Interaction Effects: Centering, Variance Inflation Factor, and Interpretation Issues

Cecil Robinson **Randall E. Schumacker**
University of Alabama

Research hypotheses that include interaction effects should be of more interest to educational researchers, especially since issues related to centering and interpretation of the variance inflation factor have been introduced. The purpose of this paper was to examine interaction effects in the context of centered versus uncentered variables and the variance inflation factor, especially upon the interpretation of interaction effects. Results indicated that centering of variables was required when examining interaction effects, uncentered variables impacted the variance inflation factor values, and separate regression equations have important interpretation outcomes in the presence of non-significant interaction effects.

Historically, hypotheses that specify testing interaction effects before examining main effects have appeared under the framework of analysis of variance. In the 1960's with the emergence of multiple regression, coding for interaction effects was introduced. Faculty who taught multiple regression therefore usually included instruction on dummy coding to obtain a test of interaction effects (Fox, 1997). Today, depending upon the textbook used, analysis of variance with A x B interaction effect may be covered without any corresponding interaction effect presentation given for multiple regression (Hinkle, Wiersma, & Jurs, 1998).

Much of the published research literature seems to only examine main effects or linear effects. In practice, A x B interactions are only found in a few published journal articles, A x B x C interactions, are less common, and A x B x C x D interactions are even more scarce. Only a few 5-way interactions have ever been published. The reason is that such higher level interaction effects are extremely difficult to interpret. Interaction effects that are categorical in nature, involve multiplicative continuous variables, or hypothesize quadratic or cubic terms are rare (Schumacker & Marcoulides, 1998).

Research hypotheses that include interaction effects should be of more interest to educational researchers, especially since issues related to centering (Aiken & West, 1991) and interpretation of the variance inflation factor (Freund, Littell, and Creighton, 2003) have been introduced. The purpose of this paper is to examine interaction effects in the context of centered versus uncentered variables and the variance inflation factor, especially upon the interpretation of interaction results.

Theoretical Framework

The effects of predictor scaling on coefficients of regression equations (centered versus uncentered solutions and higher order interaction effects (3-way interactions; categorical by continuous effects) has thoughtfully been covered by Aiken and West (1991). Their example illustrates that considerable multicollinearity is introduced into a regression equation with an interaction term when the variables are not centered. The variance inflation factor should detect the degree of multicollinearity when variables are uncentered (Freund, Little, & Creighton, 2003). The variance inflation factor as a measure of the degree of multicollinearity however has not been examined in context with centered versus uncentered variables in a regression equation containing interaction effects.

Centering

Centering is defined as subtracting the mean (a constant) from each score, X, yielding a centered score. Aiken & West (1991) demonstrated that using other transformations, additive constant, or uncentered scores can have a profound effect on interaction results. Regression with higher order terms has covariance between interaction terms (XZ) and each component (X and Z) depends in part upon the means of the individual predictors. Rescaling, changes the means, thus changes the predictor covariance, yielding different regression weights for the predictors in the higher order function. Centering is therefore an important step when testing interaction effects in multiple regression to obtain a meaningful interpretation of results.

Centering the variables places the intercept at the means of all the variables. A regression equation with an intercept is often misunderstood in the context of multicollinearity. The intercept is an estimate of the response at the origin where all independent variables are zero, thus inclusion of the intercept in the

study of collinearity is not of much interest. When variables have been centered, the intercept has no effect on the collinearity of the other variables (Belsley, Kuh, and Welsch, 1980).

Centering is also consistent with the computation of the variance inflation factor (VIF) and therefore it is suggested that VIF be computed only after first centering variables (Freund, Littell, and Creighton, 2003). Centered variables have low intercorrelation, while uncentered variables have higher intercorrelation, thus higher collinearity. The variance inflation factor is therefore an important part of examining interaction effects in multiple regression.

Variance Inflation Factor

When a full regression model is specified, multicollinearity amongst the predictor variables is possible. Multicollinearity can inflate the variance amongst the variables in the model. These inflated variances are problematic in regression because some variables add very little or even no new and independent information to the model (Belsley, Kuh & Welsch, 1980). Although Schroeder, Sjoquist and Stephen (1986) assert that there is no statistical test that can determine whether or not multicollinearity is a problem, there are ways for detecting multicollinearity (Berry & Feldman, 1985). For example, the variance inflation factor (VIF) can detect the degree of multicollinearity when variables are uncentered (Freund, Littell & Creighton, 2003). Stine (1985) also suggested a graphical approach to detecting VIF.

