

Supplemental Material

Illustration

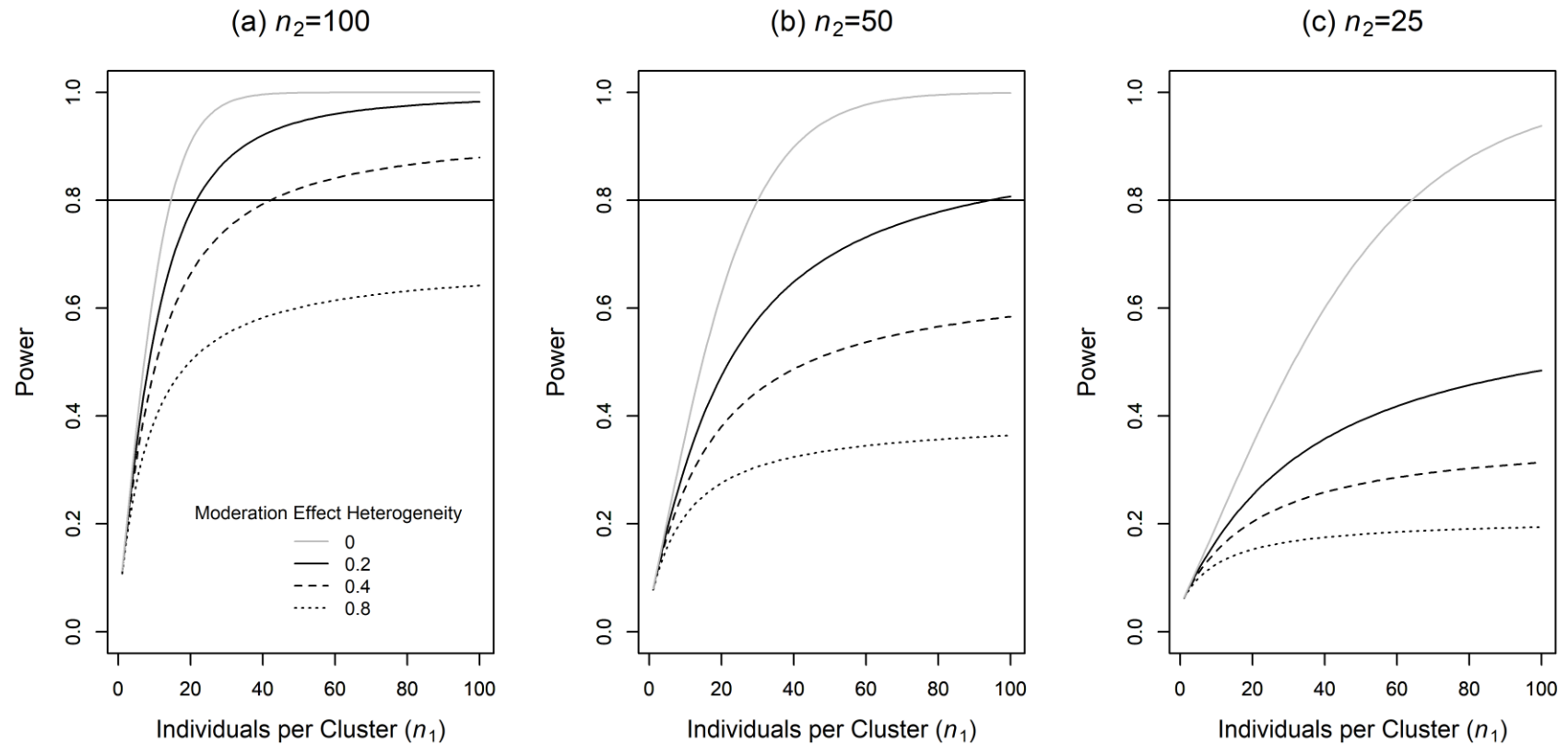
We supplement our error variance derivations using a hypothetical educational research study to illustrate the influence of parameters on power to detect moderation effects across various conditions. Consider the examination of a spatial reasoning intervention designed to increase student mathematics achievement (Lowrie et al., 2021). Students are individually randomized to the summer school intervention program or placed on a wait-list and student mathematics anxiety is a possible moderator of the intervention-outcome relationship (Ashcraft & Krause, 2007; Ashcraft & Moore, 2009). Waitlisted students continue with their summer as usual while students assigned to the summer school intervention program become nested within intervention-groups. Power rates for detecting the moderation effect of student math anxiety would increase as total sample size increases and when the magnitude of the moderation effect is larger. These general, and expected results hold across all conditions considered and so we illustrate analyses when varying the number of intervention-groups (i.e., $n_2^{(t)}$) or students per intervention-group ($n_1^{(t)}$) with $ME = 0.1$. Results found when $ME = 0.1$ replicate for $ME = 0.2$ but with larger power rates (see Supplemental Materials).

Sample Size

Regarding the relative contribution of sample sizes, changes to the number of intervention groups ($n_2^{(t)}$) would reduce error variance (σ_{ME}^2) and increase power to detect the moderation effect more than changes to the number of students per group ($n_1^{(t)}$; see Figures 2 and 3). This relationship is dependent on the values of other parameters such as moderation effect heterogeneity (ω). For example, increasing $n_1^{(t)}$ will increase power across a larger range of $n_1^{(t)}$ values when the moderation effect of math anxiety has little heterogeneity (i.e., small ω) but the same increases to $n_1^{(t)}$ will do little to increase power when $\omega = 0.8$. Conversely, increasing $n_2^{(t)}$ continuously increases power until rates approach one (see Figure 3) and this relationship remains fairly consistent across ω values.

