Regression as the Univariate General Linear Model: Examining Test Statistics, p values, Effect Sizes, and Descriptive Statistics Using R

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This paper presents regression as the univariate general linear model (GLM). Building on the work of Cohen (1968), McNeil (1974), and Zientek and Thompson (2009), the paper uses descriptive statistics to build a small, simulated dataset that readers can use to verify that multiple linear regression (MLR) subsumes the univariate parametric analyses in the GLM. Unlike other related works, we provide R syntax that demonstrates how MLR produces equivalent test statistics, *p* values, effect sizes, and descriptive statistics when compared to the univariate analyses that MLR subsumes. The paper diverges from Zientek and Thompson by presenting an expanded hierarchy for MLR and demonstrating why only the case of the chi-square test of independence where the criterion variable is dichotomous, and not the general case, is subsumed by MLR. Readers will find an accessible treatment of the GLM as well as R syntax, which they can use to report descriptive statistics, *p* values, and effect sizes associated with the univariate parametric statistics in the GLM.

In 1968, Cohen presented multiple linear regression (MLR) as the univariate general linear model (GLM). Since that time, Cohen's work has been extended to consider canonical correlation as the multivariate GLM (see Knapp, 1978) and structural equation modeling as an even more general case of the GLM (see Bagozzi, Fornell, & Larcker, 1981). As noted by Graham (2008),

> The vast majority of parametric statistical procedures in common use are part of [a single analytic family called] the General Linear Model (GLM), including the *t* test, analysis of variance (ANOVA), multiple regression, descriptive discriminant analysis (DDA), multivariate analysis of variance (MANOVA), canonical correlation analysis (CCA), and structural equation modeling (SEM). Moreover, these procedures are hierarchical in that some procedures are special cases of others. (p. 485).

In addition to the hierarchical nature of the GLM is the concept that the subsumed analyses share three characteristics. Analyses in the GLM implicitly or explicitly are correlational in nature, yield variance accounted effect sizes, and produce scores on latent variables that are derived by applying weights to measured variables (Thompson, 2006, p. 360).

Although the characteristics of the GLM seem to be straightforward, graduate students and emerging scholars are likely to benefit from being able to verify the hierarchical nature of the GLM through illustrations that compare univariate statistical analyses to MLR analyses. Not only has active learning been shown to be beneficial when learning statistics (White, 2015), research (e.g., Henson, Hull, & Williams, 2010) indicates that many graduate students and emerging scholars may have insufficient quantitative proficiency. Therefore, we offer an illustration of MLR as the univariate GLM that considers the similarities and differences in the test statistics, *p* values, effect sizes, and descriptive statistics generated. Namely, we consider ANCOVA, ANOVA, *r*, repeated measures ANOVA (RM ANOVA), independent samples *t*-test, paired-samples *t*-test, and single-sample *t*-test. Our interest in developing this work is similar to other methodologists who seek to "improve statistical practice, and thereby, improve the quality of the knowledge produced by the legions of researchers around the world who use these techniques on a daily basis" (Osborne, 2013, p. 1).

We also make five unique contributions to the literature. We demonstrate MLR as the univariate GLM for parametric analyses using R, which is a free statistical programming language that is gaining popularity in social science research and that is compatible with Unix, Windows, and Mac operating systems (R Development Core Team, 2017). Prior contributions (e.g., Zientek & Thompson, 2009) have used commercial statistical software packages (e.g., SPSS). Second, we demonstrate that the hierarchical

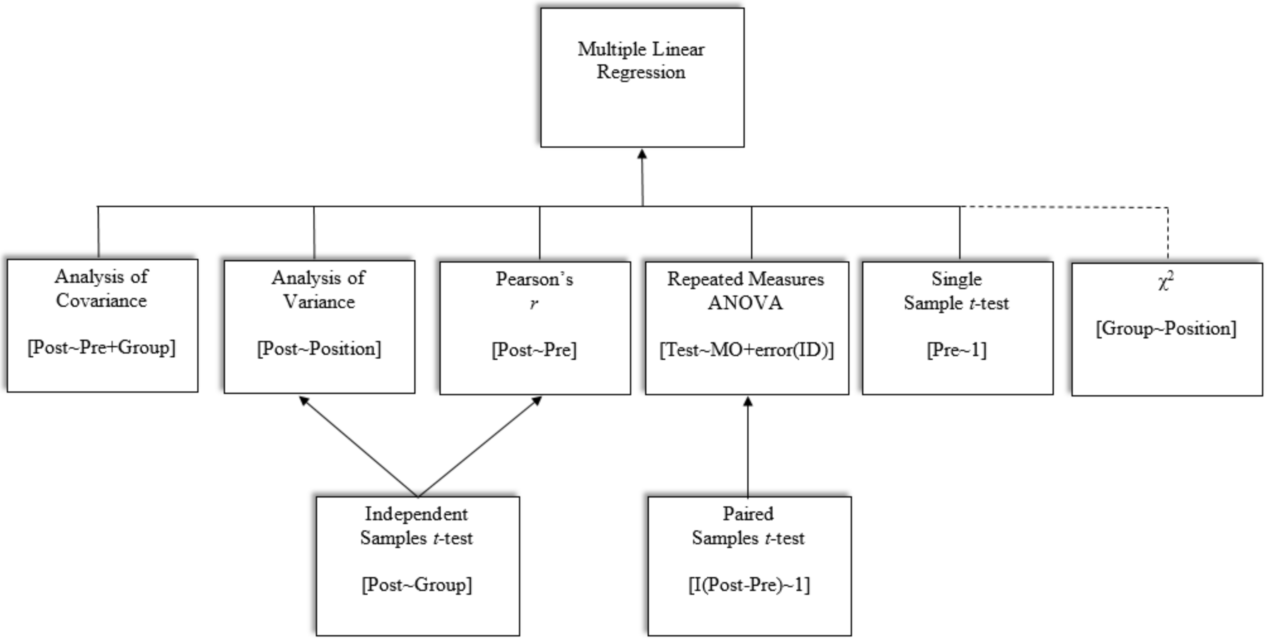

Figure 1. Multiple linear regression (MLR) as the univariate general linear model. Dotted line indicates that χ^2 test of independence is only assumed by MLR in the case of a dichotomous dependent variable. Illustrative models identified in [ ]. See formula in stats package (R Development Core Team, 2017) for formatting of model formulae.

nature of the univariate parametric statistical analyses is not as flat as portrayed in Zientek and Thompson (p. 344). Namely, we show that ANOVA and *r* subsume the independent samples *t*-test. Not only is it important to show that these analyses (i.e., ANOVA, *r*, and independent samples *t*-test) are mathematically equivalent, demonstrating that *r* subsumes the independent samples *t*-test helps undo the misconception that correlation never implies causality and that causality is a function of design, not statistics (cf. Huck, 2012). Third, we demonstrate that RM ANOVA is subsumed by MLR and subsumes the paired-samples *t*-test. Fourth, we demonstrate that the single-sample *t*-test is subsumed by MLR. Finally, we demonstrate why the general case for the chi-square test of independence cannot be subsumed by MLR and that only in the case of a dichotomous dependent variable does MLR subsume the chi-square test. Therefore, the hierarchy of analyses subsumed by MLR presented in Figure 1, which serves as a framework for our paper, diverges from the hierarchy presented by Zientek and Thompson (p. 344) in important ways.

Method

The syntax in Appendix A was used to generate the datasets in Tables 1 and 2 that serve as the basis of the analyses illustrated. The dataset contains four variables: pretest scores (Pre), posttest scores (Post), follow-up scores (FollowUp), group assignment (Control, Treatment) and position (full-time [Full], part-time [Part], seasonal [Seasonal]). The dataset was designed so that each group has equal variances (SD = 1) and equal covariances (rs = 0.6) between the pretest, posttest, and follow-up scores to satisfy statistical assumption in the illustrated analyses. In both groups, the mean pretest score is 4.0. In the control and treatment groups, the mean posttest score is, respectively, 4.0 and 6.0. In the control and treatment groups, the mean follow-up score is, respectively, 4.0 and 5.5. Table 1 was used as input to all of the analyses with the exception of the RM ANOVA analyses, where Table 2 was used. Table 1 is considered the wide representation of the data as each repeated measure (i.e., Pre, Post, and FollowUp) is represented in a separate column. Table 2 is considered the long representation of the data as the three repeated measures are contained in one column (Test), with a corresponding column that indicates the particular measurement occasion (MO), where 1, 2, and 3, respectively, indicate pretest, posttest, and follow-up.

ANCOVA. An ANCOVA was run with posttest scores, pretest scores, and group assignment, respectively, serving as the dependent, covariate, and independent variable. Pretest scores were centered at the group mean in order to have a meaningful intercept. A linear model with and without the covariate was analyzed and then compared with ANOVA to facilitate an ANCOVA analysis in R (cf. Crawley, 2013). In the MLR model, posttest scores were regressed on the pretest scores and the grouping variable. The ANCOVA models tested the hypothesis that group had a statistically and practically significant effect on posttest scores after controlling for pretest scores. Test statistics, *p* values, effect sizes, and adjusted group means were compared between the results of the two analyses.

ANOVA. A MLR and ANOVA were run with posttest scores and position, respectively, serving as the dependent and independent variables. The ANOVA models tested the hypothesis that there was a statistically and practically significant difference in posttest scores by position. Test statistics, *p* values, effect sizes, and group means were compared between the results of the two analyses.

r. A MLR and *r* were run with posttest and pretest scores, respectively, serving as the dependent and independent variables. The *r* models tested the hypothesis that there was a statistically and practically significant relationship between posttest and pretest scores. Test statistics, *p* values, and effect sizes were compared between the results of the two analyses.

RM ANOVA. A MLR and RM ANOVA were run testing the hypotheses that pretest, posttest, and follow-up scores are statistically and practically different. For MLR, test scores (see Table 2) were modeled by measurement occasion (i.e., 1=pretest, 2=posttest, 3=measurement occasion) and participant ID. For RM ANOVA, test scores were modeled by measurement occasion and individual error (cf. Fox & Weisberg, 2011). Test statistics, *p* values, effect sizes, and measurement occasion means were compared between the results of the two analyses.

Table 1. Simulated Dataset - Wide

id	Pre	Post	FollowUp	Group	Position
1	4.30	5.83	5.12	Control	Full
2	3.89	3.69	4.39	Control	Full
3	3.81	2.97	4.59	Control	Full
4	5.59	5.42	3.96	Control	Full
5	1.27	2.84	1.97	Control	Part
6	3.22	3.06	3.55	Control	Part
7	4.37	3.04	2.82	Control	Part
8	4.29	3.62	3.48	Control	Part
9	3.49	4.41	3.84	Control	Part
10	4.68	4.48	4.24	Control	Part
11	5.07	5.55	6.26	Control	Seasonal
12	3.64	4.15	3.07	Control	Seasonal
13	4.92	4.11	4.12	Control	Seasonal
14	3.56	3.99	4.24	Control	Seasonal
15	3.91	2.85	4.35	Control	Full
16	4.50	6.23	5.40	Treatment	Full
17	5.14	7.08	7.52	Treatment	Full
18	3.72	6.92	5.33	Treatment	Full
19	3.48	5.76	4.32	Treatment	Full
20	3.55	6.11	5.19	Treatment	Full
21	5.92	6.69	5.96	Treatment	Part
22	2.83	6.17	4.34	Treatment	Part
23	4.23	7.18	6.75	Treatment	Part
24	5.04	6.21	5.83	Treatment	Part
25	4.47	4.97	4.59	Treatment	Seasonal
26	3.62	4.63	4.64	Treatment	Seasonal
27	2.42	4.07	4.67	Treatment	Seasonal
28	3.39	6.40	4.97	Treatment	Seasonal
29	2.75	4.56	5.93	Treatment	Seasonal
30	4.95	7.01	7.06	Treatment	Seasonal

Independent Samples *t*-test. A MLR, ANOVA, *r*, and independent samples *t*-test were run with posttest scores and group, respectively, serving as the dependent and independent variables. The numeric representation of group served as the independent variable for *r*. The independent samples *t*-test models tested the hypotheses that there was a statistically and practically significant mean difference in posttest scores by group. Test statistics, *p* values, effect sizes, and group means were compared among the results of the four analyses.