VIF measures the impact of multicollinearity among the X's in a regression model on the precision of estimation. It expresses the degree to which multicollinearity amongst the predictors degrades the precision of an estimate. VIF is a statistic used to measure possible multicollinearity amongst the predictor or explanatory variables. VIF is computed as $(1/(1-R^2))$ for each of the k – 1 independent variable equations. For example, given 4 independent predictor variables, the independent regression equations are formed by using each k-1 independent variable as the dependent variable:

$$X_1 = X_2\ X_3\ X_4$$
$$X_2 = X_1\ X_3\ X_4$$
$$X_3 = X_1\ X_2\ X_4$$

Each independent variable model will return an R^2 value and VIF value. The term to exclude in the model is then based on the value of VIF. If X_j is highly correlated with the remaining predictors, its variance inflation factor will be very large. A general rule is that the VIF should not exceed 10 (Belsley, Kuh, & Welsch, 1980). When X_j is orthogonal to the remaining predictors, its variance inflation factor will be 1.

Method and Procedures

The rationale for the data analysis was that three concepts: self-efficacy (Bandura 1997), hope (Snyder, 1995) and optimism (Scheier & Carver, 1992); comprise a cognitive set that form a belief system which influenced academic achievement. We hypothesized an interaction effect between ethnicity and each predictor variable: self-efficacy, hope, and optimism in predicting academic achievement. The self-efficacy, hope, and optimism variables were centered and uncentered for comparison purposes. In addition, the variance inflation factor was calculated to determine the degree of multicollinearity present in the data results.

Participants

High school students (N = 209) from an ethnically diverse, working-class public high school in the southeast United States participated in this study. The ethnic breakdown of the participants was 105 African-American and 104 Caucasian American.

Materials

Students reported their gender, ethnicity, age, year in school, academic achievement (1 = mostly A's to 6 = half C's or lower), and educational goals (1 = not important to 8 = very important). The academic hope, academic self-efficacy, and optimism variables represented a cognitive set of competence measures in predicting academic achievement among this diverse high school population.

The *Academic Hope Scale* (AHS) measured academic hope components and is a sub-scale from the Domain Specific Hope Scale-Revised which measures hope in life areas including social relationships, family life, physical health, psychological health, work, romantic relationships, leisure activities, and religions/spiritual life with moderately high score reliability of .89 and above (Campbell & Kwon, 2001).

The *Academic Self-Efficacy Scale* (ASES) measured student beliefs about how they react to different academic tasks to succeed in academic achievement. The ASES is an excerpt from Bandura's Multidimensional Self-Efficacy Scale and has moderately high score reliability ranging from .69 to .85 (Zimmerman, Bandura, & Martinez-Pons, 1992).

The *Life Orientation Test* (LOT) measured dispositional optimism, or one's expectancies that he or she will experience positive outcomes with score reliability of .76 (Scheier & Carver, 1985).

Results

A multiple regression analysis was conducted using SPSS 16.0 with centered and uncentered variables. The interaction effects along with the variance inflation factors are reported separately.

Hypothesis 1: Is there a statistically significant interaction between academic hope and ethnicity in predicting academic achievement?

Hypothesis 2: Is there a statistically significant interaction between academic self efficacy and ethnicity in predicting academic achievement?

Hypothesis 3: Is there a statistically significant interaction between optimism and ethnicity in predicting academic achievement?

The results of all three hypotheses clearly indicated the importance of centering variables when including an interaction term, as noted by Aiken and West (1991). More importantly, the variance inflation factor was also affected if variables were not centered; thus falsely indicating multicollinearity.

Interpretation of Interaction Effects

Interpretation of results would be erroneous for interaction effects with uncentered variables. Centering reduces VIF to acceptable levels with academic hope interaction significant, but academic self-efficacy and optimism interaction not significant. We provide separate regression results for each ethnic group for interpretation of the interaction effects using centered variables. To compute the separate regression models, academic hope, self-efficacy and optimism scores were centered on the means for each of the ethnic groups. To compare and interpret the results of the separate regression models, an F-value was computed to test for differences between the separate regression models, and a t-value was computed to test for differences between the separate regression coefficients (Kleinbaum & Kupper, 1978).