Figure 2

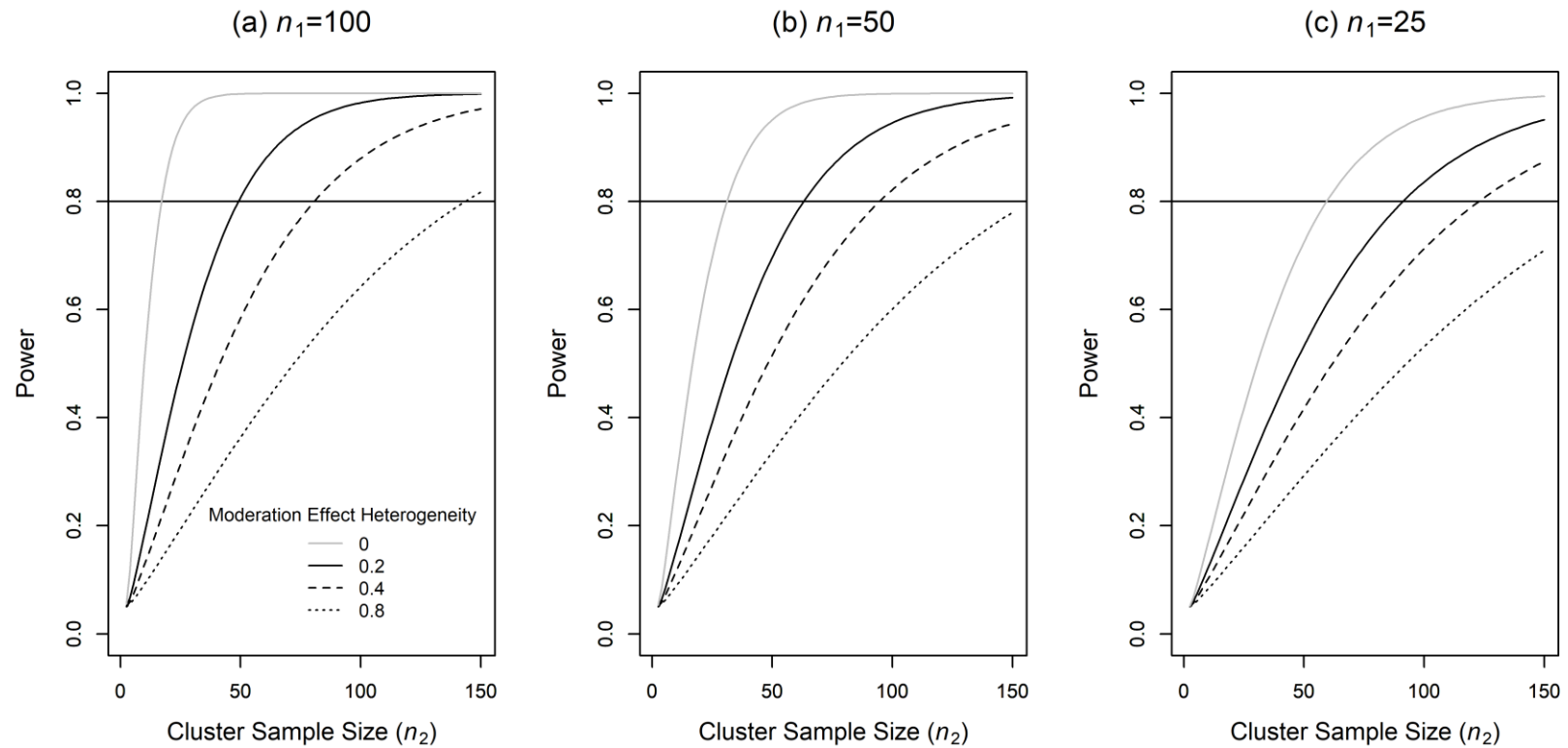
Statistical Power to Detect Moderation Effects with Heterogeneity in a Two/One Partially Nested Design as Individuals per Cluster Increases and Cluster Sample Size is (a) 100, (b) 50 and (c) 25



Note. This figure represents a continuous moderator when sample sizes are equivalent in the treatment and control condition, without covariates with $ME=0.1$ and $\rho=0.2$.

Figure 3

Statistical Power to Detect Moderation Effects with Heterogeneity in a Two/One Partially Nested Design as Cluster Sample Size Increases and Individual per Cluster Sample Size is (a) 100, (b) 50 and (c) 25



Note. This figure represents a continuous moderator when sample sizes are equivalent in the treatment and control condition, without covariates with $ME=0.1$ and $\rho=0.2$.

Moderation Effect Heterogeneity

Greater amounts of heterogeneity in the moderation effect of student math anxiety would require larger sample sizes to detect the effect (see Tables 1 and 2). Put differently, power to detect the moderating effect of student math anxiety will generally decline as ω increases. For example, in a study without covariates and $\omega = 0.8$, achieving 80% power will almost always require more than 150 intervention groups ($n_2^{(t)} > 150$). In comparison, moderation effects when $\omega = 0$ could be consistently detected (i.e., >80% power) with less than 50 intervention groups ($n_2^{(t)} \leq 50$).

We did not consider the influence of increased moderator or outcome variance on power because they are rather immutable and random slope variance (τ_{11}^2) is considered through ω ($\omega = \tau_{11}^2 / \tau_{00}^2$). However, we can examine the influence of outcome variance decomposition on planning our hypothetical study. For outcome variance in our hypothetical study, imagine we standardized variance terms so that $\tau_{00}^2 + \sigma_{y^{(t)}}^2 = 1$, $\rho = \tau_{00}^2 / \tau_{00}^2 + \sigma_{y^{(t)}}^2$, and a pilot study indicated $\rho = 0.1$ ($\sigma_{y^{(t)}}^2 = 0.1$ with $\tau_{00}^2 = 0.9$) or $\rho = 0.2$ ($\sigma_{y^{(t)}}^2 = 0.2$ with $\tau_{00}^2 = 0.8$). Power to detect the moderation effect in our study would be higher when $\rho = 0.1$ (see Table 1) and increasing the number of students per intervention group ($n_1^{(t)}$) would have a stronger influence on power when $\rho = 0.1$.

Variance Explained by Covariates

Covariates that explain variance in the outcome ($R_{y^{(t)}}^2, R_{y^{(c)}}^2$) increase the power to detect moderation effects (see Table 2) or reduce the sample size required to consistently detect a moderation effect. In our example, we could include a mathematics achievement test score from the previous year (i.e., pre-test) as a covariate to explain variance in a student's current mathematics achievement. If this or other covariates explained 40% of the variance in $\sigma_{y^{(t)}}^2$ and $\sigma_{y^{(c)}}^2$ ($R^2 = 0.4$), power rates would increase by 10% or more and by 15% or more if $R^2 = 0.7$. In terms of adequate sample sizes, our hypothetical study with prognostic covariates would achieve nearly 80% power to detect the moderation effect with 25 intervention groups when $\rho = 0.1$, $\omega = 0.2$, and $n_1^{(t)} \geq 100$.

Binary Moderator

In our example study we have only considered a continuous moderator, but our formulas accommodate binary moderators as well. For example, it may be of interest to examine treatment effect heterogeneity for Black students (e.g., Shin, 2012). The relationships between power and the parameters discussed remain unchanged with a binary moderator. The notable exception being substantially lower power rates across all conditions when compared to a continuous moderator. In our analyses this difference is a result of the reduction in moderator variance. Specifically, the maximum variance of a binary moderator is 0.25 and the variance of our continuous moderator was set to 1.0. With a binary moderator, $ME = 0.1$, and no prognostic covariates included in the analytic model, adequate power (i.e., 80%) would only be achieved when $n_2^{(t)} \geq 100$ and $n_1^{(t)} \geq 100$ (i.e., more than 100 intervention groups with at least 100 students per group) even under conditions that maximize power (e.g., small ρ and ω values). When $ME = 0.2$ adequate power would still likely require $n_2^{(t)} \geq 50$ and small ρ and ω values. The benefits of covariates do extend to detecting moderation effects from binary moderators. With covariates such that $R^2 = 0.4$ or $R^2 = 0.7$, a wide range of sample sizes and study design conditions would achieve 80% power with a binary moderator. The inclusion of a covariate such as a pre-test is a practical necessity in our example study scenario examining moderation effects with a binary variable.