Paired-Samples *t*-test. A MLR, RM ANOVA, and paired-samples *t*-test were run testing the hypotheses that posttest scores are statistically and practically different than pretest scores. For MLR, the difference between posttest and pretest scores served as the dependent variable, which was modeled only by the intercept. For RM ANOVA, test scores were modeled by measurement occasion (i.e., 1=pretest,

Table 2. Simulated Dataset – Long

id	Group	Position	MO	Test
1	Control	Full	1	4.30
2	Control	Full	1	3.89
3	Control	Full	1	3.81
4	Control	Full	1	5.59
5	Control	Part	1	1.27
6	Control	Part	1	3.22
7	Control	Part	1	4.37
8	Control	Part	1	4.29
9	Control	Part	1	3.49
10	Control	Part	1	4.68
11	Control	Seasonal	1	5.07
12	Control	Seasonal	1	3.64
13	Control	Seasonal	1	4.92
14	Control	Seasonal	1	3.56
15	Control	Full	1	3.91
16	Treatment	Full	1	4.50
17	Treatment	Full	1	5.14
18	Treatment	Full	1	3.72
19	Treatment	Full	1	3.48
20	Treatment	Full	1	3.55
21	Treatment	Part	1	5.92
22	Treatment	Part	1	2.83
23	Treatment	Part	1	4.23
24	Treatment	Part	1	5.04
25	Treatment	Seasonal	1	4.47
26	Treatment	Seasonal	1	3.62
27	Treatment	Seasonal	1	2.42
28	Treatment	Seasonal	1	3.39
29	Treatment	Seasonal	1	2.75
30	Treatment	Seasonal	1	4.95
1	Control	Full	2	5.83
2	Control	Full	2	3.69
3	Control	Full	2	2.97
4	Control	Full	2	5.42
5	Control	Part	2	2.84
6	Control	Part	2	3.06
7	Control	Part	2	3.04
8	Control	Part	2	3.62
9	Control	Part	2	4.41
10	Control	Part	2	4.48
11	Control	Seasonal	2	5.55
12	Control	Seasonal	2	4.15
13	Control	Seasonal	2	4.11
14	Control	Seasonal	2	3.99
15	Control	Full	2	2.85
16	Treatment	Full	2	6.23
17	Treatment	Full	2	7.08
18	Treatment	Full	2	6.92
19	Treatment	Full	2	5.76
20	Treatment	Full	2	6.11
21	Treatment	Part	2	6.69
22	Treatment	Part	2	6.17
23	Treatment	Part	2	7.18
24	Treatment	Part	2	6.21
25	Treatment	Seasonal	2	4.97
26	Treatment	Seasonal	2	4.63
27	Treatment	Seasonal	2	4.07
28	Treatment	Seasonal	2	6.40
29	Treatment	Seasonal	2	4.56
30	Treatment	Seasonal	2	7.01
1	Control	Full	3	5.12
2	Control	Full	3	4.39
3	Control	Full	3	4.59
4	Control	Full	3	3.96
5	Control	Part	3	1.97
6	Control	Part	3	3.55
7	Control	Part	3	2.82
8	Control	Part	3	3.48
9	Control	Part	3	3.84
10	Control	Part	3	4.24
11	Control	Seasonal	3	6.26
12	Control	Seasonal	3	3.07
13	Control	Seasonal	3	4.12
14	Control	Seasonal	3	4.24
15	Control	Full	3	4.35
16	Treatment	Full	3	5.40
17	Treatment	Full	3	7.52
18	Treatment	Full	3	5.33
19	Treatment	Full	3	4.32
20	Treatment	Full	3	5.19
21	Treatment	Part	3	5.96
22	Treatment	Part	3	4.34
23	Treatment	Part	3	6.75
24	Treatment	Part	3	5.83
25	Treatment	Seasonal	3	4.59
26	Treatment	Seasonal	3	4.64
27	Treatment	Seasonal	3	4.67
28	Treatment	Seasonal	3	4.97
29	Treatment	Seasonal	3	5.93
30	Treatment	Seasonal	3	7.06

Note. MO=measurement occasion (1 = Pre; 2 = Post; 3 = FollowUp).

Table 3. Transformation Formulae

Transformation	Formula	Reference
$t \rightarrow F$	t^2	Thompson (2006)
$F \rightarrow t$	$\sqrt{F}$	
$MR^2 \rightarrow r$	$\sqrt{MR^2}$	Thompson (2006)
$\eta^2 \rightarrow r$	$\sqrt{\eta^2}$	
$r \rightarrow R^2$ $r \rightarrow \eta^2$	r^2	
$d \rightarrow r$	$d/\sqrt{d^2 + (N^2 - 2N)/(n_1 n_2)}$	McGrath and Meyer (2006)
$r \rightarrow d$	$\sqrt{-r^2(N^2 - 2N)/(n_1 n_2)(r^2 - 1)}$	
$t_c \rightarrow d_c$	$t_c\sqrt{2(1 - r)/n}$	Dunlap et al. (1996)
$t \rightarrow d$	$t/\sqrt{n}$	Cohen (1988)
$\chi^2 \rightarrow F$	$\chi^2/[(rows - 1) * (columns - 1)]$	Knapp (1978)
$F \rightarrow \chi^2$	$F * (rows - 1) * (columns - 1)$	
$Cramer's\ V \rightarrow MR^2$	$Cramer's\ V^2$	Cohen (1988)
$MR^2 \rightarrow Cramer's\ V$	$\sqrt{MR^2}$	

2=posttest) and individual error (cf. Fox & Weisberg, 2011). For paired-samples *t*-test, the pretest and posttest scores, respectively, served as the independent and dependent variables. The paired-samples *t*-test models tested the hypothesis that there was a statistically and practically significant mean difference between posttest and pretest scores. Test statistics, *p* values, effect sizes, and differences between measurement occasion means were compared among the results of the three analyses.

Single-Sample *t*-test. A MLR and a single-sample *t*-test were run testing the hypotheses that pretest scores are statistically and practically different from 0. For MLR, pretest scores served as the dependent variable, which was modeled only by the intercept. The single-sample *t*-test models tested the hypothesis that the pretest scores were statistically and practically significantly different from 0. Test statistics, *p* values, effect sizes, and means were compared between the results of the two analyses.

χ^2. Two sets of analyses were run using both chi-square test of independence and MLR. In the first set of analyses, position and group, respectively, served as the dependent and independent variables. The first set of analyses tested the hypothesis that group had a statistically and practically significant effect on position. In the second set of analyses, group and position, respectively, served as the dependent and independent variables. The second set of analyses tested the hypothesis that position had a statistically and practically significant effect on group. In both sets of analyses, the numeric representation of the dependent variable was used for MLR. Test statistics, *p* values, and effect sizes were compared between the results of the two analyses.

Results

Appendix B contains the R output that resulted from running the syntax in Appendix A. The following sections reference relevant line numbers in Appendix B when presenting the results for each of the analyses demonstrated. Table 3 provides a consolidation of the formulae used to transform statistics and effect sizes.

ANCOVA. Table 4 and Appendix B (lines 95 – 223) present the results of the ANCOVA analyses. The *p* values for the two analyses (i.e., ANCOVA, MLR) were the same (i.e., 3.22454e-07; see Appendix B, lines 137 – 141). For ANCOVA, the test statistic produced is an *F* statistic, whereas a *t* statistic is produced for the group *b* weight. As t^2 is equal to *F* (Thompson, 2006), the *t* statistic of 6.723161 is equivalent to the *F* statistic of 45.20089 (see Appendix B, lines 143 – 159).

Partial η^2 is the typical effect size reported for ANCOVA, where the variance associated with the covariate (pretest in this case) is excluded from the denominator and only variance associated with the grouping variable (group in this case) is included in the numerator (cf. Thompson, 2006). When using MLR, the partial η^2 can be produced by using commonality analysis coefficients (Zientek, Nimon, &

Brown, 2016), which can be produced in R using the `calc.yhat` function (Nimon, Oswald, & Roberts, 2013). In the two analyses, the effect sizes produced were identical (i.e., .6260434; see Appendix B, lines 161 – 186).

Table 4. ANCOVA Results

ANCOVA		MLR	
Statistic	Value	Statistic	Value
p	<.01	p	<.01
F	45.20	t	6.72
η_p^2	.63	η_p^2	.63
Adjusted $M_{Control}$	4.00	Adjusted $M_{Ccontrol}$	4.00
Adjusted $M_{Treatment}$	6.00	Adjusted $M_{Treatment}$	6.00

In ANCOVA, the group means typically reported are means that have been adjusted for their covariate rather than the observed means (Nimon & Henson, 2015; Tracz, Nelson, Newman, & Beltran, 2005), although in this case there was no difference between observed and adjusted means because the covariate was mean centered. In both analyses, the "adjusted" posttest means were, respectively, 4 and 6 for the control and treatment group. While R provides a function that yields adjusted means via the effect function (Fox, 2003), adjusted means when using MLR require that the intercept and regression weights be used (see Appendix B, lines 188 – 222). In summary, group had a statistically and practically significant effect on posttest scores after controlling for pretest scores (t = 6.72, F [1, 27] = 45.2, p < .01; η_p^2 = .63; Adjusted $M_{Control}$ = 4.0, Adjusted $M_{Treatment}$ = 6.0).

ANOVA. Table 5 and Appendix B (lines 224 − 314) present the results of the ANOVA and MLR. For the two analyses, results yielded the same values for the test statistic (F = .33; see Appendix B, lines 265 – 270), p value (p =. 72; see Appendix B, lines 259 – 263), and effect size ($\eta^2 = MR^2$ = .02; see Appendix B, lines 272 – 276). While the effect size values are identical, the effect size reported for an ANOVA analysis is η^2 and the effect size reported for the MLR analysis is MR^2. As both effect sizes are variance-explained statistics, they indicate how much variance in posttest scores was accounted for by group membership. Group means for each analysis were identical with M_{Full} = 5.29, M_{Part} = 4.77, and M_{Seasonal} = 4.94. Because ANOVA does not provide group means, the values were obtained by calculating descriptive statistics (see Appendix B, lines 278 − 284). For MLR analyses, the group mean values were obtained by using the intercept and regression coefficients (see Appendix B, lines 285 – 314). In summary, there were no statistically or practically significant mean differences in posttest scores by position (F [2, 27] = .33, p = .72; $\eta^2 = MR^2$ = .02; M_{Full} = 5.29, M_{Part} = 4.77, $M_{Seasonal}$ = 4.94).