The results in Table 4 demonstrate that separate regression models provide clarity of interpretation that single regression models with interaction term may not provide. First, although the results of Academic Hope confirm the interaction effect between ethnicity and Academic Hope (Table 1), the separate regression models highlight a magnitude of the interaction effect not present in the single regression model. Namely, Academic Hope explains nine times more variance in the Caucasian American student population than the African-American student population. Second, the difference in variance between ethnic groups is not examined in the single model with the interaction term, and as such may introduce Type II error. Although the interaction terms were not significant for Academic Self-Efficacy (Table 2) or Optimism (Table 3) in the single regression models, testing for differences in the amount of variance accounted for by each of the separate regression models reveals significant differences between Caucasian American and African-American students. Therefore, significant differences existed for academic self-efficacy and optimism not captured by the interaction effect in the single regression equation.

Summary and Conclusion

Historically, interaction effects and main effects were conducted using analysis of variance. In this context, a summary table reported the F values with a significant interaction effect being plotted to visually display an ordinal or disordinal interaction amongst the cell means. Multiple regression can also analyze various types of interaction effects, but how interaction effects are computed is important. We strongly recommend that variables be centered and the variance inflation factor reported otherwise erroneous results could occur and be misinterpreted.

Table 1. *Uncentered and Centered Regression Models of Academic Hope and Ethnicity Predicting Grade Point Average*

	B	*SE B*	*B*	*t*	*p*	*VIF*
Uncentered Regression Model						
Intercept	0.804	0.351		2.291	0.023	
Ethnicity	1.266	0.582	0.844	2.175	0.031	37.064
Academic Hope Scale (AHS)	0.057	0.009	0.507	6.168	0.000	1.668
Ethnicity x AHS	-0.036	0.015	-0.978	-2.453	**0.015**	**39.198**
Centered Regression Model						
Intercept	2.988	0.068		43.633	0.000	
Ethnicity	-0.134	0.097	-0.089	-1.379	0.170	1.027
Academic Hope Scale (AHS)	0.057	0.009	0.507	6.168	0.000	1.668
Ethnicity x AHS	-0.036	0.015	-0.201	-2.453	**0.015**	**1.653**

Note: R^2 = 0.168; F (3, 205) = 13.802, p = 0.000 for both uncentered and centered regression models

Table 2. *Uncentered and Centered Regression Models of Academic Self-Efficacy and Ethnicity Predicting Grade Point Average*

	B	*SE B*	*β*	*t*	*p*	*VIF*
Uncentered Regression Model						
Intercept	1.135	0.368		3.083	0.002	
Ethnicity	0.672	0.558	0.447	1.204	0.230	32.654
Academic Self Efficacy (ASE)	0.031	0.006	0.436	4.956	0.000	1.831
Ethnicity x ASE	-0.013	0.009	-0.535	-1.403	**0.162**	**34.380**
Centered Regression Model						
Intercept	2.957	0.069		42.590	0.000	
Ethnicity	-0.098	0.098	-0.065	-0.998	0.320	1.008
Academic Self Efficacy (ASE)	0.031	0.006	0.436	4.956	0.000	1.831
Ethnicity x ASE	-0.013	0.009	-0.123	-1.403	**0.162**	**1.825**

Note: R^2 = 0.133; F (3, 205) = 10.51, p =0 .000 for both uncentered and centered regression models

Table 3. *Uncentered and Centered Regression Models of Optimism and Ethnicity Predicting Grade Point Average*

	B	*SE B*	*β*	*t*	*p*	*VIF*
Uncentered Regression Model						
Intercept	1.758	0.383		4.591	0.000	
Ethnicity	0.223	0.538	0.149	0.415	0.679	28.346
Optimism (Opt)	0.044	0.014	0.300	3.108	0.002	2.061
Ethnicity x Opt	-0.011	0.020	-0.201	-0.547	**0.585**	**29.795**
Centered Regression Model						
Intercept	2.936	0.072		40.972	0.000	
Ethnicity	-0.066	0.101	-0.044	-0.654	0.514	1.001
Optimism (Opt)	0.044	0.014	0.300	3.108	0.002	2.061
Ethnicity x Opt	-0.011	0.020	-0.053	-0.547	0.**585**	**2.059**

Note: R^2 = 0.071; F (3, 205) = 5.247, p = 0.002 for both uncentered and centered regression models

Although multiple regression can analyze interaction effects, our results demonstrate that interaction terms not significant in regression models can produce significantly different regression models when computed separately. Therefore, we also suggest that separate regression equations be computed for each level of the interaction variable to provide a more robust interpretation of the interaction effect. This is a different research question than testing for an interaction effect, but traditional research methods dictate that non-significant interaction does not warrant further exploration, however, our results suggest otherwise.