Unbalanced Study Arms

Thus far our example power analyses have assumed a balanced sample across study arms with $n^{(c)} = n_1^{(t)} \times n_2^{(t)}$ but our formulas accommodate unbalanced sampling. Power rates will likely be similar across these designs. For example, if our study had access to a large control group sample (i.e., students placed on summer as usual wait list) such that it was twice the size of the treatment group ($2n^{(c)}$) or oversampled the treatment group such that the control group was half its size ($n^{(c)} / 2$) power to detect the moderation effect would be within a few percentage points (given $n_2^{(t)}$ and total sample size are equivalent). The influence of other study design parameters remains consistent with unbalanced sampling across study arms. Specifically, increasing ρ and ω values would decrease power to detect moderation effects while increases in ME and sample size, particularly $n_2^{(t)}$, would increase power.

Tables
Analytic Model without Covariates

Table 1A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design without Covariates and ME=0.1

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ω	n_2	n_1	$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	0.98	0.98	0.00	1.00	1.00	0.00
		50	0.95	0.95	0.00	0.99	0.98	0.00
		25	0.83	0.84	-0.01	0.89	0.89	0.00
		10	0.54	0.56	-0.02	0.56	0.57	-0.01
	50	100	0.82	0.81	0.00	0.94	0.94	0.00
		50	0.71	0.70	0.00	0.82	0.81	0.00
		25	0.54	0.54	-0.01	0.60	0.61	-0.01
		10	0.30	0.31	-0.01	0.31	0.32	-0.01
	25	100	0.50	0.50	0.00	0.69	0.67	0.02
		50	0.40	0.40	0.00	0.50	0.50	0.00
		25	0.28	0.29	-0.01	0.32	0.33	-0.01
		10	0.16	0.17	-0.02	0.15	0.17	-0.02
0.4	100	100	0.89	0.88	0.00	0.98	0.98	0.00
		50	0.83	0.83	0.00	0.94	0.94	0.00
		25	0.72	0.72	0.00	0.82	0.83	-0.01
		10	0.47	0.49	-0.02	0.52	0.54	-0.02
	50	100	0.60	0.60	0.00	0.81	0.81	0.00
		50	0.52	0.52	0.00	0.69	0.69	0.00
		25	0.41	0.42	-0.01	0.51	0.52	-0.01
		10	0.26	0.27	-0.01	0.29	0.30	-0.01
	25	100	0.33	0.33	0.00	0.50	0.50	0.01
		50	0.28	0.28	0.00	0.39	0.39	0.00
		25	0.22	0.23	-0.01	0.28	0.28	-0.01
		10	0.15	0.15	-0.01	0.16	0.17	-0.01
0.6	100	100	0.76	0.75	0.01	0.94	0.94	0.00
		50	0.71	0.71	0.00	0.88	0.88	0.00
		25	0.61	0.61	-0.01	0.75	0.76	-0.01
		10	0.42	0.44	-0.02	0.49	0.51	-0.02
	50	100	0.47	0.46	0.01	0.69	0.69	0.00
		50	0.41	0.42	0.00	0.61	0.60	0.01
		25	0.35	0.35	0.00	0.47	0.47	0.00
		10	0.23	0.24	-0.01	0.27	0.28	-0.02
	25	100	0.25	0.24	0.00	0.40	0.39	0.00
		50	0.24	0.23	0.01	0.33	0.33	0.00
		25	0.19	0.19	0.00	0.25	0.26	-0.01
		10	0.13	0.14	-0.01	0.15	0.15	-0.01

Table 1A(continued)

0.8	100	100	0.65	0.64	0.01	0.88	0.88	0.00
		50	0.60	0.61	0.00	0.82	0.82	0.00
		25	0.54	0.55	0.00	0.70	0.71	-0.01
		10	0.38	0.40	-0.01	0.46	0.48	-0.02
	50	100	0.37	0.37	0.00	0.59	0.59	0.00
		50	0.35	0.34	0.01	0.51	0.52	0.00
		25	0.29	0.30	0.00	0.41	0.42	-0.01
		10	0.21	0.22	-0.01	0.25	0.26	-0.01
	25	100	0.21	0.20	0.01	0.34	0.33	0.01
		50	0.19	0.19	0.00	0.28	0.28	0.00
		25	0.17	0.17	0.00	0.21	0.22	-0.01
		10	0.13	0.13	0.00	0.14	0.15	-0.01

Table 2A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design without Covariates and ME=0.2

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ω	n_2	n_1	$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	1.00	1.00	0.00	1.00	1.00	0.00
		10	0.99	0.99	0.00	0.99	0.99	0.00
	50	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	0.99	0.99	0.00	0.99	0.99	0.00
		10	0.83	0.84	-0.01	0.85	0.85	0.00
	25	100	0.98	0.97	0.00	1.00	1.00	0.00
		50	0.93	0.93	0.00	0.98	0.98	0.00
		25	0.81	0.80	0.01	0.87	0.86	0.01
		10	0.50	0.51	-0.02	0.52	0.52	0.00
0.4	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	1.00	1.00	0.00	1.00	1.00	0.00
		10	0.97	0.97	0.00	0.98	0.98	0.00
	50	100	0.99	0.99	0.00	1.00	1.00	0.00
		50	0.98	0.98	0.00	1.00	1.00	0.00
		25	0.95	0.94	0.00	0.98	0.98	0.00
		10	0.75	0.77	-0.02	0.81	0.82	-0.01
	25	100	0.87	0.86	0.01	0.98	0.97	0.01
		50	0.80	0.79	0.01	0.93	0.92	0.01
		25	0.68	0.67	0.01	0.80	0.79	0.00
		10	0.44	0.45	-0.01	0.49	0.49	0.00
0.6	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	1.00	1.00	0.00	1.00	1.00	0.00
		10	0.94	0.95	-0.01	0.97	0.98	0.00
	50	100	0.96	0.96	0.00	1.00	1.00	0.00
		50	0.94	0.94	0.00	0.99	0.99	0.00
		25	0.88	0.88	0.00	0.96	0.96	0.00
		10	0.69	0.71	-0.02	0.77	0.79	-0.02
	25	100	0.74	0.72	0.01	0.93	0.92	0.01
		50	0.68	0.67	0.01	0.86	0.86	0.01
		25	0.59	0.58	0.01	0.73	0.73	0.01
		10	0.40	0.41	-0.01	0.46	0.47	0.00

Table 2A (continued)