Table 5. ANOVA Results

ANOVA		MLR	
Statistic	Value	Statistic	Value
p	.72	p	.72
F	.33	F	.33
η^2	.02	MR^2	.02
M_{Full}	5.29	M_{Full}	5.29
M_{Part}	4.77	M_{Part}	4.77
$M_{Seasonal}$	4.94	$M_{Seasonal}$	4.94

***r*.** Table 6 and Appendix B (lines 316 – 405) present the results of the r and MLR analyses. The p values for the two analyses were the same (i.e., 0.02192; see Appendix B, lines 356 – 360). For r, the test statistic produced is a t statistic, whereas an F statistic is produced for the MLR. As t^2 is equal to F (Thompson, 2006), the t statistic of 2.426894 is equivalent to the F statistic of 5.889816 (see Appendix B, lines 362 – 384). The effect size reported for r is the correlation coefficient r, whereas MR^2 is reported for the MLR. As with the test statistic, the r^2 is equal to R^2 (Thompson, 2006). As such, the r of .416885 is equivalent to the MR^2 of .1737931 (see Appendix B, lines 386 – 405). In summary, there was a statistically and practically significant relationship between pretest and posttest scores (t = 2.43, F [1, 28] = 5.89, p = .02; r = .42, MR^2 = .17).

Table 6. *r* Results

r		MLR	
Statistic	Value	Statistic	Value
p	.02	p	.02
t	2.43	F	5.89
r	.42	MR^2	.17

RM ANOVA. Table 7 and Appendix B (lines 407 – 490) present the results of the RM ANOVA and MLR analysis. The p values (p = 3.799596e-05; see Appendix B, lines 436 – 441), test statistics (F = 12.19; see Appendix B, lines 443 – 447), and effect sizes (η_p^2 = .30; see Appendix B, lines 449 – 453) were all identical between the two analyses. In both RM ANOVA and MLR, the partial η^2 is calculated by dividing the amount of variance associated with measurement occasion by the sum of the amount of variance associated with measurement occasion and error (cf. Nimon & Williams, 2009). Group means for each analysis were identical with M_{Pre} = 4.00, M_{Post} = 5.00, and $M_{FollowUp}$ = 4.75. While RM ANOVA does not provide means for each measurement occasion (e.g., pretest, posttest, follow-up), the mean values can be obtained be calculating descriptive statistics (see Appendix B, lines 455 – 460). For the MLR analyses, measurement occasion mean values can be obtained by using the intercept and regression coefficients from a model that regresses the dependent variable on the measurement occasion (see Appendix B, lines 461 – 490). In summary, there were statistically and practically significant mean differences among pretest, posttest, and follow-up scores (F [2, 58] = 12.19, p < .01; η_p^2 = .30; M_{Pre}= 4.0, M_{Post} = 5.0, $M_{FollowUp}$ = 4.75).

Table 7. RM ANOVA Results

RM ANOVA		MLR	
Statistic	Value	Statistic	Value
p	<.01	p	<.01
F	12.19	F	12.19
η_p^2	.30	η_p^2	.30
M_{Pre}	4.00	M_{Pre}	4.00
M_{Post}	5.00	M_{Post}	5.00
$M_{FollowUp}$	4.75	$M_{FollowUp}$	4.75

Independent Samples *t*-test. Table 8 and Appendix B (lines 492 – 703) present the results of the independent samples *t*-test analyses. The *p* values for all four analyses (i.e., *t*-test, MLR, ANOVA, and *r*) were the same (i.e., 7.537174e-06; see Appendix B, lines 558 – 566). The *t*-test and *r* provide a *t*-test statistic, whereas MLR and ANOVA provide an *F* statistic. As t^2 is equal to *F* (Thompson, 2006), the *t* statistics of -5.477226 and 5.47726 are equivalent to the *F* statistic of 30.0 (see Appendix B, lines 568 – 605). Note that the test statistic for the *t*-test is negative while positive for *r* (see Appendix B, lines 571 & 579). This is because the mean for the first group (Control) was less than the mean for the second group (Treatment), and there was a positive relationship between posttest scores and the numeric representation of group since group was coded as 1 and treatment was coded as 2.

Table 8. Independent Samples *t*-test Results

t-test		MLR		ANOVA		*r*	
Statistic	Value	Statistic	Value	Statistic	Value	Statistic	Value
p	<.01	p	<.01	p	<.01	p	<.01
t	-5.48	F	30.00	F	30.00	t	5.48
d	2.00	MR^2	.52	η^2	.52	r	.72
$M_{Control}$	4.00	$M_{Control}$	4.00	$M_{Control}$	4.00	$M_{Control}$	4.00
$M_{Treatment}$	6.00	$M_{Treatment}$	6.00	$M_{Treatment}$	6.00	$M_{Treatment}$	6.00

The typical effect size reported for an independent samples *t*-test is Cohen's *d,* where the mean difference is divided by the pooled standard deviation (Cohen, 1988). The MLR, ANOVA, and *r*, respectively, yielded MR^2, η^2, and *r*. Whereas the R^2 and the η^2 are already in a comparable dimension and equal at .517241, the Cohen's *d* of 2 was converted to *r* (Lakens, 2013; McGrath & Meyer, 2006), resulting in .719195, which is equivalent to the MR^2 of .517241 for reasons previously stated (see Appendix B, lines 607 – 655). Group means for each analysis are also identical where $M_{Control}$ = 4.0 and $M_{Treatment}$ = 6.0. As ANOVA and *r* do not provide group means or information to compute group means, descriptive statistics were calculated (see Appendix B, lines 696 – 703). For MLR analyses, group mean values were obtained by using the intercept and regression coefficients (see Appendix B, lines 671 – 695). In summary, there was a statistically and practically significant mean difference in posttest scores by group (t [28] = $\pm$5.48, F [1, 28] = 30.00, p < .01; d = 2.00, r = .72, $MR^2 = \eta^2$ = .52*;* $M_{Control}$ = 4.00, $M_{Treatment}$ = 6.00).

Paired-Samples *t*-test. Table 9 and Appendix B (lines 705 – 813) present the results of the paired-samples *t*-test analyses. The *p* values for all three analyses (i.e., *t*-test, MLR, and RM ANOVA) were the same (i.e., .000327; see Appendix B, lines 757 – 764). The *t*-test and the MLR provide a *t* statistic whereas RM ANOVA provides an *F* statistic. As t^2 is equal to *F* (Thompson, 2006), the *t* statistic of 4.074684 is equivalent to the *F* statistic of 16.60305 (see Appendix B, lines 766 – 794). The effect size for each analysis was also identical (i.e., Cohen's *d* = .803388; see Appendix B, lines 796 – 804). Cohen's *d* was calculated using the formula for matched groups (Dunlap, Cortina, Vaslow, & Burke, 1996). The mean difference between posttest and pretest

Table 9. Paired-Samples *t*-test Results

t-test		MLR		RM ANOVA	
Statistic	Value	Statistic	Value	Statistic	Value
p	<.01	p	<.01	p	<.01
t	4.07	t	4.07	F	16.60
d	.80	d	.80	d	.80
$M_{Post\text{-}Pre}$	1.00	$M_{Post\text{-}Pre}$	1.00	$M_{Post\text{-}Pre}$	1.00

scores was identical for each analysis (i.e., $M_{Post\text{-}Pre}$ = 1; see Appendix B, lines 806 – 813). Because ANOVA does not provide group means, descriptive statistics were calculated on posttest minus pretest scores. For MLR, the intercept provided the mean difference between posttest and pretest scores. In summary, there was a statistically and practically significant mean difference between posttest and pretest scores (t [29] = 4.07, F [1, 29] = 16.60, $p < .01$; d = .80; $M_{Post\text{-}Pre}$ = 1.00).

Single-Sample *t*-test. Table 10 and Appendix B (lines 815 – 876) present the results of the single-sample *t*-test analyses. The p values for both analyses (i.e., *t*-test, MLR) were the same (i.e., 8.45791e-20; see Appendix B, lines 852 – 856). The *t*-test and the MLR produced identical t statistics (i.e., 22.2967; see Appendix B, lines 858-863). The effect size for each analysis was also identical (i.e., Cohen's d = 4.070802; see Appendix B, lines 865 – 869). For the single-sample *t*-test, Cohen's d was calculated by dividing the M by the SD of pretest scores. For MLR, Cohen's d was calculated using the t statistic and formula for one-sample *t*-test (Cohen, 1988, p. 72). The mean pretest score was identical for each analysis (i.e., M_{Pre} = 4.00; see Appendix B, lines 871 – 876). For MLR, the intercept provided the mean pretest score. In summary, the mean pretest score was statistically and practically significant different from 0 (t [29] = 22.30, $p < .01$; d = 4.07; M_{Pre} = 4.00).

Table 10. Single-Sample *t*-test Results

t-test		MLR	
Statistic	Value	Statistic	Value
p	<.01	p	<.01
t	22.30	t	22.30
d	4.07	d	4.07
M_{Pre}	4.00	M_{Pre}	4.00

χ^2 Table 11 and Appendix B (lines 878 – 1072) present the results of the χ^2 and MLR analyses. To demonstrate that MLR does not subsume χ^2 analyses in all cases, we first modeled position by group, which considered a 3x2 association. Using MLR to analyze a 3x2 association is not valid for multiple reasons. First, MLR does not accept categorical data as a dependent variable. Second, modeling the numeric representation of a variable with more than two categorical levels (e.g., position) is not appropriate and returned erroneous results, as depicted in Table 11 and Appendix B (lines 897 – 966).

Table 11. .χ^2 Results

Position ~ Group				Group ~ Position			
χ^2		MLR (Incorrect)		χ^2		MLR (Correct)	
Statistic	Value	Statistic	Value	Statistic	Value	Statistic	Value
p	.67	p	.67	p	.67	p	.69
χ^2	.80	F	.19	χ^2	.80	F	.37
Cramer's V	.16	MR^2	.01	Cramer's V	.16	MR^2	.03

To demonstrate that MLR does subsume χ^2 analyses in certain cases, we modeled group by position (Group~Position), which considered a 2x3 association where group was treated as a dichotomous dependent variable (see Appendix B, lines 968 – 1072). The group by position results are provided in the Group~Position column of Table 11. The chi-square test returned 2 degrees of freedom (df = [rows – 1] [columns – 1]) and the MLR returned df_{error} = 27, where the latter took into consideration the number of predictors (k = 2) and sample size (n = 30). As well, the two approaches to the χ^2 analysis delivered different p values (see Appendix B, lines 1026 – 1030). This difference in p value is attributed to the fact that chi-square probability calculations are not sensitive to sample size (McNeil, 1974). In this example, the probability statistic from the MLR (i.e., .694) can be considered more accurate than from the chi-square (i.e., .670) due to the small sample size of 30. The chi-square probability value "becomes more exact when larger sample sizes are observed" (McNeil, p. 53).

Similar to the other analyses, different test statistics were returned. The chi-square test yielded a χ^2 statistic (i.e., .80), and the MLR yielded an F statistic (i.e., .37). When converted using Knapp's (1978) formula and its derivative (see Table 3), these test statistics are approximately equal (see Appendix B, lines 1032 – 1054). Effect sizes produced by the analyses are also different but equivalent. The chi-square test produced a Cramer's V (i.e., .163), and the MLR produced an MR^2 (i.e., .027). Once the Cramer's V is squared, the observed effect sizes are identical (Cohen, 1988, see Appendix B, lines 1056 – 1072). In summary, position did not have a statistically or practically significant effect on group (χ^2 [2] = .80, p = .67; F [2, 27] = .37, p = .694; Cramer's V = .16, MR^2 =.03).

Discussion

The content presented in this article affords graduate students and emerging scholars a cogent illustration of how MLR subsumes univariate analyses in the GLM. In addition to the illustration, the present paper extends current literature by demonstrating how (a) independent samples *t*-test is subsumed by both ANOVA and Pearson's *r*, (b) RM ANOVA is subsumed by MLR and subsequently subsumes paired-samples *t*-test, (c) MLR subsumes single-samples *t*-test, and (d) MLR subsumes chi-square only in special cases. Researchers may utilize the content herein as a reference guide since it provides (a) a more rigorous visual representation of the univariate GLM, (b) an explanation of the test statistics and effect sizes yielded by comparable statistical analyses, (c) a complete table of transformation formulae with pertinent references, (d) example write-ups that accompany each set of analyses, and (e) replicable syntax that may be copied, modified, and applied to other research studies.

