectively

n	*R* .00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50	.4368	.4320	.4142	.4192	.2140
	.9502	**.9492**	**.9736**	**.9812**	**.9896**
	.3950	*.3934*	*.3658*	*.3626*	*.1138*
	.9502	***.9492***	***.9736***	***.9810***	***.9762***
100	.4352	.4342	.3996	.2780	.2120
	.9994	**.9986**	**.9992**	**1.000**	**1.000**
	.3894	*.3762*	*.3446*	*.1986*	*.1240*
	.9994	***.9986***	***.9992***	***1.000***	***.9996***
150	.4180	.4246	.4006	.3226	.0112
	1.000	**1.000**	**1.000**	**1.000**	**.9970**
	.3612	*.3606*	*.3390*	*.2532*	*0*
	1.000	***1.000***	***1.000***	***1.000***	*.7200*
200	.3906	.3948	.3888	.3160	.0402
	1.000	**1.000**	**1.000**	**1.000**	**1.000**
	.3280	*.3278*	*.3194*	*.2352*	*.0048*
	1.000	***1.000***	***1.000***	***1.000***	***1.000***

Note: S_T are in Roman font, and S_W are italicized. Proportions ≥ .8 are in bold.

Table 16. Proportions of Elements Fitting the Rule for the *p* = 3 Vector in Table 12 Population Effects: Two Small and One Medium Effect, Respectively

n	*R* .00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50	.3536	.3498	.3420	.3750	.3926
	.3508	.3458	.3356	.3708	.3934
	.5908	.5476	.5414	.6380	**.8838**
	.3374	*.3368*	*.3266*	*.3574*	*.3364*
	.3346	*.3312*	*.3198*	*.3526*	*.3416*
	.5902	*.5474*	*.5412*	*.6362*	***.8616***
100	.5610	.5604	.5676	.5564	.0216
	.5470	.5688	.5618	.5506	.0248
	.8906	**.8570**	**.8614**	**.9428**	.5264
	.5404	*.5428*	*.5502*	*.5332*	*0*
	.5252	*.5492*	*.5466*	*.5268*	*0*
	.8906	***.8570***	***.8614***	***.9426***	*.0238*
150	.6490	.6574	.6514	.5828	.0068
	.6478	.6462	.6560	.5826	.0058
	.9766	**.9674**	**.9624**	**.9970**	.5626
	.6288	*.6424*	*.6320*	*.5510*	*0*
	.6294	*.6304*	*.6402*	*.5502*	*0*
	.9764	***.9674***	***.9624***	***.9970***	*.0134*
200	.6688	.6980	.6872	.6562	.0296
	.6712	.7160	.6748	.6550	.0254
	.9974	**.9944**	**.9960**	**.9976**	**.8472**
	.6740	*.6802*	*.6660*	*.6342*	*.0018*
	.6454	*.6978*	*.6526*	*.6296*	*.0012*
	.9974	***.9944***	***.9960***	***.9976***	*.4308*

Note: S_T are in Roman font, and S_W are italicized. Proportions ≥ .8 are in bold.

Table 17. Proportions of Elements Fitting the Rule for the p = 4 Vector in Table 13 Population Effects: One Small and Three Large Effects, Respectively

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.2302	.2954	.3280	.2772	.1922
		.9906	**.9936**	**.9914**	**.9940**	**.9866**
		.9912	**.9942**	**.9918**	**.9940**	**.9856**
		.9912	**.9934**	**.9918**	**.9944**	**.9840**
		.1326	*.2106*	*.2466*	*.1906*	*.0810*
		.9796	***.9908***	***.9894***	***.9916***	***.9534***
		.9780	***.9902***	***.9892***	***.9904***	***.9560***
		.9804	***.9900***	***.9892***	***.9914***	***.9558***
100		.1874	.2412	.2280	.1754	.0072
		.9998	**.9998**	**.9998**	**.9992**	**.9564**
		1.000	**1.000**	**1.000**	**1.000**	**.9558**
		1.000	**1.000**	**.9998**	**.9998**	**.9590**
		.0876	*.1620*	*.1540*	*.0900*	*0*
		.9992	***.9998***	***.9998***	***.9994***	*.0084*
		.9986	***.9998***	***1.000***	***.9994***	*.0078*
		.9998	***1.000***	***.9998***	***.9992***	*.0080*
150		.1154	.1712	.2168	.1746	.0860
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
		.0418	*.0886*	*.1346*	*.0900*	*.0218*
		1.000	***1.000***	***1.000***	***1.000***	***.9994***
		1.000	***.9998***	***1.000***	***1.000***	***.9996***
		.9998	***1.000***	***1.000***	***1.000***	***.9998***
200		.0998	.1128	.1870	.0816	.0004
		1.000	**1.000**	**1.000**	**1.000**	**.9956**
		1.000	**1.000**	**1.000**	**1.000**	**.9982**
		1.000	**1.000**	**1.000**	**1.000**	**.9976**
		.0328	*.0374*	*.1006*	*.0202*	*0*
		1.000	***1.000***	***1.000***	***1.000***	*.0302*
		1.000	***1.000***	***1.000***	***1.000***	*.0304*
		1.000	***1.000***	***1.000***	***1.000***	*.0308*