0.8	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	0.99	0.99	0.00	1.00	1.00	0.00
		25	0.98	0.98	0.00	1.00	1.00	0.00
		10	0.91	0.92	-0.01	0.96	0.97	-0.01
	50	100	0.91	0.91	0.01	0.99	0.99	0.00
		50	0.88	0.88	0.00	0.98	0.98	0.00
		25	0.81	0.82	0.00	0.94	0.94	0.00
		10	0.64	0.66	-0.02	0.74	0.76	-0.02
	25	100	0.62	0.61	0.01	0.87	0.86	0.01
		50	0.59	0.57	0.01	0.80	0.79	0.01
		25	0.51	0.51	0.00	0.68	0.67	0.01
		10	0.36	0.37	-0.01	0.44	0.44	0.00

Table 3A. Comparison of Formula Based Type I Error and Standard Error of the Moderation Effect and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design without Covariates

ρ	n_2	n_1	Rejection Rate	Formula α	Difference	True Standard Error of ME	Formula Standard Error of ME	Difference
0.2	100	100	0.044	0.050	-0.006	0.0133	0.0137	-0.0003
		50	0.047	0.050	-0.004	0.0191	0.0193	-0.0002
		25	0.045	0.050	-0.005	0.0272	0.0273	-0.0001
		10	0.047	0.050	-0.003	0.0436	0.0432	0.0004
	50	100	0.041	0.050	-0.009	0.0189	0.0195	-0.0006
		50	0.039	0.050	-0.011	0.0268	0.0276	-0.0009
		25	0.041	0.050	-0.009	0.0383	0.0391	-0.0008
		10	0.042	0.050	-0.008	0.0619	0.0619	0.0001
	25	100	0.032	0.050	-0.018	0.0268	0.0282	-0.0014
		50	0.035	0.050	-0.015	0.0382	0.0399	-0.0017
		25	0.032	0.050	-0.018	0.0539	0.0565	-0.0026
		10	0.033	0.050	-0.017	0.0864	0.0895	-0.0031
0.1	100	100	0.047	0.050	-0.004	0.0138	0.0141	-0.0002
		50	0.042	0.050	-0.008	0.0195	0.0199	-0.0004
		25	0.044	0.050	-0.007	0.0275	0.0281	-0.0006
		10	0.045	0.050	-0.005	0.0440	0.0445	-0.0005
	50	100	0.040	0.050	-0.010	0.0196	0.0201	-0.0005
		50	0.042	0.050	-0.008	0.0279	0.0284	-0.0006
		25	0.038	0.050	-0.012	0.0390	0.0402	-0.0013
		10	0.042	0.050	-0.008	0.0631	0.0637	-0.0005
	25	100	0.036	0.050	-0.014	0.0281	0.0290	-0.0010
		50	0.033	0.050	-0.018	0.0389	0.0411	-0.0022
		25	0.033	0.050	-0.017	0.0562	0.0582	-0.0020
		10	0.034	0.050	-0.016	0.0894	0.0923	-0.0029

Table 4A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator Without a Random Slope ($\omega = 0$) in a Balanced Design without Covariates

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ρ	n_2	n_1	$ME = 0.1$			$ME = 0.2$		
0.2	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	0.96	0.95	0.00	1.00	1.00	0.00
		10	0.62	0.63	-0.01	1.00	1.00	0.00
	50	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	0.95	0.94	0.01	1.00	1.00	0.00
		25	0.71	0.71	0.00	1.00	1.00	0.00
		10	0.34	0.36	-0.01	0.88	0.89	0.00
	25	100	0.94	0.92	0.02	1.00	1.00	0.00
		50	0.69	0.67	0.02	1.00	1.00	0.00
		25	0.39	0.40	-0.01	0.94	0.92	0.01
		10	0.17	0.19	-0.02	0.57	0.58	-0.01
0.1	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	0.94	0.94	0.00	1.00	1.00	0.00
		10	0.59	0.61	-0.01	0.99	0.99	0.00
	50	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	0.94	0.93	0.01	1.00	1.00	0.00
		25	0.69	0.69	0.00	1.00	1.00	0.00
		10	0.33	0.34	-0.01	0.87	0.87	0.00
	25	100	0.93	0.91	0.02	1.00	1.00	0.00
		50	0.67	0.65	0.02	1.00	1.00	0.00
		25	0.36	0.38	-0.01	0.92	0.91	0.02
		10	0.17	0.18	-0.02	0.54	0.55	-0.01

Table 5A

Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Binary Moderator in a Balanced Design without Covariates and ME=0.1

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ω	n_2	n_1	$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	0.84	0.84	0.00	0.89	0.89	0.00
		50	0.62	0.62	0.00	0.66	0.66	0.00
		25	0.40	0.41	-0.01	0.40	0.41	-0.01
		10	0.18	0.20	-0.02	0.17	0.19	-0.02
	50	100	0.54	0.54	0.00	0.61	0.61	0.00
		50	0.36	0.36	0.00	0.38	0.38	0.00
		25	0.22	0.22	-0.01	0.21	0.22	-0.01
		10	0.11	0.12	-0.01	0.10	0.11	-0.01
	25	100	0.30	0.30	0.00	0.33	0.33	0.00
		50	0.18	0.19	-0.01	0.19	0.21	-0.02
		25	0.12	0.13	-0.01	0.11	0.13	-0.02
		10	0.06	0.08	-0.02	0.07	0.08	-0.01
0.4	100	100	0.71	0.71	0.00	0.83	0.83	0.00
		50	0.54	0.54	0.00	0.62	0.62	0.00
		25	0.37	0.37	0.00	0.38	0.38	0.00
		10	0.19	0.19	-0.01	0.18	0.19	-0.01
	50	100	0.43	0.43	0.00	0.53	0.53	0.00
		50	0.31	0.31	0.00	0.34	0.35	0.00
		25	0.20	0.20	0.00	0.20	0.21	-0.01
		10	0.11	0.12	-0.01	0.10	0.11	-0.01
	25	100	0.22	0.22	0.00	0.29	0.29	0.00
		50	0.16	0.17	-0.01	0.18	0.19	-0.01
		25	0.11	0.12	-0.01	0.11	0.12	-0.02
		10	0.07	0.08	-0.02	0.06	0.08	-0.02
0.6	100	100	0.62	0.62	0.00	0.76	0.76	0.00
		50	0.48	0.48	0.00	0.57	0.57	0.00
		25	0.34	0.35	-0.01	0.36	0.36	0.00
		10	0.17	0.19	-0.01	0.17	0.18	-0.01
	50	100	0.35	0.35	0.00	0.47	0.46	0.01
		50	0.26	0.26	-0.01	0.32	0.32	0.00
		25	0.18	0.19	-0.01	0.20	0.20	-0.01
		10	0.11	0.12	-0.01	0.10	0.11	-0.01
	25	100	0.18	0.19	-0.01	0.25	0.25	-0.01
		50	0.15	0.15	-0.01	0.16	0.17	-0.01