Novice readers of academic literature describing the GLM may interpret the arguments presented as doctrine without fully understanding and exploring the underlying concepts. This article attempts to guide the novice reader through the hierarchical nature of the univariate GLM by demonstrating the analyses which may be replicated and compared using the syntax and output provided. If readers undergo the replication process afforded, they should recognize that MLR does, in fact, subsume the univariate parametric analyses within the GLM. Through this exploration, replication, and independent study, readers will likely better understand the arguments and concepts that connect the univariate GLM analyses.

The statistical analyses presented in this paper are often described as independent tools that are used for specific purposes. In reality, and due to their inherent incorporation within the GLM, MLR is not unidimensional in its application. We expect that prudent researchers will understand the similarities, differences, and limitations (e.g., chi-square's sensitivity to sample size) of the univariate GLM analyses and will apply the appropriate analysis to best match their research design and data.

The paper indirectly reinforces the concept that statistics do not determine causality. Although MLR is often maligned for not yielding experimental evidence (e.g., Nisbett, 2016), readers should understand that data from an experimental design could be analyzed with MLR and therefore yield experimental evidence. Also, the paper demonstrates why statements such as "*correlation* never *implies causality"* are wrong (cf. Huck, 2012). Only aspects of research design determine causality, not the statistics used to analyze the data yielded from the research design.

This paper is not without limitations. First, it considered only the univariate GLM and did not demonstrate how canonical correlation subsumes the multivariate and univariate analyses. Nor did it demonstrate SEM as the most general form of the GLM or consider other univariate analyses including split-plot ANOVA. Second, the paper provided only R syntax to accompany the analyses. Third, the data used to demonstrate the GLM were simulated and designed to meet the statistical assumptions of the analyses. As such, the syntax did not include checks for the statistical assumptions for each analysis. Future research could consider building on the work presented in this paper by addressing the aforementioned limitations.

References

Bagozzi, R. P., Fornell, C., & Larcker, D. F. (1981). Canonical correlation analysis as a special case of a structural relations model. *Multivariate Behavioral Research, 16*, 437–454. doi:10.1207/s15327906mbr1604_2

Cohen, J. (1968). Multiple regression as a general data-analytic system. *Psychological Bulletin*, *70*, 426–433. doi:10.1037/h0026714

Cohen, J., (1988). *Statistical power and analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Erlbaum.

Crawley, M. J. (2013). *The R book*. West Sussex, England: Wiley.

Dunlap, W. P., Cortina, J. M., Vaslow, J. B., & Burke, M. J. (1996). Meta-analysis of experiments with matched groups or repeated measures designs. *Psychological Methods*, *1*, 170–177. doi:10.1037/1082-989x.1.2.170

Fox, J. (2003). Effect displays in R for generalised linear models. *Journal of Statistical Software, 8*, 1-27. doi:10.18637/jss.v008.i15

Fox, J., & Weisberg, S. (2011). *An R companion to applied regression* (2nd ed.). Thousand Oaks, CA: Sage.

Graham, J. M. (2008). The general linear model as structural equation modeling. *Journal of Educational and Behavioral Statistics*, *33*, 485-506. doi: 10.3102/1076998607306151

Henson, R. K., Hull, D. M., & Williams, C. S. (2010). Methodology in our education research culture: Toward a stronger collective quantitative proficiency. *Educational Researcher*, *39*, 229-240. doi:10.3102/0013189x10365102

Huck, S. W. (2012). *Reading statistics and research* (6th ed.). Boston, MA: Allyn & Bacon.

Knapp, T. R. (1978). Canonical correlation analysis: A general parametric significance-testing system. *Psychological Bulletin*, *85*, 410-416. doi:10.1037/0033-2909.85.2.410

Lakens, D. (2013). Calculating and reporting effect sizes to facilitate cumulative science: A practical primer for t-tests and ANOVAs. *Frontiers in Psychology*, *4*, 863. doi:10.3389/fpsyg.2013.00863

McGrath R. E., & Meyer G. J. (2006). When effect sizes disagree: The case of r and d. *Psychological Methods*, *11*, 386–401. doi: 10.1037/1082-989X.11.4.386

McNeil, K. (1974). The multiple linear regression approach to “chi square” hypotheses. *Journal of Experimental Education*, *43*, 53-55. doi: 10.1080/00220973.1974.10806319

Nimon, K., & Henson, R. (2015). Validity of a residualized dependent variable after pretest covariance correction: Still the same variable? *Journal of Experimental Education*, *18*, 405-422. doi:10.1080/00220973.2014.907228

Nimon, K., Oswald, F. L., & Roberts, J. K. (2013). *Yhat: Interpreting regression effects*. R Packaged version, 2-0, http://CRAN. R-project.org/package= yhat

Nimon, K., & Williams, C. (2009). Performance improvement through repeated measures: A primer for educators considering univariate and multivariate design. *Research in Higher Education Journal*, *2*, 117–136.

Nisbett, R. (2016). *The crusade against multiple regression analysis: A conversation with Richard Nisbett*. Retrieved from https://www.edge.org/conversation/richard_nisbett-the-crusade-against-multiple-regression-analysis

Osborne, J. W. (2013). Normality of residuals is a continuous variable, and does seem to influence the trustworthiness of confidence intervals: A response to, and appreciation of, Williams, Grajales, and Kurkiewicz (2013). *Practical Assessment, Research & Evaluation*, *18*(*12*), 1-9.

R Development Core Team. (2017*). R: A language and environment for statistical computing* [Computer software]. Vienna, Austria: R Foundation for Statistical Computing. Retrieved from http://www.R-project.org/

Thompson, B. (2006). *Foundations of behavioral statistics*. New York: Guilford.

Tracz, S. M., Nelson, L. L., Newman, I., & Beltran, A. (2005). The misuse of ANCOVA: The academic and political implications of Type VI errors in studies of achievement and socioeconomic status. *Multiple Linear Regression Viewpoints*, *31*, 16-21.

White, D. (2015). Active learning and threshold concepts in multiple testing that can further develop student critical statistical thinking. *Teaching Statistics*, *37*, 48-53. doi:10.1111/test.12069

Zientek, L. R., Nimon, K., & Brown, B. (2016). Analyzing data from a pretest-posttest control group design: The importance of statistical assumptions. *European Journal of Training and Development*, 40, 638-659. doi:10.1108/ejtd-08-2015-0066

Zientek, L. R., & Thompson, B. (2009). Matrix summaries improve research reports: Secondary analyses using published literature. *Educational Researcher*, *38*, 343-352. doi: 10.3102/0013189X09339056.