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

As for the idea that SCs based upon pooled within group variance ($\mathbf{s_W}$) might outperform those based on total variance ($\mathbf{s_T}$) in the presence of significant differences among group means (Dalgliesh, 1994; Huberty, 1975), the results of this study indicate that the opposite is true. For two-group MANOVA where the MANOVA was significant and subsequent descriptive discriminant analysis (DDA) conducted, $\mathbf{s_T}$ consistently outperformed $\mathbf{s_W}$. However, when the rule fit well (SC vector proportions fitting the rule $\geq$ .8), differences between $\mathbf{s_T}$ and $\mathbf{s_W}$ proportions were minimal. This minimal difference when the rule fits well is important given that SPSS DDA output includes $\mathbf{s_W}$ coefficients and not $\mathbf{s_T}$. If the conditions the researcher is testing are conditions where SC rule of thumb fit is high, the researcher using SPSS for DDA need not be concerned about not having $\mathbf{s_T}$ coefficients available.

For vectors containing either all small, medium or large effects, there was a three-way interaction such that as group sample size, *n*, and *p* variable intercorrelation, **R**, increases for a *p* x 1 vector of effects, the proportion of vectors fitting the rule increases. However, as the number of continuous variables, *p*, increases, the proportion of vectors fitting the rule decreases. This may be an issue of power; as the number of *p* variables increases, multivariate power generally decreases (Stevens, 2002).

Table 18. Proportions of Elements Fitting the Rule for the *p* = 5 Vector in Table 14 Population Effects: Two Small, One Medium, Two Large Effects, Respectively

n	*R*	.00 - .20	.21 - .40	.41 - .60	.61 - .80	.81 - .99
50		.3098	.3270	.2794	.1978	.0470
		.3010	.3334	.2860	.1964	.0452
		.8294	**.8458**	**.8314**	.7452	.4154
		.9896	**.9768**	**.9906**	**.9866**	**.8960**
		.9904	**.9766**	**.9902**	**.9880**	**.8980**
		.2206	*.2546*	*.1956*	*.0906*	*0*
		.2162	*.2506*	*.2000*	*.0940*	*0*
		.7626	*.7980*	*.7500*	*.5602*	*.0012*
		.9872	***.9762***	***.9868***	***.9650***	*.0774*
		.9884	***.9766***	***.9860***	***.9676***	*.0758*
100		.2096	.2676	.2348	.2110	.1086
		.2140	.2668	.2452	.2104	.1186
		.9168	**.9424**	**.9290**	**.9182**	**.8294**
		1.000	**1.000**	**.9998**	**.9998**	**.9994**
		1.000	**1.000**	**.9998**	**.9998**	**.9998**
		.1214	*.1912*	*.1454*	*.1186*	*.0302*
		.1200	*.1876*	*.1480*	*.1218*	*.0326*
		.8482	***.9096***	***.8776***	***.8466***	*.6020*
		.9998	***1.000***	***.9996***	***.9996***	***.9956***
		1.000	***1.000***	***.9996***	***.9996***	***.9960***
150		.1634	.2440	.1882	.2060	.0038
		.1608	.2430	.1980	.2032	.0028
		.9574	**.9776**	**.9628**	**.9752**	.4410
		1.000	**1.000**	**1.000**	**1.000**	**.9916**
		1.000	**1.000**	**1.000**	**1.000**	**.9926**
		.0810	*.1584*	*.1040*	*.1268*	*0*
		.0740	*.1562*	*.1130*	*.1222*	*0*
		.8970	***.9560***	***.9232***	***.9430***	*.0006*
		1.000	***1.000***	***1.000***	***1.000***	*.1784*
		1.000	***1.000***	***1.000***	***1.000***	*.1862*
200		.1392	.1836	.1470	.1056	.0160
		.1338	.1922	.1494	.1042	.0166
		.9792	**.9864**	**.9808**	**.9678**	**.8188**
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
		1.000	**1.000**	**1.000**	**1.000**	**1.000**
		.0624	*.1032*	*.0680*	*.0388*	*.0002*
		.0612	*.1036*	*.0740*	*.0344*	*.0002*
		.9424	***.9678***	***.9520***	***.9100***	*.3130*
		1.000	***1.000***	***1.000***	***1.000***	***.9956***
		1.000	***1.000***	***1.000***	***1.000***	***.9964***

Note: S_T are in Roman font, and S_W are italicized. Proportions $\geq$ .8 are in bold.

For vectors with mixed medium and large effects, the *n* x **R** interaction was also evident, but not for the highest level of intercorrelation, **R** = .81 - .99. For these mixed vectors, the fit of the rule dropped for certain group sample sizes *n* when intercorrelation was highest. Furthermore, the drop was more pronounced for $\mathbf{s_W}$ than $\mathbf{s_T}$. Proportions fitting the rule for mixed vectors containing small effects appeared less stable at high intercorrelation, with drops evident for certain *n* cells with intercorrelation **R** = .61 - .99 (e.g., Table 12). However, as previously noted, proportions of vectors fitting the rule were low where mixed vectors included small effects. Thus, the rule of thumb works better where *p* variable intercorrelation is low to moderate. If a researcher reduces collinearity in the continuous variable set in order to achieve a more parsimonious model for conducting MANOVA (Stevens, 2002), then the researcher might still confidently apply the rule of thumb to a *post hoc* DDA, provided that anticipated effects are medium and/or large.

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