		25	0.10	0.11	-0.02	0.10	0.12	-0.02
		10	0.07	0.08	-0.01	0.06	0.08	-0.02
0.8	100	100	0.54	0.53	0.00	0.71	0.71	0.00
		50	0.43	0.44	-0.01	0.54	0.54	0.00
		25	0.31	0.31	0.00	0.34	0.35	-0.01
		10	0.17	0.18	-0.01	0.17	0.18	-0.01
	50	100	0.30	0.30	0.00	0.42	0.42	0.00
		50	0.24	0.24	0.00	0.30	0.30	0.00
		25	0.18	0.18	0.00	0.19	0.20	-0.01
		10	0.10	0.12	-0.01	0.10	0.11	-0.01
	25	100	0.16	0.17	0.00	0.21	0.22	-0.01
		50	0.13	0.14	0.00	0.16	0.16	-0.01
		25	0.10	0.11	-0.01	0.10	0.12	-0.02
		10	0.07	0.08	-0.01	0.06	0.08	-0.02

Table 6A

Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Binary Moderator in a Balanced Design without Covariates and ME=0.2

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ω	n_2	n_1	$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	1.00	1.00	0.00	1.00	1.00	0.00
		25	0.93	0.93	0.00	0.93	0.93	0.00
		10	0.59	0.60	-0.01	0.57	0.57	-0.01
	50	100	0.99	0.98	0.00	1.00	0.99	0.00
		50	0.90	0.90	0.00	0.92	0.91	0.01
		25	0.67	0.67	0.00	0.67	0.66	0.00
		10	0.32	0.34	-0.02	0.30	0.32	-0.01
	25	100	0.82	0.81	0.01	0.88	0.86	0.02
		50	0.60	0.59	0.01	0.62	0.61	0.01
		25	0.36	0.36	-0.01	0.35	0.36	-0.01
		10	0.16	0.18	-0.02	0.15	0.17	-0.02
	0.4	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	0.99	0.99	0.00	1.00	0.99	0.00
		25	0.90	0.90	0.00	0.91	0.91	-0.01
		10	0.59	0.60	-0.01	0.56	0.57	-0.01
0.4	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	0.99	0.99	0.00	1.00	0.99	0.00
		25	0.90	0.90	0.00	0.91	0.91	-0.01
		10	0.59	0.60	-0.01	0.56	0.57	-0.01
	50	100	0.95	0.94	0.01	0.98	0.98	0.00
		50	0.83	0.83	0.00	0.88	0.88	0.00
		25	0.62	0.62	0.00	0.65	0.64	0.01
		10	0.31	0.33	-0.02	0.30	0.32	-0.02
	25	100	0.69	0.68	0.01	0.81	0.80	0.01
		50	0.52	0.52	0.00	0.58	0.58	0.01
		25	0.33	0.34	-0.01	0.34	0.35	-0.01
		10	0.16	0.18	-0.02	0.15	0.17	-0.03
	0.6	100	0.99	1.00	0.00	1.00	1.00	0.00
		50	0.97	0.97	0.00	0.99	0.99	0.00
		25	0.86	0.87	0.00	0.90	0.90	0.00
		10	0.56	0.57	-0.01	0.55	0.56	-0.01
0.6	100	100	0.99	1.00	0.00	1.00	1.00	0.00
		50	0.97	0.97	0.00	0.99	0.99	0.00
		25	0.86	0.87	0.00	0.90	0.90	0.00
		10	0.56	0.57	-0.01	0.55	0.56	-0.01
	50	100	0.89	0.88	0.01	0.97	0.96	0.00
		50	0.78	0.77	0.01	0.85	0.85	0.01
		25	0.57	0.57	0.00	0.61	0.61	0.00
		10	0.31	0.32	-0.01	0.30	0.31	-0.01
	25	100	0.59	0.58	0.00	0.73	0.73	0.01
		50	0.45	0.45	0.00	0.53	0.53	0.00
		25	0.32	0.32	0.00	0.33	0.33	0.00

		10	0.16	0.17	-0.02	0.15	0.17	-0.02
0.8	100	100	0.99	0.98	0.00	1.00	1.00	0.00
		50	0.95	0.95	0.00	0.98	0.98	0.00
		25	0.83	0.84	0.00	0.89	0.88	0.01
		10	0.54	0.55	-0.02	0.54	0.54	0.00
	50	100	0.82	0.82	0.01	0.94	0.94	0.00
		50	0.71	0.70	0.00	0.82	0.81	0.01
		25	0.54	0.54	0.00	0.59	0.59	0.00
		10	0.30	0.31	-0.01	0.29	0.30	-0.02
	25	100	0.51	0.50	0.00	0.67	0.66	0.02
		50	0.41	0.41	0.00	0.50	0.50	0.00
		25	0.29	0.29	-0.01	0.33	0.33	0.00
		10	0.15	0.16	-0.01	0.14	0.16	-0.02

Table 7A. Comparison of Formula Based Type I Error and Standard Error of the Moderation Effect and Monte Carlo Simulation Results for a Binary Moderator in a Balanced Design without Covariates

ρ	n_2	n_1	Rejection Rate	Formula α	Difference	True Standard Error of ME	Formula Standard Error of ME	Difference
0.2	100	100	0.041	0.050	-0.009	0.0267	0.0273	-0.0007
		50	0.046	0.050	-0.004	0.0384	0.0386	-0.0002
		25	0.043	0.050	-0.007	0.0538	0.0547	-0.0009
		10	0.043	0.050	-0.007	0.0859	0.0866	-0.0007
	50	100	0.042	0.050	-0.008	0.0379	0.0391	-0.0012
		50	0.040	0.050	-0.010	0.0539	0.0553	-0.0014
		25	0.041	0.050	-0.009	0.0769	0.0782	-0.0013
		10	0.038	0.050	-0.012	0.1221	0.1243	-0.0022
	25	100	0.035	0.050	-0.015	0.0537	0.0563	-0.0027
		50	0.034	0.050	-0.016	0.0760	0.0798	-0.0039
		25	0.035	0.050	-0.015	0.1087	0.1129	-0.0041
		10	0.030	0.050	-0.020	0.1720	0.1801	-0.0081
0.1	100	100	0.046	0.050	-0.004	0.0275	0.0281	-0.0006
		50	0.043	0.050	-0.007	0.0393	0.0398	-0.0005
		25	0.047	0.050	-0.003	0.0563	0.0563	0.0001
		10	0.039	0.050	-0.011	0.0884	0.0916	-0.0032
	50	100	0.042	0.050	-0.008	0.0391	0.0402	-0.0011
		50	0.042	0.050	-0.008	0.0553	0.0569	-0.0016
		25	0.042	0.050	-0.008	0.0790	0.0807	-0.0017
		10	0.038	0.050	-0.013	0.1251	0.1301	-0.0050
	25	100	0.031	0.050	-0.019	0.0549	0.0581	-0.0033
		50	0.030	0.050	-0.020	0.0780	0.0822	-0.0043
		25	0.032	0.050	-0.018	0.1106	0.1168	-0.0062
		10	0.033	0.050	-0.017	0.1767	0.1871	-0.0104

Table 8A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Binary Moderator Without a Random Slope ($\omega = 0$) in a Balanced Design without Covariates.