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APPENDIX A

R Software to Replicate Reported Analyses

```
###Install necessary packages (first time only)
install.packages("yhat")
install.packages("car")
install.packages("effects")
install.packages("MASS")
install.packages("psych")
install.packages("lsr")

###Load necessary packages
library(yhat)
library(car)
library(effects)
library(MASS)
library(psych)
library(lsr)

###Create simulated dataset

###Set seed
set.seed (1234)

###Control Simulated Data
ctlcov<-matrix(c( 1,  .6, .6, .6, 1, .6, .6, .6, 1), 3, 3)
rownames(ctlcov)<-colnames(ctlcov)<-c("Pre","Post","FollowUp")
ctldata<-mvrnorm(n=15,c(4.00,4.00,4.00),ctlcov,empirical=TRUE)
ctldata<-data.frame(ctldata)
ctldata$Group<-0

###Experimental Simulated Data
expcov<-matrix(c( 1,  .6, .6, .6, 1, .6, .6, .6, 1), 3, 3)
rownames(expcov)<-colnames(expcov)<-c("Pre","Post","FollowUp")
expdata<-mvrnorm(n=15,c(4.00,6.00,5.5),expcov,empirical=TRUE)
expdata<-data.frame(expdata)
expdata$Group<-1

###Merged Simulated Data
ds<-rbind(ctldata,expdata)
ds$Group<-as.factor(ds$Group)
levels(ds$Group)<-c("Control","Treatment")
ds$Position<-as.factor(c(rep("Full",4),rep("Part",6),rep("Seasonal",4),
                               rep("Full",6),rep("Part",4),rep("Seasonal",6)))

###Describe dataset
head(ds)
describe(ds)

###Run descriptive statistics by group
describe(subset(ds,Group=="Control"))
describe(subset(ds,Group=="Treatment"))

ds1<-ds
ds1$Group<-as.numeric(ds1$Group)-1
cor(subset(ds1,Group==0,select= -c(Group,Position)))
cor(subset(ds1,Group==1,select= -c(Group,Position)))

###Create long version of data for 3-wave repeated measures ANOVA
dslong3<-
reshape(ds,varying=c("Pre","Post","FollowUp"),v.names="Test",timevar="MO",times=c(1,2,3),directio
n="long")
dslong3$id<-as.factor(dslong3$id)
dslong3$MO<-as.factor(dslong3$MO)

###Create long version of data for 2-wave repeated measures ANOVA
dslong2<-subset(dslong3,MO!=3)
```

```
###Create long version of data for 1-wave repeated measures ANOVA
dslong1<-subset(dslong3,MO==1)

###ANCOVA SUBSUMED BY MLR###

###Center predictor to have meaningful intercept
ds$Prec<-ds$Pre-mean(ds$Pre)

###ANCOVA on Post by Group with Pre
lm.out1<-lm(Post~Prec,data=ds)
lm.out2<-lm(Post~Prec+Group,data=ds)
anova(lm.out1,lm.out2)

###ANCOVA via MLR
summary(lm.out2)

###Compare p values
anova(lm.out1,lm.out2)[2,"Pr(>F)"]#ANCOVA
summary(lm.out2)$coefficients["GroupTreatment","Pr(>|t|)"]#MLR

###Compare test statistics
anova(lm.out1,lm.out2)[2,"F"]#ANCOVA
summary(lm.out2)$coefficients["GroupTreatment","t value"]#MLR

###Transform t statistics to F statistics
anova(lm.out1,lm.out2)[2,"F"]#ANOVA
summary(lm.out2)$coefficients["GroupTreatment","t value"]^2#MLR

###Transform F statistics to t statistics
sqrt(anova(lm.out1,lm.out2)[2,"F"])#ANOVA
summary(lm.out2)$coefficients["GroupTreatment","t value"]#MLR

###Compare effect sizes
###ANCOVA
(aout<-Anova(lm.out2,type="III"))
aout["Group","Sum Sq"]/(aout["Group","Sum Sq"]+aout["Residuals","Sum Sq"])
###MLR
(rout<-calc.yhat(lm.out2,prec=11)$APSRelatedMetrics)
   rout["Group","Commonality"]/
(1-rout["Total","Commonality"]+
   rout["Group","Commonality"])

###Compare adjusted means
###ANCOVA
effect("Group",lm.out2,data=ds)

###MLR
summary(lm.out2)
summary(lm.out2)$coefficients["(Intercept)","Estimate"]
summary(lm.out2)$coefficients["(Intercept)","Estimate"]+
summary(lm.out2)$coefficients["GroupTreatment","Estimate"]

###ANOVA SUBSUMED BY MLR###

###ANOVA on Post by Position
(aout<-anova(aov(Post~Position,data=ds)))

###MLR on Post by Position using MLR
lm.out<-lm(Post~Position,data=ds)
summary(lm.out)

###Compare p values
aout["Position","Pr(>F)"]#ANOVA
anova(lm.out)["Position","Pr(>F)"]#MLR

###Compare test statistics
aout["Position","F value"]#ANOVA
summary(lm.out)$fstatistic["value"]#MLR

###Compare effect sizes
aout["Position","Sum Sq"]/sum(aout[,"Sum Sq"])#ANOVA
```

```
summary(lm.out)$r.squared#MLR

###Compare group means
###ANOVA
aggregate(ds$Post~ds$Position,ds,mean)
###MLR
summary(lm.out)
summary(lm.out)$coefficients["(Intercept)","Estimate"]
summary(lm.out)$coefficients["(Intercept)","Estimate"]+
summary(lm.out)$coefficients["PositionPart","Estimate"]
summary(lm.out)$coefficients["(Intercept)","Estimate"]+
summary(lm.out)$coefficients["PositionSeasonal","Estimate"]

###r SUBSUMED BY MLR###

###correlation between Post and Pre using Pearson's
(cor.out<-cor.test(ds$Post,ds$Pre))

###correlation between Post and Group using MLR
lm.out<-lm(Post~Pre,data=ds)
summary(lm.out)

###Compare p values
cor.out$p.value#Pearson's r
anova(lm.out)["Pre","Pr(>F)"]#MLR

###Compare test statistics
cor.out$statistic#Pearsons's r
summary(lm.out)$fstatistic["value"]#MLR

###Transform t to F
cor.out$statistic^2#Pearsons r
summary(lm.out)$fstatistic["value"]#MLR

###Transform F to t
cor.out$statistic#Pearson's r
sqrt(summary(lm.out)$fstatistic["value"])#MLR

###Compare effect sizes
cor.out$estimate#Pearson's r
summary(lm.out)$r.squared#MLR

###Transform r to R2
cor.out$estimate^2#Pearson's r
summary(lm.out)$r.squared#MLR

###Transform R2 to r
cor.out$estimate#Pearson's r
sqrt(summary(lm.out)$r.squared)#MLR

###REPEATED MEASURES ANOVA SUBSUMED BY MLR###

###Repeated Measures ANOVA on Pre, Post, and Followup
aoutrm<-aov(Test~MO+Error(id),data=dslong3)
summary(aoutrm)

###MLR on Pre, Post, and Followup
(aoutmlr<-anova(lm(Test~MO+id,data=dslong3)))

###Compare p values
((a<-unlist(summary(aoutrm)[["Error: Within"]]))["Pr(>F)1"])#ANOVA
aoutmlr["MO","Pr(>F)"]

###Compare test statistic
a[["F value1"]]#RM ANOVA
aoutmlr["MO","F value"]#MLR

###Compare effect sizes
a[["Sum Sq1"]]/(a[["Sum Sq1"]]+a[["Sum Sq2"]])#RM ANOVA
aoutmlr["MO","Sum Sq"]/(aoutmlr["MO","Sum Sq"]+aoutmlr["Residuals","Sum Sq"])#MLR
```

```
###Compare measurement occasion means
aggregate(Test~MO,dslong3,mean)#RM ANOVA
lm.out<-lm(Test~MO,data=dslong3)
summary(lm.out)
summary(lm.out)$coefficients["(Intercept)","Estimate"]
summary(lm.out)$coefficients["(Intercept)","Estimate"]+
summary(lm.out)$coefficients["MO2","Estimate"]
summary(lm.out)$coefficients["(Intercept)","Estimate"]+
summary(lm.out)$coefficients["MO3","Estimate"]

###INDEPENDENT T TEST SUBSUMED BY MLR, ANOVA, AND r###

###t-test on Post by Group
(t.out<-t.test(Post~Group,data=ds,paired=FALSE,var.equal=TRUE))

###MLR on Post by Group
lm.out<-lm(Post~Group,data=ds)
summary(lm.out)

###ANOVA on Post by Group
(aout<-anova(aov(Post~Group,data=ds)))

###correlation between Post and Group using Pearson's r
(cor.out<-cor.test(ds$Post,as.numeric(ds$Group)))

###Compare p values
t.out$p.value#t-test
anova(lm.out)["Group","Pr(>F)"]#MLR
aout["Group","Pr(>F)"]#ANOVA
cor.out$p.value#Pearson's r

###Compare test statistic
t.out$statistic#t-test
summary(lm.out)$fstatistic["value"]#MLR
aout["Group","F value"]#ANOVA
cor.out$statistic#Pearson's r

###Transform t to F
t.out$statistic^2#t-test
summary(lm.out)$fstatistic["value"]#MLR
aout["Group","F value"]#ANOVA
cor.out$statistic^2#Pearson's r

###Transform F to t
abs(t.out$statistic)#t-test
sqrt(summary(lm.out)$fstatistic["value"])#MLR
sqrt(aout["Group","F value"])#ANOVA
cor.out$statistic#Pearson's r

###Compare effect sizes
(d<-cohensD(ds$Post~ds$Group))#t-test
(r2<-summary(lm.out)$r.squared)#MLR
(e2<-aout["Group","Sum Sq"]/sum(aout[,"Sum Sq"]))#ANOVA
cor.out$estimate#Pearson's r

###Transform d, eta-squared, and R2 to r
d/sqrt(d**2+((nrow(ds)**2-2*nrow(ds))/(table(ds$Group)[1]*table(ds$Group)[2])))#t-test
(tr1<-sqrt(r2))#MLR
(tr2<-sqrt(aout["Group","Sum Sq"]/sum(aout[,"Sum Sq"])))#ANOVA
(tr3<-cor.out$estimate)#Pearson's r

###Transform R2, eta-squared, and r to d
d#t-test
sqrt((-tr1**2*(nrow(ds)**2-2*nrow(ds)))/(table(ds$Group)[1]*table(ds$Group)[2]*(tr1**2-1)))#MLR
sqrt((-tr2**2*(nrow(ds)**2-2*nrow(ds)))/(table(ds$Group)[1]*table(ds$Group)[2]*(tr2**2-1)))#ANOVA
sqrt((-tr3**2*(nrow(ds)**2-2*nrow(ds)))/(table(ds$Group)[1]*table(ds$Group)[2]*(tr3**2-
1)))#Pearson's r

###Transform d and r to R2/eta-squared
(d/sqrt(d**2+((nrow(ds)**2-2*nrow(ds))/(table(ds$Group)[1]*table(ds$Group)[2]))))**2#t-test
r2#MLR
```

```
e2#ANOVA
tr3**2#Pearson's r

###Compare group means
t.out#t-test
summary(lm.out)#MLR
summary(lm.out)$coefficients["(Intercept)","Estimate"]
summary(lm.out)$coefficients["(Intercept)","Estimate"]+
summary(lm.out)$coefficients["GroupTreatment","Estimate"]
aggregate(ds$Post~ds$Group,ds,mean)#ANOVA
aggregate(ds$Post~ds$Group,ds,mean)#Pearson's r

###PAIRED T TEST SUBSUMED BY MLR and ANOVA###

###t-test on Pre and Post
(t.out<-t.test(ds$Post,ds$Pre,paired=TRUE))

###MLR on Pre and Post
lm.out<-lm(I(Post-Pre)~1,data=ds)
summary(lm.out)

###Repeated Measures ANOVA on Pre and Post
aout<-aov(Test~MO+Error(id),data=dslong2)
summary(aout)

###Compare p values
t.out$p.value#t-test
summary(lm.out)$coefficients["(Intercept)","Pr(>|t|)"]#MLR
unlist(summary(aout)[["Error: Within"]])["Pr(>F)1"]#ANOVA

###Compare test statistic
(t1<-t.out$statistic)#t-test
(t2<-summary(lm.out)$coefficients["(Intercept)","t value"])#MLR
(f<-unlist(summary(aout)[["Error: Within"]])["F value1"])#ANOVA

###Transform t to F
t1**2#t-test
t2**2#MLR
f#ANOVA

###Transform F to t
t1#t-test
t2#MLR
sqrt(f)#ANOVA

###Compare effect sizes
t1*sqrt(2*(1-cor(ds$Post,ds$Pre))/nrow(ds))#t-test
t2*sqrt(2*(1-cor(ds$Post,ds$Pre))/nrow(ds))#MLR
sqrt(f)*sqrt(2*(1-cor(ds$Post,ds$Pre))/nrow(ds))#MLR

###Compare group means
t.out$estimate#t-test
summary(lm.out)$coefficients["(Intercept)","Estimate"]#MLR
describe(ds$Post-ds$Pre)$mean#ANOVA

###SINGLE SAMPLE T TEST SUBSUMED BY MLR###

###t-test on Pre
(t.out<-t.test(ds$Pre))

###MLR on on Pre
lm.out<-lm(Pre~1,data=ds)
summary(lm.out)

###Compare p values
t.out$p.value#t-test
summary(lm.out)$coefficients["(Intercept)","Pr(>|t|)"]#MLR

###Compare test statistic
(t1<-t.out$statistic)#t-test
(t2<-summary(lm.out)$coefficients["(Intercept)","t value"])#MLR
```

```
###Compare effect sizes
(d<-cohensD(ds$Pre))#t-test
(t2/sqrt(length(ds$Post)))#MLR

###Compare group means
t.out$estimate#t-test
summary(lm.out)$coefficients["(Intercept)","Estimate"]#MLR

###chi-square via MLR###

###descriptive statistics on Position by Group
(x.out<-table(ds$Position,ds$Group))

###chi-test on Position by Group
chisq.test(x.out,correct=FALSE)

###MLR on Position by Group
lm.out<-lm(Position~Group,data=ds)
summary(lm.out)

###MLR on Position by Group - Try treating categories as numbers
lm.out<-lm(as.numeric(ds$Position)~Group,data=ds)
summary(lm.out)

###Compare p values
chisq.test(x.out,correct=FALSE)$p.value#chi-square
anova(lm.out)["Group","Pr(>F)"]#MLR

###Compare test statistic
chisq.test(x.out,correct=FALSE)$statistic#chi-square
summary(lm.out)$fstatistic["value"]#MLR

###Compare effect sizes
cramersV(x.out)#chi-square
summary(lm.out)$r.squared#MLR

###Transform Cramer's v to R2
cramersV(x.out)**2#chi-square
summary(lm.out)$r.squared#MLR

###Transform R2 to Cramer's v
cramersV(x.out)#chi-square
sqrt(summary(lm.out)$r.squared)#MLR

###chi-square via MLR###

###descriptive statistics on Group by Position
(x.out<-table(ds$Group, ds$Position))

###chi-test on Group by Position
chisq.test(x.out,correct=FALSE)

###MLR on Group by Position
lm.out<-lm(Group~Position,data=ds)
summary(lm.out)

###MLR on Group by Position - Try treating categories as numbers
lm.out<-lm(as.numeric(ds$Group)~Position,data=ds)
summary(lm.out)

###Compare p values
chisq.test(x.out,correct=FALSE)$p.value#chi-square
anova(lm.out)["Position","Pr(>F)"]#MLR

###Compare test statistic
(x2<-chisq.test(x.out,correct=FALSE)$statistic)#chi-square
(F<-summary(lm.out)$fstatistic["value"])#MLR

###Transform x2 to F
x2/((length(levels(ds$Position))-1)*(length(levels(ds$Group))-1))#chi-square
```

```
summary(lm.out)$fstatistic["value"]#MLR

###Transform F to x2
x2#chi-square
F*((length(levels(ds$Position))-1)*(length(levels(ds$Group))-1))#MLR

###Compare effect sizes
cramersV(x.out)#chi-square
summary(lm.out)$r.squared#MLR

###Transform Cramer's v to R2
cramersV(x.out)**2#chi-square
summary(lm.out)$r.squared#MLR

###Transform R2 to Cramer's v
cramersV(x.out)#chi-square
sqrt(summary(lm.out)$r.squared)#MLR
```