Table 4. *Summary of Separate Regression Analyses Predicting Grade Point Average*

	African-American (n = 105)					Caucasian American (n = 104)				
	b	*SE b*	*β*	*t*	*p*	*b*	*SE b*	*β*	*t*	*p*
Academic Hope										
Intercept	2.855	0.069		41.557	0.000	2.988	0.068		43.725	0.000
Academic Hope Scale	0.020	0.012	0.170	**1.750**	**0.083**	0.057	0.009	0.522	**6.181**	**0.000**
Note: R^2 = 0.029; F(1,103) = 3.1, p = 0.08 for African-American students; R^2 = 0.272; F(1,102) = 38.2, p =0.0001 for Caucasian American students; Comparison of regression models, ***F* (1, 207) = 68.57, *p* < 0.001**; Comparison of regression coefficients, t = 3.7, p < 0.001										
Academic Self-Efficacy										
Intercept	2.859	0.067		42.930	0.000	2.957	.072		41.113	0.000
Academic Self-Efficacy	0.018	0.007	0.256	**2.687**	**0.008**	0.031	.006	0.428	**4.784**	**0.000**
Note: R^2 = 0.065; F (1,103) = 7.22, p = 0.008 for African-American students; R^2 = 0.183; F (1,102) = 22.89, p =0.0001 for Caucasian American students; Comparison of regression models, ***F* (1, 207) = 29.75, *p* <0 .001**; Comparison of regression coefficients, t = 2.0, p < 0.05										
Optimism										
Intercept	2.870	0.067		43.152	0.000	2.936	0.076		38.518	0.000
Optimism	0.033	0.013	0.247	**2.586**	**0.011**	0.044	0.015	0.278	**2.922**	**0.004**
Note: R^2 = 0.048; F (1,103) = .24, p = 0.63 for African-American students; R^2 = 0.124; F (1,102) = 1.58, p =0.21 for Caucasian American students; Comparison of regression models, ***F* (1, 207) = 17.87, *p* <0 .001**; Comparison of regression coefficients, t = .76, p > 0.10										

References

Aiken, L.S. & West, S.G. (1991). *Multiple Regression: Testing and Interpreting Interactions*. Sage Publications: Thousand Oaks, CA.

Bandura, A. (1997). *Self-efficacy: The Exercise of Self-Control.* New York: Freeman.

Belsley, D. A., Kuh, E. & Welsch, R. E. (1980). *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*. New York: John Wiley.

Berry, W. D. & Feldman, S. (1985). *Multiple Regression in Practice*. London: Sage Publications.

Campbell, D.G. & Kwon, P. (2001). Domain-specific hope and personal style: Toward an integrative understanding of dysphoria. *Journal of Social and Clinical Psychology, 20*(4), 498-520.

Fox, J. (1997). *Applied Regression Analysis, Linear Models, and Related Methods*. Sage Publications: Thousand Oaks, CA.

Freund, R.J., Littell, R.C., and Creighton, L. (2003). *Regression Using JMP*. Cary, NC: SAS Institute, Inc.

Hinkle, D.E., Wiersma, W., & Jurs, S.G. (1998). *Applied Statistics for the Behavioral Sciences, 4th Edition.* Houghton Mifflin Company: NY.

Kleinbaum, D. G., & Kupper, L. L. (1978). *Applied Regression Analysis and Other Multivariable Methods*. North Scituate, MA: Duxbury Press.

Scheier, M.F., & Carver, C.S. (1985). Optimism, coping, and health: Assessment and implications of generalized outcome expectancies. *Health Psychology, 4,* 219-247.

Scheier, M.F., & Carver, C.S. (1992). Effects of optimism or psychological and physical well-being: Theoretical overview and empirical update. *Cognitive Therapy and Research, 16,* 201-228

Schroeder, L. D., Sjoquist, D. L. & Stephan, P. E. (1986). *Understanding Regression Analysis.* Beverly Hills, CA: Sage Publications.

Schumacker, R.E. & Marcoulides, G.A. (1998). *Interaction and Nonlinear Effects in Structural Equation Modeling*. Erlbaum Associates: Mahway, NJ.

Snyder, C. R. (1995). Conceptualizing, Measuring, and Nurturing Hope. *Journal of Counseling & Development, 73*, 355-360.

Stine, Robert, A. (1995, February). Graphical interpretation of variance inflation factors. *The American* Statistician*, 49*(1), 53-56.

Zimmerman, B., Bandura, A., & Martinez-Pons, M. (1992). Self-motivation for academic attainment: The role of self-efficacy beliefs and personal goal setting. *American Educational Research Journal, 29,* 663-676.

Send correspondence to: Randall E. Schumacker
University of Alabama
Email: rschumacker@ua.edu