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ρ	n_2	n_1	<i>ME = 0.1</i>			<i>ME = 0.2</i>		
0.2	100	100	0.96	0.95	0.95	1.00	1.00	1.00
		50	0.73	0.73	0.73	1.00	1.00	1.00
		25	0.43	0.44	0.44	0.96	0.95	0.95
		10	0.20	0.20	0.21	0.63	0.63	0.64
	50	100	0.72	0.71	0.71	1.00	1.00	1.00
		50	0.43	0.44	0.44	0.95	0.95	0.95
		25	0.23	0.24	0.24	0.71	0.71	0.71
		10	0.11	0.12	0.12	0.34	0.35	0.35
	25	100	0.38	0.41	0.39	0.94	0.93	0.92
		50	0.21	0.23	0.23	0.68	0.68	0.67
		25	0.12	0.14	0.14	0.39	0.40	0.39
		10	0.07	0.08	0.08	0.17	0.19	0.19
0.1	100	100	0.94	0.94	0.94	1.00	1.00	1.00
		50	0.71	0.71	0.71	1.00	1.00	1.00
		25	0.42	0.42	0.42	0.95	0.94	0.94
		10	0.17	0.19	0.19	0.57	0.57	0.58
	50	100	0.69	0.69	0.69	1.00	1.00	1.00
		50	0.41	0.41	0.41	0.94	0.93	0.93
		25	0.22	0.23	0.23	0.69	0.69	0.69
		10	0.11	0.12	0.12	0.32	0.33	0.33
	25	100	0.37	0.39	0.38	0.93	0.92	0.91
		50	0.20	0.22	0.22	0.66	0.66	0.65
		25	0.11	0.13	0.13	0.37	0.39	0.38
		10	0.06	0.08	0.08	0.15	0.18	0.17

Table 9A

Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in an Unbalanced Design without Covariates and ME=0.1

ω	n_2	n_1	p	n_c	Total	Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
						$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	0.5	5000	15000	0.97	0.97	0.000	1.00	0.99	0.00
		50		2500	7500	0.89	0.89	0.005	0.93	0.94	-0.01
		25		1250	3750	0.72	0.72	0.000	0.76	0.76	0.00
		10		500	1500	0.42	0.44	-0.011	0.42	0.44	-0.03
	50	100		2500	7500	0.77	0.77	0.008	0.87	0.88	-0.01
		50		1250	3750	0.59	0.61	-0.023	0.70	0.70	0.00
		25		625	1875	0.44	0.44	0.002	0.45	0.45	0.00
		10		250	750	0.22	0.23	-0.013	0.21	0.24	-0.04
	25	100		1250	3750	0.45	0.45	-0.006	0.59	0.58	0.01
		50		625	1875	0.32	0.33	-0.017	0.37	0.39	-0.02
		25		312.5	937.5	0.21	0.21	-0.002	0.24	0.25	-0.01
		10		125	375	0.11	0.13	-0.014	0.11	0.13	-0.03
	100	100	2	20000	30000	1.00	0.99	0.005	1.00	1.00	0.00
		50		10000	15000	0.96	0.97	-0.009	1.00	0.99	0.00
		25		5000	7500	0.90	0.90	-0.004	0.93	0.95	-0.02
		10		2000	3000	0.65	0.68	-0.027	0.68	0.69	-0.01
	50	100		10000	15000	0.85	0.85	0.001	0.98	0.96	0.01
		50		5000	7500	0.78	0.78	0.003	0.89	0.88	0.01
		25		2500	3750	0.63	0.64	-0.012	0.70	0.69	0.01
		10		1000	1500	0.36	0.39	-0.023	0.40	0.41	-0.02
	25	100		5000	7500	0.54	0.53	0.006	0.74	0.74	0.01
		50		2500	3750	0.45	0.45	0.004	0.58	0.58	0.00
		25		1250	1875	0.35	0.34	0.001	0.39	0.40	-0.01
		10		500	750	0.20	0.21	-0.012	0.20	0.22	-0.01

0.8	100	100	0.5	5000	15000	0.61	0.61	0.01	0.85	0.85	0.00
		50		2500	7500	0.57	0.57	0.00	0.75	0.75	0.00
		25		1250	3750	0.48	0.46	0.02	0.60	0.61	-0.01
		10		500	1500	0.31	0.32	-0.02	0.34	0.35	-0.01
	50	100		2500	7500	0.37	0.37	0.00	0.56	0.55	0.02
		50		1250	3750	0.31	0.31	0.00	0.47	0.47	0.00
		25		625	1875	0.26	0.26	0.00	0.34	0.36	-0.02
		10		250	750	0.15	0.17	-0.02	0.21	0.22	-0.01
	25	100		1250	3750	0.20	0.19	0.00	0.30	0.30	0.00
		50		625	1875	0.18	0.17	0.01	0.26	0.26	-0.01
		25		312.5	937.5	0.15	0.15	0.00	0.20	0.20	-0.01
		10		125	375	0.10	0.11	-0.01	0.11	0.12	-0.01
	100	100	2	20000	30000	0.67	0.66	0.01	0.89	0.90	-0.01
		50		10000	15000	0.60	0.60	0.00	0.86	0.86	0.00
		25		5000	7500	0.59	0.60	0.00	0.76	0.77	-0.01
		10		2000	3000	0.39	0.41	-0.02	0.51	0.52	-0.02
	50	100		10000	15000	0.39	0.38	0.01	0.62	0.60	0.01
		50		5000	7500	0.37	0.37	0.00	0.55	0.54	0.01
		25		2500	3750	0.34	0.33	0.00	0.46	0.47	-0.01
		10		1000	1500	0.22	0.24	-0.02	0.30	0.32	-0.02
	25	100		5000	7500	0.22	0.20	0.01	0.35	0.34	0.02
		50		2500	3750	0.18	0.20	-0.01	0.28	0.29	-0.01
		25		1250	1875	0.17	0.17	0.00	0.24	0.25	-0.01
		10		500	750	0.12	0.14	-0.02	0.20	0.18	0.02

Note. p is the proportion of the treatment sample in the control

Analytic Model with Covariates

Table 10A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design with Covariates and $R^2 = 0.4$