APPENDIX B
R Output for Illustrative Examples

```
> ###Load necessary packages
> library(yhat)
> library(car)
> library(effects)
> library(MASS)
> library(lsr)
>
> ###Create simulated dataset
>
> ###Set seed
> set.seed (1234)
>
> ###Control Simulated Data
> ctlcov<-matrix(c( 1,  .6, .6, .6, 1, .6, .6, .6, 1), 3, 3)
> rownames(ctlcov)<-colnames(ctlcov)<-c("Pre","Post","FollowUp")
> ctldata<-mvrnorm(n=15,c(4.00,4.00,4.00),ctlcov,empirical=TRUE)
> ctldata<-data.frame(ctldata)
> ctldata$Group<-0
>
> ###Experimental Simulated Data
> expcov<-matrix(c( 1,  .6, .6, .6, 1, .6, .6, .6, 1), 3, 3)
> rownames(expcov)<-colnames(expcov)<-c("Pre","Post","FollowUp")
> expdata<-mvrnorm(n=15,c(4.00,6.00,5.5),expcov,empirical=TRUE)
> expdata<-data.frame(expdata)
> expdata$Group<-1
>
> ###Merged Simulated Data
> ds<-rbind(ctldata,expdata)
> ds$Group<-as.factor(ds$Group)
> levels(ds$Group)<-c("Control","Treatment")
> ds$Position<-as.factor(c(rep("Full",4),rep("Part",6),rep("Seasonal",4),
+  rep("Full",6),rep("Part",4),rep("Seasonal",6)))
>
> ###Describe dataset
> head(ds)
       Pre     Post FollowUp   Group Position
1 4.296838 5.828809 5.122568 Control     Full
2 3.889897 3.688667 4.390797 Control     Full
3 3.813139 2.967588 4.589542 Control     Full
4 5.587708 5.424477 3.962162 Control     Full
5 1.267185 2.839900 1.970185 Control     Part
6 3.217834 3.056502 3.548479 Control     Part
> describe(ds)
          vars  n mean   sd median trimmed  mad  min  max range  skew kurtosis   se
Pre          1 30 4.00 0.98   3.90    4.04 0.81 1.27 5.92  4.65 -0.42     0.33 0.18
Post         2 30 5.00 1.41   4.80    5.00 1.84 2.84 7.18  4.34 -0.02    -1.41 0.26
FollowUp     3 30 4.75 1.24   4.59    4.72 1.01 1.97 7.52  5.55  0.19    -0.19 0.23
Group*       4 30 1.50 0.51   1.50    1.50 0.74 1.00 2.00  1.00  0.00    -2.07 0.09
Position*    5 30 2.00 0.83   2.00    2.00 1.48 1.00 3.00  2.00  0.00    -1.60 0.15
>
> ###Run descriptive statistics by group
> describe(subset(ds,Group=="Control"))
          vars  n mean  sd median trimmed  mad  min  max range  skew kurtosis   se
Pre          1 15 4.00 1.0   3.91    4.09 0.63 1.27 5.59  4.32 -0.98     1.33 0.26
Post         2 15 4.00 1.0   3.99    3.95 1.38 2.84 5.83  2.99  0.46    -1.16 0.26
FollowUp     3 15 4.00 1.0   4.12    3.98 0.69 1.97 6.26  4.29  0.15     0.18 0.26
Group*       4 15 1.00 0.0   1.00    1.00 0.00 1.00 1.00  0.00   NaN      NaN 0.00
Position*    5 15 1.93 0.8   2.00    1.92 1.48 1.00 3.00  2.00  0.10    -1.53 0.21
> describe(subset(ds,Group=="Treatment"))
          vars  n mean   sd median trimmed  mad  min  max range  skew kurtosis   se
Pre          1 15 4.00 1.00   3.72    3.97 1.16 2.42 5.92  3.50  0.18    -1.12 0.26
Post         2 15 6.00 1.00   6.21    6.06 1.05 4.07 7.18  3.11 -0.58    -1.13 0.26
FollowUp     3 15 5.50 1.00   5.33    5.44 0.98 4.32 7.52  3.20  0.59    -0.96 0.26
Group*       4 15 2.00 0.00   2.00    2.00 0.00 2.00 2.00  0.00   NaN      NaN 0.00
Position*    5 15 2.07 0.88   2.00    2.08 1.48 1.00 3.00  2.00 -0.12    -1.79 0.23
>
> ds1<-ds
> ds1$Group<-as.numeric(ds1$Group)-1
```

```
> cor(subset(ds1,Group==0,select= -c(Group,Position)))
         Pre Post FollowUp
Pre      1.0  0.6      0.6
Post     0.6  1.0      0.6
FollowUp 0.6  0.6      1.0
> cor(subset(ds1,Group==1,select= -c(Group,Position)))
         Pre Post FollowUp
Pre      1.0  0.6      0.6
Post     0.6  1.0      0.6
FollowUp 0.6  0.6      1.0
>
> ###Create long version of data for 3-wave repeated measures ANOVA
> dslong3<-
reshape(ds,varying=c("Pre","Post","FollowUp"),v.names="Test",timevar="MO",times=c(1,2,3),directio
n="long")
> dslong3$id<-as.factor(dslong3$id)
> dslong3$MO<-as.factor(dslong3$MO)
>
> ###Create long version of data for 2-wave repeated measures ANOVA
> dslong2<-subset(dslong3,MO!=3)
>
> ###Create long version of data for 1-wave repeated measures ANOVA
> dslong1<-subset(dslong3,MO==1)
>
> ###ANCOVA SUBSUMED BY MLR###
>
> ###Center predictor to have meaningful intercept
> ds$Prec<-ds$Pre-mean(ds$Pre)
>
> ###ANCOVA on Post by Group with Pre
> lm.out1<-lm(Post~Prec,data=ds)
> lm.out2<-lm(Post~Prec+Group,data=ds)
> anova(lm.out1,lm.out2)
Analysis of Variance Table

Model 1: Post ~ Prec
Model 2: Post ~ Prec + Group
  Res.Df   RSS Df Sum of Sq      F    Pr(>F)
1     28 47.92
2     27 17.92  1        30 45.201 3.225e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> ###ANCOVA via MLR
> summary(lm.out2)

Call:
lm(formula = Post ~ Prec + Group, data = ds)

Residuals:
     Min       1Q   Median       3Q      Max
-1.31704 -0.53236  0.06803  0.47765  1.65071

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      4.0000     0.2103  19.016  < 2e-16 ***
Prec             0.6000     0.1540   3.897 0.000581 ***
GroupTreatment   2.0000     0.2975   6.723 3.22e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8147 on 27 degrees of freedom
Multiple R-squared:  0.691,     Adjusted R-squared:  0.6681
F-statistic: 30.19 on 2 and 27 DF,  p value: 1.3e-07

>
> ###Compare p values
> anova(lm.out1,lm.out2)[2,"Pr(>F)"]#ANCOVA
[1] 3.22454e-07
> summary(lm.out2)$coefficients["GroupTreatment","Pr(>|t|)"]#MLR
[1] 3.22454e-07
```

```
>
> ###Compare test statistics
> anova(lm.out1,lm.out2)[2,"F"]#ANCOVA
[1] 45.20089
> summary(lm.out2)$coefficients["GroupTreatment","t value"]#MLR
[1] 6.723161
>
> ###Transform t statistics to F statistics
> anova(lm.out1,lm.out2)[2,"F"]#ANOVA
[1] 45.20089
> summary(lm.out2)$coefficients["GroupTreatment","t value"]^2#MLR
[1] 45.20089
>
> ###Transform F statistics to t statistics
> sqrt(anova(lm.out1,lm.out2)[2,"F"])#ANOVA
[1] 6.723161
> summary(lm.out2)$coefficients["GroupTreatment","t value"]#MLR
[1] 6.723161
>
> ###Compare effect sizes
> ###ANCOVA
> (aout<-Anova(lm.out2,type="III"))
Anova Table (Type III tests)

Response: Post
            Sum Sq Df F value    Pr(>F)
(Intercept) 240.00  1 361.607 < 2.2e-16 ***
Prec         10.08  1  15.188 0.0005807 ***
Group        30.00  1  45.201 3.225e-07 ***
Residuals    17.92 27
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> aout["Group","Sum Sq"]/(aout["Group","Sum Sq"]+aout["Residuals","Sum Sq"])
[1] 0.6260434
> ###MLR
> (rout<-calc.yhat(lm.out2,prec=11)$APSRelatedMetrics)
           Commonality  % Total        R2  Prec.Inc Group.Inc
Prec         0.1737931 0.251497 0.1737931        NA 0.5172414
Group        0.5172414 0.748503 0.5172414 0.1737931        NA
Prec,Group   0.0000000 0.000000 0.6910345        NA        NA
Total        0.6910345 1.000000        NA        NA        NA
>    rout["Group","Commonality"]/
+ (1-rout["Total","Commonality"]+
+    rout["Group","Commonality"])
[1] 0.6260434
>
> ###Compare adjusted means
> ###ANCOVA
> effect("Group",lm.out2,data=ds)

 Group effect
Group
  Control Treatment
        4         6
> ###MLR
> summary(lm.out2)

Call:
lm(formula = Post ~ Prec + Group, data = ds)

Residuals:
     Min       1Q   Median       3Q      Max
-1.31704 -0.53236  0.06803  0.47765  1.65071

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      4.0000     0.2103  19.016  < 2e-16 ***
Prec             0.6000     0.1540   3.897 0.000581 ***
GroupTreatment   2.0000     0.2975   6.723 3.22e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```

Residual standard error: 0.8147 on 27 degrees of freedom
Multiple R-squared:  0.691,     Adjusted R-squared:  0.6681 
F-statistic: 30.19 on 2 and 27 DF,  p-value: 1.3e-07

> summary(lm.out2)$coefficients["(Intercept)","Estimate"]
[1] 4
> summary(lm.out2)$coefficients["(Intercept)","Estimate"]+
+ summary(lm.out2)$coefficients["GroupTreatment","Estimate"]
[1] 6
> 
> ###ANOVA SUBSUMED BY MLR###
> 
> ###ANOVA on Post by Position
> (aout<-anova(aov(Post~Position,data=ds)))
Analysis of Variance Table

Response: Post
          Df Sum Sq Mean Sq F value Pr(>F)
Position   2  1.376  0.6880  0.3281 0.7232
Residuals 27 56.624  2.0972               
> 
> ###MLR on Post by Position using MLR
> lm.out<-lm(Post~Position,data=ds)
> summary(lm.out)

Call:
lm(formula = Post ~ Position, data = ds)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.4393 -0.9360 -0.1363  1.2848  2.4130 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)        5.2859     0.4580  11.543 6.01e-12 ***
PositionPart      -0.5154     0.6476  -0.796    0.433    
PositionSeasonal  -0.3424     0.6476  -0.529    0.601    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.448 on 27 degrees of freedom
Multiple R-squared:  0.02372,   Adjusted R-squared:  -0.04859 
F-statistic: 0.3281 on 2 and 27 DF,  p-value: 0.7232

> 
> ###Compare p values
> aout["Position","Pr(>F)"]#ANOVA
[1] 0.7231535
> anova(lm.out)["Position","Pr(>F)"]#MLR
[1] 0.7231535
> 
> ###Compare test statistics
> aout["Position","F value"]#ANOVA
[1] 0.3280564
> summary(lm.out)$fstatistic["value"]#MLR
    value 
0.3280564 
> 
> ###Compare effect sizes
> aout["Position","Sum Sq"]/sum(aout[,"Sum Sq"])#ANOVA
[1] 0.02372397
> summary(lm.out)$r.squared#MLR
[1] 0.02372397
> 
> ###Compare group means
> ###ANOVA
> aggregate(ds$Post~ds$Position,ds,mean)
  ds$Position  ds$Post
1        Full 5.285930
2        Part 4.770533
```

```
3    Seasonal 4.943537
> ###MLR
> summary(lm.out)

Call:
lm(formula = Post ~ Position, data = ds)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.4393 -0.9360 -0.1363  1.2848  2.4130 

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)    
(Intercept)        5.2859     0.4580  11.543 6.01e-12 ***
PositionPart      -0.5154     0.6476  -0.796    0.433    
PositionSeasonal  -0.3424     0.6476  -0.529    0.601    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.448 on 27 degrees of freedom
Multiple R-squared:  0.02372,   Adjusted R-squared:  -0.04859 
F-statistic: 0.3281 on 2 and 27 DF,  p-value: 0.7232

> summary(lm.out)$coefficients["(Intercept)","Estimate"]
[1] 5.28593
> summary(lm.out)$coefficients["(Intercept)","Estimate"]+
+ summary(lm.out)$coefficients["PositionPart","Estimate"]
[1] 4.770533
> summary(lm.out)$coefficients["(Intercept)","Estimate"]+
+ summary(lm.out)$coefficients["PositionSeasonal","Estimate"]
[1] 4.943537
> 
> ###r SUBSUMED BY MLR###
> 
> ###correlation between Post and Pre using Pearson's
> (cor.out<-cor.test(ds$Post,ds$Pre))

        Pearson's product-moment correlation

data:  ds$Post and ds$Pre
t = 2.4269, df = 28, p-value = 0.02192
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.06662175 0.67567416
sample estimates:
     cor 
0.416885 

> 
> ###correlation between Post and Group using MLR
> lm.out<-lm(Post~Pre,data=ds)
> summary(lm.out)

Call:
lm(formula = Post ~ Pre, data = ds)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.1839 -0.8869 -0.1131  1.0334  2.0902 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept)   2.6000     1.0174   2.556   0.0163 *
Pre           0.6000     0.2472   2.427   0.0219 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.308 on 28 degrees of freedom
Multiple R-squared:  0.1738,    Adjusted R-squared:  0.1443 
F-statistic:  5.89 on 1 and 28 DF,  p-value: 0.02192
```

```
>
> ###Compare p values
> cor.out$p.value#Pearson's r
[1] 0.02191639
> anova(lm.out)["Pre","Pr(>F)"]#MLR
[1] 0.02191639
>
> ###Compare test statistics
> cor.out$statistic#Pearsons's r
       t
2.426894
> summary(lm.out)$fstatistic["value"]#MLR
   value
5.889816
>
> ###Transform t to F
> cor.out$statistic^2#Pearsons r
       t
5.889816
> summary(lm.out)$fstatistic["value"]#MLR
   value
5.889816
>
> ###Transform F to t
> cor.out$statistic#Pearson's r
       t
2.426894
> sqrt(summary(lm.out)$fstatistic["value"])#MLR
   value
2.426894
>
> ###Compare effect sizes
> cor.out$estimate#Pearson's r
     cor
0.416885
> summary(lm.out)$r.squared#MLR
[1] 0.1737931
>
> ###Transform r to R2
> cor.out$estimate^2#Pearson's r
      cor
0.1737931
> summary(lm.out)$r.squared#MLR
[1] 0.1737931
>
> ###Transform R2 to r
> cor.out$estimate#Pearson's r
     cor
0.416885
> sqrt(summary(lm.out)$r.squared)#MLR
[1] 0.416885
>
> ###REPEATED MEASURES ANOVA SUBSUMED BY MLR###