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ω	n_2	n_1	$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	0.99	0.99	0.00	1.00	1.00	0.00
		50	0.98	0.98	0.00	1.00	1.00	0.00
		25	0.93	0.93	0.00	0.97	0.97	0.00
		10	0.70	0.73	-0.03	0.74	0.76	-0.02
	50	100	0.87	0.86	0.01	0.98	0.97	0.00
		50	0.80	0.79	0.01	0.92	0.92	0.00
		25	0.66	0.66	-0.01	0.77	0.77	0.00
		10	0.41	0.43	-0.02	0.45	0.47	-0.02
	25	100	0.57	0.56	0.01	0.77	0.76	0.01
		50	0.48	0.48	0.00	0.64	0.62	0.01
		25	0.37	0.38	0.00	0.44	0.45	-0.01
		10	0.21	0.23	-0.02	0.24	0.25	-0.01
0.4	100	100	0.91	0.90	0.00	0.99	0.99	0.00
		50	0.88	0.88	0.00	0.98	0.98	0.00
		25	0.80	0.80	-0.01	0.92	0.92	0.00
		10	0.60	0.62	-0.03	0.69	0.71	-0.02
	50	100	0.63	0.63	0.00	0.86	0.86	0.01
		50	0.58	0.58	0.00	0.78	0.78	0.00
		25	0.50	0.50	0.00	0.65	0.66	-0.01
		10	0.34	0.35	-0.02	0.40	0.42	-0.01
	25	100	0.36	0.35	0.01	0.56	0.55	0.01
		50	0.32	0.32	0.00	0.48	0.47	0.00
		25	0.27	0.27	0.00	0.37	0.38	0.00
		10	0.18	0.19	-0.02	0.21	0.22	-0.01
0.6	100	100	0.78	0.78	0.00	0.96	0.96	0.00
		50	0.74	0.74	0.00	0.93	0.93	0.00
		25	0.68	0.69	-0.01	0.86	0.86	0.00
		10	0.53	0.54	-0.02	0.64	0.66	-0.02
	50	100	0.48	0.48	0.01	0.73	0.73	0.00
		50	0.45	0.45	0.00	0.68	0.67	0.01
		25	0.40	0.40	0.00	0.56	0.56	0.00
		10	0.29	0.31	-0.01	0.36	0.38	-0.01
	25	100	0.26	0.26	0.00	0.44	0.43	0.01
		50	0.24	0.24	0.00	0.38	0.38	0.01
		25	0.21	0.21	-0.01	0.31	0.31	0.00
		10	0.16	0.17	-0.01	0.20	0.21	-0.01

Table 10A (continued)

0.8	100	100	0.66	0.66	0.00	0.91	0.91	0.00
		50	0.64	0.65	0.00	0.87	0.87	0.00
		25	0.58	0.59	-0.01	0.79	0.80	0.00
		10	0.47	0.48	-0.01	0.60	0.62	-0.02
	50	100	0.40	0.39	0.00	0.63	0.63	0.01
		50	0.37	0.37	0.00	0.57	0.57	0.00
		25	0.34	0.33	0.00	0.49	0.49	0.00
		10	0.26	0.26	0.00	0.33	0.35	-0.01
	25	100	0.21	0.21	0.00	0.35	0.35	0.00
		50	0.20	0.20	0.00	0.32	0.31	0.00
		25	0.18	0.18	0.00	0.26	0.27	0.00
		10	0.14	0.15	-0.01	0.18	0.19	-0.02

Note. ME=0.1

Table 11A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design with Covariates and $R^2 = 0.7$

ω	n_2	n_1	Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
			$\rho = 0.2$			$\rho = 0.1$		
0.2	100	100	1.00	1.00	0.00	1.00	1.00	0.00
		50	0.99	0.99	0.00	1.00	1.00	0.00
		25	0.98	0.98	0.00	1.00	1.00	0.00
		10	0.89	0.90	-0.01	0.93	0.95	-0.01
	50	100	0.91	0.90	0.01	0.99	0.99	0.00
		50	0.87	0.87	0.00	0.97	0.97	0.00
		25	0.80	0.80	0.00	0.91	0.91	0.00
		10	0.59	0.61	-0.02	0.68	0.70	-0.01
	25	100	0.61	0.60	0.01	0.85	0.84	0.01
		50	0.56	0.56	0.01	0.77	0.76	0.01
		25	0.47	0.47	0.00	0.64	0.63	0.01
		10	0.32	0.34	-0.01	0.38	0.40	-0.01
	10	100	0.93	0.92	0.00	1.00	1.00	0.00
		50	0.90	0.91	0.00	0.99	0.99	0.00
		25	0.87	0.87	0.00	0.97	0.98	0.00
		10	0.75	0.77	-0.02	0.87	0.88	-0.01
0.4	100	100	0.93	0.92	0.00	1.00	1.00	0.00
		50	0.90	0.91	0.00	0.99	0.99	0.00
		25	0.87	0.87	0.00	0.97	0.98	0.00
		10	0.75	0.77	-0.02	0.87	0.88	-0.01
	50	100	0.67	0.66	0.01	0.90	0.90	0.00
		50	0.63	0.62	0.00	0.86	0.86	0.00
		25	0.57	0.58	-0.01	0.78	0.78	0.00
		10	0.45	0.47	-0.01	0.58	0.59	-0.02
	25	100	0.38	0.37	0.01	0.61	0.61	0.01
		50	0.35	0.34	0.01	0.56	0.55	0.01
		25	0.32	0.32	0.00	0.47	0.47	0.00
		10	0.24	0.25	0.00	0.32	0.33	-0.01
	10	100	0.80	0.79	0.01	0.97	0.97	0.00
		50	0.77	0.78	-0.01	0.96	0.96	0.00
		25	0.74	0.74	0.00	0.93	0.93	0.00
		10	0.64	0.66	-0.02	0.81	0.82	-0.02
0.6	100	100	0.80	0.79	0.01	0.97	0.97	0.00
		50	0.77	0.78	-0.01	0.96	0.96	0.00
		25	0.74	0.74	0.00	0.93	0.93	0.00
		10	0.64	0.66	-0.02	0.81	0.82	-0.02
	50	100	0.51	0.51	0.01	0.77	0.77	0.00
		50	0.49	0.48	0.01	0.73	0.73	0.00
		25	0.44	0.44	0.00	0.66	0.66	0.00
		10	0.37	0.38	-0.01	0.52	0.53	-0.01
	25	100	0.27	0.27	0.00	0.47	0.46	0.01
		50	0.27	0.26	0.01	0.44	0.43	0.01
		25	0.23	0.24	-0.01	0.38	0.38	0.01
		10	0.20	0.21	-0.01	0.26	0.27	-0.01

Table 11A (continued)