>
> ###Repeated Measures ANOVA on Pre, Post, and Followup
> aoutrm<-aov(Test~MO+Error(id),data=dslong3)
> summary(aoutrm)

Error: id
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals 29  92.22    3.18

Error: Within
          Df Sum Sq Mean Sq F value  Pr(>F)
MO         2  16.25   8.125   12.19 3.8e-05 ***
Residuals 58  38.65   0.666
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> ###MLR on Pre, Post, and Followup
> (aoutmlr<-anova(lm(Test~MO+id,data=dslong3)))
```

```
Analysis of Variance Table

Response: Test
          Df Sum Sq Mean Sq F value    Pr(>F)
MO         2 16.250  8.1250 12.1928 3.800e-05 ***
id        29 92.225  3.1802  4.7723 2.128e-07 ***
Residuals 58 38.650  0.6664
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> ###Compare p values
> ((a<-unlist(summary(aoutrm)[["Error: Within"]]))["Pr(>F)1"])#ANOVA
     Pr(>F)1
3.799596e-05
> aoutmlr["MO","Pr(>F)"]
[1] 3.799596e-05
>
> ###Compare test statistic
> a[["F value1"]]#RM ANOVA
[1] 12.19276
> aoutmlr["MO","F value"]#MLR
[1] 12.19276
>
> ###Compare effect sizes
> a[["Sum Sq1"]]/(a[["Sum Sq1"]]+a[["Sum Sq2"]])#RM ANOVA
[1] 0.2959927
> aoutmlr["MO","Sum Sq"]/(aoutmlr["MO","Sum Sq"]+aoutmlr["Residuals","Sum Sq"])#MLR
[1] 0.2959927
>
> ###Compare measurement occasion means
> aggregate(Test~MO,dslong3,mean)#RM ANOVA
  MO Test
1  1 4.00
2  2 5.00
3  3 4.75
> lm.out<-lm(Test~MO,data=dslong3)
> summary(lm.out)

Call:
lm(formula = Test ~ MO, data = dslong3)

Residuals:
    Min      1Q  Median      3Q     Max
-2.7798 -0.7439 -0.1371  0.9388  2.7708

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   4.0000     0.2239  17.863  < 2e-16 ***
MO2           1.0000     0.3167   3.158  0.00219 **
MO3           0.7500     0.3167   2.368  0.02008 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.227 on 87 degrees of freedom
Multiple R-squared:  0.1105,    Adjusted R-squared:   0.09
F-statistic: 5.401 on 2 and 87 DF,  p-value: 0.00615

> summary(lm.out)$coefficients["(Intercept)","Estimate"]
[1] 4
> summary(lm.out)$coefficients["(Intercept)","Estimate"]+
+ summary(lm.out)$coefficients["MO2","Estimate"]
[1] 5
> summary(lm.out)$coefficients["(Intercept)","Estimate"]+
+ summary(lm.out)$coefficients["MO3","Estimate"]
[1] 4.75
>
> ###INDEPENDENT T TEST SUBSUMED BY MLR, ANOVA, AND r###
>
> ###t-test on Post by Group
> (t.out<-t.test(Post~Group,data=ds,paired=FALSE,var.equal=TRUE))
```

```
        Two Sample t-test

data:  Post by Group
t = -5.4772, df = 28, p-value = 7.537e-06
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.747973 -1.252027
sample estimates:
  mean in group Control mean in group Treatment
                      4                       6

>
> ###MLR on Post by Group
> lm.out<-lm(Post~Group,data=ds)
> summary(lm.out)

Call:
lm(formula = Post ~ Group, data = ds)

Residuals:
    Min      1Q  Median      3Q     Max
-1.9256 -0.9559  0.1309  0.6400  1.8288

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      4.0000     0.2582  15.492 2.90e-15 ***
GroupTreatment   2.0000     0.3651   5.477 7.54e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1 on 28 degrees of freedom
Multiple R-squared:  0.5172,    Adjusted R-squared:    0.5
F-statistic:    30 on 1 and 28 DF,  p-value: 7.537e-06

>
> ###ANOVA on Post by Group
> (aout<-anova(aov(Post~Group,data=ds)))
Analysis of Variance Table

Response: Post
          Df Sum Sq Mean Sq F value    Pr(>F)
Group      1     30      30      30 7.537e-06 ***
Residuals 28     28       1
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> ###correlation between Post and Group using Pearson's r
> (cor.out<-cor.test(ds$Post,as.numeric(ds$Group)))

        Pearson's product-moment correlation

data:  ds$Post and as.numeric(ds$Group)
t = 5.4772, df = 28, p-value = 7.537e-06
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.4844481 0.8573274
sample estimates:
     cor
0.719195

>
> ###Compare p values
> t.out$p.value#t-test
[1] 7.537174e-06
> anova(lm.out)["Group","Pr(>F)"]#MLR
[1] 7.537174e-06
> aout["Group","Pr(>F)"]#ANOVA
[1] 7.537174e-06
> cor.out$p.value#Pearson's r
[1] 7.537174e-06
>
```

```
> ###Compare test statistic
> t.out$statistic#t-test
        t
-5.477226
> summary(lm.out)$fstatistic["value"]#MLR
value
   30
> aout["Group","F value"]#ANOVA
[1] 30
> cor.out$statistic#Pearson's r
       t
5.477226
>
> ###Transform t to F
> t.out$statistic^2#t-test
 t
30
> summary(lm.out)$fstatistic["value"]#MLR
value
   30
> aout["Group","F value"]#ANOVA
[1] 30
> cor.out$statistic^2#Pearson's r
 t
30
>
> ###Transform F to t
> abs(t.out$statistic)#t-test
       t
5.477226
> sqrt(summary(lm.out)$fstatistic["value"])#MLR
   value
5.477226
> sqrt(aout["Group","F value"])#ANOVA
[1] 5.477226
> cor.out$statistic#Pearson's r
       t
5.477226
>
> ###Compare effect sizes
> (d<-cohensD(ds$Post~ds$Group))#t-test
[1] 2
> (r2<-summary(lm.out)$r.squared)#MLR
[1] 0.5172414
> (e2<-aout["Group","Sum Sq"]/sum(aout[,"Sum Sq"]))#ANOVA
[1] 0.5172414
> cor.out$estimate#Pearson's r
     cor
0.719195
>
> ###Transform d, eta-squared, and R2 to r
> d/sqrt(d**2+((nrow(ds)**2-2*nrow(ds))/(table(ds$Group)[1]*table(ds$Group)[2])))#t-test
 Control
0.719195
> (tr1<-sqrt(r2))#MLR
[1] 0.719195
> (tr2<-sqrt(aout["Group","Sum Sq"]/sum(aout[,"Sum Sq"])))#ANOVA
[1] 0.719195
> (tr3<-cor.out$estimate)#Pearson's r
     cor
0.719195
>
> ###Transform R2, eta-squared, and r to d
> d#t-test
[1] 2
> sqrt((-tr1**2*(nrow(ds)**2-2*nrow(ds)))/(table(ds$Group)[1]*table(ds$Group)[2]*(tr1**2-1)))#MLR
Control
      2
> sqrt((-tr2**2*(nrow(ds)**2-2*nrow(ds)))/(table(ds$Group)[1]*table(ds$Group)[2]*(tr2**2-
1)))#ANOVA
Control
```

```
      2
> sqrt((-tr3**2*(nrow(ds)**2-2*nrow(ds)))/(table(ds$Group)[1]*table(ds$Group)[2]*(tr3**2-
1)))#Pearson's r
cor
  2
>
> ###Transform d and r to R2/eta-squared
> (d/sqrt(d**2+((nrow(ds)**2-2*nrow(ds))/(table(ds$Group)[1]*table(ds$Group)[2]))))**2#t-test
  Control
0.5172414
> r2#MLR
[1] 0.5172414
> e2#ANOVA
[1] 0.5172414
> tr3**2#Pearson's r
      cor
0.5172414
>
> ###Compare group means
> t.out#t-test

        Two Sample t-test

data:  Post by Group
t = -5.4772, df = 28, p-value = 7.537e-06
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -2.747973 -1.252027
sample estimates:
  mean in group Control mean in group Treatment
                      4                       6

> summary(lm.out)#MLR

Call:
lm(formula = Post ~ Group, data = ds)

Residuals:
    Min      1Q  Median      3Q     Max
-1.9256 -0.9559  0.1309  0.6400  1.8288

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      4.0000     0.2582  15.492 2.90e-15 ***
GroupTreatment   2.0000     0.3651   5.477 7.54e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1 on 28 degrees of freedom
Multiple R-squared:  0.5172,    Adjusted R-squared:    0.5
F-statistic:    30 on 1 and 28 DF,  p-value: 7.537e-06

> summary(lm.out)$coefficients["(Intercept)","Estimate"]
[1] 4
> summary(lm.out)$coefficients["(Intercept)","Estimate"]+
+ summary(lm.out)$coefficients["GroupTreatment","Estimate"]
[1] 6
> aggregate(ds$Post~ds$Group,ds,mean)#ANOVA
   ds$Group ds$Post
1   Control       4
2 Treatment       6
> aggregate(ds$Post~ds$Group,ds,mean)#Pearson's r
   ds$Group ds$Post
1   Control       4
2 Treatment       6
>
> ###PAIRED T TEST SUBSUMED BY MLR and ANOVA###
>
> ###t-test on Pre and Post
> (t.out<-t.test(ds$Post,ds$Pre,paired=TRUE))
```

```
        Paired t-test

data:  ds$Post and ds$Pre
t = 4.0747, df = 29, p-value = 0.0003265
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.4980643 1.5019357
sample estimates:
mean of the differences
                      1

>
> ###MLR on Pre and Post
> lm.out<-lm(I(Post-Pre)~1,data=ds)
> summary(lm.out)

Call:
lm(formula = I(Post - Pre) ~ 1, data = ds)

Residuals:
     Min       1Q   Median       3Q      Max
-2.33317 -1.16276 -0.02756  0.90826  2.33647

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   1.0000     0.2454   4.075 0.000327 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.344 on 29 degrees of freedom

>
> ###Repeated Measures ANOVA on Pre and Post
> aout<-aov(Test~MO+Error(id),data=dslong2)
> summary(aout)

Error: id
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals 29   59.8   2.062

Error: Within
          Df Sum Sq Mean Sq F value   Pr(>F)
MO         1   15.0  15.000    16.6 0.000327 ***
Residuals 29   26.2   0.903
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> ###Compare p values
> t.out$p.value#t-test
[1] 0.0003265097
> summary(lm.out)$coefficients["(Intercept)","Pr(>|t|)"]#MLR
[1] 0.0003265097
> unlist(summary(aout)[["Error: Within"]])["Pr(>F)1"]#ANOVA
     Pr(>F)1
0.0003265097
>
> ###Compare test statistic
> (t1<-t.out$statistic)#t-test
       t
4.074684
> (t2<-summary(lm.out)$coefficients["(Intercept)","t value"])#MLR
[1] 4.074684
> (f<-unlist(summary(aout)[["Error: Within"]])["F value1"])#ANOVA
F value1
16.60305
>
> ###Transform t to F
> t1**2#t-test
       t
16.60305
> t2**2#MLR
```

```
[1] 16.60305
> f#ANOVA
F value1
16.60305
>
> ###Transform F to t
> t1#t-test
       t
4.074684
> t2#MLR
[1] 4.074684
> sqrt(f)#ANOVA
F value1
4.074684
>
> ###Compare effect sizes
> t1*sqrt(2*(1-cor(ds$Post,ds$Pre))/nrow(ds))#t-test
        t
0.8033882
> t2*sqrt(2*(1-cor(ds$Post,ds$Pre))/nrow(ds))#MLR
[1] 0.8033882
> sqrt(f)*sqrt(2*(1-cor(ds$Post,ds$Pre))/nrow(ds))#MLR
 F value1
0.8033882
>
> ###Compare group means
> t.out$estimate#t-test
mean of the differences
                      1
> summary(lm.out)$coefficients["(Intercept)","Estimate"]#MLR
[1] 1
> describe(ds$Post-ds$Pre)$mean#ANOVA
[1] 1
>
> ###SINGLE SAMPLE T TEST SUBSUMED BY MLR###
>
> ###t-test on Pre
> (t.out<-t.test(ds$Pre))

        One Sample t-test

data:  ds$Pre
t = 22.2967, df = 29, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 3.633088 4.366912
sample estimates:
mean of x
        4

>
> ###MLR on on Pre
> lm.out<-lm(Pre~1,data=ds)
> summary(lm.out)

Call:
lm(formula = Pre ~ 1, data = ds)

Residuals:
    Min      1Q  Median      3Q     Max
-2.7328 -0.4991 -0.1004  0.6342  1.9180

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   4.0000     0.1794    22.3   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9826 on 29 degrees of freedom

>
```

```
> ###Compare p values
> t.out$p.value#t-test
[1] 8.45791e-20
> summary(lm.out)$coefficients["(Intercept)","Pr(>|t|)"]#MLR
[1] 8.45791e-20
>
> ###Compare test statistic
> (t1<-t.out$statistic)#t-test
      t
22.2967
> (t2<-summary(lm.out)$coefficients["(Intercept)","t value"])#MLR
[1] 22.2967
>
> ###Compare effect sizes
> (d<-cohensD(ds$Pre))#t-test
[1] 4.070802
> (t2/sqrt(length(ds$Post)))#MLR
[1] 4.070802
>
> ###Compare group means
> t.out$estimate#t-test
mean of x
        4
> summary(lm.out)$coefficients["(Intercept)","Estimate"]#MLR
[1] 4
>
> ###chi-square via MLR###
>
> ###descriptive statistics on Position by Group
> (x.out<-table(ds$Position,ds$Group))

           Control Treatment
  Full           5         5
  Part           6         4
  Seasonal       4         6
>
> ###chi-test on Position by Group
> chisq.test(x.out,correct=FALSE)

        Pearson's Chi-squared test

data:  x.out
X-squared = 0.8, df = 2, p-value = 0.6703

>
> ###MLR on Position by Group
> lm.out<-lm(Position~Group,data=ds)
Warning messages:
1: In model.response(mf, "numeric") :
  using type = "numeric" with a factor response will be ignored
2: In Ops.factor(y, z$residuals) : - not meaningful for factors
> summary(lm.out)

Call:
lm(formula = Position ~ Group, data = ds)

Residuals:
Error in quantile.default(resid) : factors are not allowed
In addition: Warning message:
In Ops.factor(r, 2) : ^ not meaningful for factors
>
> ###MLR on Position by Group - Try treating categories as numbers
> lm.out<-lm(as.numeric(ds$Position)~Group,data=ds)
> summary(lm.out)

Call:
lm(formula = as.numeric(ds$Position) ~ Group, data = ds)

Residuals:
     Min       1Q   Median       3Q      Max
-1.06667 -0.93333  0.06667  0.93333  1.06667
```

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)      1.9333     0.2175   8.889 1.21e-09 ***
GroupTreatment   0.1333     0.3076   0.433    0.668
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8423 on 28 degrees of freedom
Multiple R-squared:  0.006667,  Adjusted R-squared:  -0.02881
F-statistic: 0.1879 on 1 and 28 DF,  p-value: 0.668

>
> ###Compare p values
> chisq.test(x.out,correct=FALSE)$p.value#chi-square
[1] 0.67032
> anova(lm.out)["Group","Pr(>F)"]#MLR
[1] 0.6679755
>
> ###Compare test statistic
> chisq.test(x.out,correct=FALSE)$statistic#chi-square
X-squared
      0.8
> summary(lm.out)$fstatistic["value"]#MLR
    value
0.1879195
>
> ###Compare effect sizes
> cramersV(x.out)#chi-square
[1] 0.1632993
> summary(lm.out)$r.squared#MLR
[1] 0.006666667
>
> ###Transform Cramer's v to R2
> cramersV(x.out)**2#chi-square
[1] 0.02666667
> summary(lm.out)$r.squared#MLR
[1] 0.006666667
>
> ###Transform R2 to Cramer's v
> cramersV(x.out)#chi-square
[1] 0.1632993
> sqrt(summary(lm.out)$r.squared)#MLR
[1] 0.08164966
>
> ###chi-square via MLR###
>
> ###descriptive statistics on Group by Position
> (x.out<-table(ds$Group, ds$Position))

            Full Part Seasonal
  Control      5    6        4
  Treatment    5    4        6
>
> ###chi-test on Group by Position
> chisq.test(x.out,correct=FALSE)

        Pearson's Chi-squared test

data:  x.out
X-squared = 0.8, df = 2, p-value = 0.6703

>
> ###MLR on Group by Position
> lm.out<-lm(Group~Position,data=ds)
Warning messages:
1: In model.response(mf, "numeric") :
  using type = "numeric" with a factor response will be ignored
2: In Ops.factor(y, z$residuals) : - not meaningful for factors
> summary(lm.out)
```

```
Call:
lm(formula = Group ~ Position, data = ds)

Residuals:
Error in quantile.default(resid) : factors are not allowed
In addition: Warning message:
In Ops.factor(r, 2) : ^ not meaningful for factors
>
> ###MLR on Group by Position - Try treating categories as numbers
> lm.out<-lm(as.numeric(ds$Group)~Position,data=ds)
> summary(lm.out)

Call:
lm(formula = as.numeric(ds$Group) ~ Position, data = ds)

Residuals:
   Min     1Q Median     3Q    Max
  -0.6   -0.5    0.0    0.5    0.6

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)        1.5000     0.1644   9.122 9.81e-10 ***
PositionPart      -0.1000     0.2325  -0.430    0.671
PositionSeasonal   0.1000     0.2325   0.430    0.671
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.52 on 27 degrees of freedom
Multiple R-squared:  0.02667,   Adjusted R-squared:  -0.04543
F-statistic: 0.3699 on 2 and 27 DF,  p-value: 0.6943

>
> ###Compare p values
> chisq.test(x.out,correct=FALSE)$p.value#chi-square
[1] 0.67032
> anova(lm.out)["Position","Pr(>F)"]#MLR
[1] 0.694275
>
> ###Compare test statistic
> (x2<-chisq.test(x.out,correct=FALSE)$statistic)#chi-square
X-squared
      0.8
> (F<-summary(lm.out)$fstatistic["value"])#MLR
   value
0.369863
>
> ###Transform x2 to F
> x2/((length(levels(ds$Position))-1)*(length(levels(ds$Group))-1))#chi-square
X-squared
      0.4
> summary(lm.out)$fstatistic["value"]#MLR
   value
0.369863
>
> ###Transform F to x2
> x2#chi-square
X-squared
      0.8
> F*((length(levels(ds$Position))-1)*(length(levels(ds$Group))-1))#MLR
   value
0.739726
>
> ###Compare effect sizes
> cramersV(x.out)#chi-square
[1] 0.1632993
> summary(lm.out)$r.squared#MLR
[1] 0.02666667
>
> ###Transform Cramer's v to R2
> cramersV(x.out)**2#chi-square
[1] 0.02666667
```

```
> summary(lm.out)$r.squared#MLR
[1] 0.02666667
>
> ###Transform R2 to Cramer's v
> cramersV(x.out)#chi-square
[1] 0.1632993
> sqrt(summary(lm.out)$r.squared)#MLR
[1] 0.1632993
>
```