0.8	100	100	0.69	0.68	0.01	0.92	0.92	0.00
		50	0.67	0.67	0.00	0.91	0.90	0.00
		25	0.64	0.64	0.00	0.87	0.87	0.00
		10	0.56	0.57	-0.01	0.75	0.77	-0.02
	50	100	0.40	0.39	0.00	0.66	0.66	0.01
		50	0.39	0.38	0.00	0.63	0.62	0.00
		25	0.37	0.36	0.00	0.58	0.58	0.00
		10	0.32	0.32	-0.01	0.45	0.47	-0.01
	25	100	0.22	0.22	0.00	0.37	0.37	0.00
		50	0.21	0.21	-0.01	0.34	0.35	0.00
		25	0.19	0.20	0.00	0.32	0.32	0.01
		10	0.17	0.18	-0.01	0.24	0.25	-0.01

Note. ME=0.1

Table 12A. Comparison of Formula Based Type I Error and Standard Error of the Moderation Effect and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design with Covariates and $R^2 = 0.4$

ρ	n_2	n_1	Rejection Rate	Formula α	Difference	True Standard Error of ME	Formula Standard Error of ME	Difference
0.2	100	100	0.041	0.050	-0.009	0.0103	0.0106	-0.0003
		50	0.044	0.050	-0.007	0.0147	0.0150	-0.0002
		25	0.041	0.050	-0.009	0.0207	0.0212	-0.0005
		10	0.045	0.050	-0.005	0.0335	0.0335	0.0001
	50	100	0.042	0.050	-0.008	0.0146	0.0151	-0.0005
		50	0.040	0.050	-0.010	0.0207	0.0214	-0.0007
		25	0.042	0.050	-0.008	0.0298	0.0303	-0.0005
		10	0.041	0.050	-0.009	0.0472	0.0480	-0.0008
	25	100	0.036	0.050	-0.014	0.0212	0.0218	-0.0007
		50	0.033	0.050	-0.017	0.0297	0.0309	-0.0011
		25	0.033	0.050	-0.018	0.0419	0.0438	-0.0019
		10	0.035	0.050	-0.015	0.0675	0.0694	-0.0018
0.1	100	100	0.041	0.050	-0.009	0.0107	0.0109	-0.0002
		50	0.048	0.050	-0.002	0.0154	0.0154	0.0000
		25	0.043	0.050	-0.007	0.0215	0.0218	-0.0003
		10	0.043	0.050	-0.007	0.0343	0.0345	-0.0001
	50	100	0.040	0.050	-0.011	0.0150	0.0156	-0.0006
		50	0.042	0.050	-0.008	0.0216	0.0220	-0.0005
		25	0.046	0.050	-0.004	0.0307	0.0312	-0.0005
		10	0.041	0.050	-0.009	0.0491	0.0494	-0.0003
	25	100	0.034	0.050	-0.016	0.0216	0.0225	-0.0009
		50	0.035	0.050	-0.015	0.0302	0.0319	-0.0017
		25	0.033	0.050	-0.018	0.0433	0.0451	-0.0018
		10	0.036	0.050	-0.015	0.0696	0.0714	-0.0018

Table 13A. Comparison of Formula Based Type I Error and Standard Error of the Moderation Effect and Monte Carlo Simulation Results for a Continuous Moderator in a Balanced Design with Covariates and $R^2 = 0.7$

ρ	n_2	n_1	Rejection Rate	Formula α	Difference	True Standard Error of ME	Formula Standard Error of ME	Difference
0.2	100	100	0.044	0.050	-0.006	0.0148	0.0150	-0.0002
		50	0.046	0.050	-0.004	0.0209	0.0212	-0.0002
		25	0.045	0.050	-0.005	0.0298	0.0299	-0.0002
		10	0.045	0.050	-0.005	0.0477	0.0475	0.0002
	50	100	0.039	0.050	-0.011	0.0206	0.0214	-0.0008
		50	0.039	0.050	-0.012	0.0296	0.0303	-0.0006
		25	0.040	0.050	-0.010	0.0417	0.0428	-0.0011
		10	0.040	0.050	-0.010	0.0668	0.0681	-0.0012
	25	100	0.034	0.050	-0.016	0.0295	0.0309	-0.0014
		50	0.031	0.050	-0.019	0.0414	0.0437	-0.0023
		25	0.037	0.050	-0.013	0.0595	0.0618	-0.0024
		10	0.035	0.050	-0.015	0.0960	0.0986	-0.0026
0.1	100	100	0.047	0.050	-0.003	0.0152	0.0154	-0.0002
		50	0.044	0.050	-0.006	0.0215	0.0218	-0.0003
		25	0.043	0.050	-0.007	0.0304	0.0308	-0.0004
		10	0.039	0.050	-0.012	0.0485	0.0501	-0.0016
	50	100	0.041	0.050	-0.009	0.0216	0.0220	-0.0004
		50	0.037	0.050	-0.013	0.0302	0.0312	-0.0010
		25	0.041	0.050	-0.009	0.0429	0.0442	-0.0013
		10	0.037	0.050	-0.013	0.0683	0.0712	-0.0029
	25	100	0.031	0.050	-0.019	0.0300	0.0318	-0.0019
		50	0.034	0.050	-0.016	0.0431	0.0450	-0.0020
		25	0.031	0.050	-0.019	0.0604	0.0639	-0.0035
		10	0.032	0.050	-0.018	0.0972	0.1025	-0.0053

Table 14A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Binary Moderator in a Balanced Design with Covariates

			Rejection Rate	Predicted Power	Difference	Rejection Rate	Predicted Power	Difference
ω	n_2	n_1	$R^2 = 0.4$			$R^2 = 0.7$		
0.2	100	100	0.97	0.97	0.00	1.00	1.00	0.00
		50	0.84	0.84	0.00	0.97	0.97	0.00
		25	0.59	0.59	0.00	0.83	0.83	0.00
		10	0.26	0.28	-0.02	0.47	0.48	-0.01
	50	100	0.77	0.76	0.01	0.92	0.92	0.00
		50	0.54	0.54	0.00	0.77	0.77	0.00
		25	0.32	0.33	-0.01	0.54	0.54	0.00
		10	0.14	0.15	-0.02	0.24	0.26	-0.02
	25	100	0.46	0.45	0.00	0.64	0.62	0.02
		50	0.28	0.29	-0.01	0.46	0.46	0.01
		25	0.16	0.18	-0.02	0.28	0.28	-0.01
		10	0.08	0.10	-0.02	0.13	0.15	-0.02
0.4	100	100	0.92	0.92	0.00	0.97	0.98	0.00
		50	0.79	0.78	0.01	0.92	0.92	0.00
		25	0.54	0.54	-0.01	0.76	0.76	0.00
		10	0.27	0.27	-0.01	0.44	0.45	-0.01
	50	100	0.66	0.65	0.01	0.79	0.78	0.01
		50	0.47	0.47	0.00	0.65	0.65	0.00
		25	0.30	0.30	-0.01	0.47	0.47	0.00
		10	0.14	0.16	-0.02	0.24	0.25	-0.01
	25	100	0.37	0.37	0.00	0.47	0.46	0.00
		50	0.25	0.26	0.00	0.36	0.36	0.00
		25	0.16	0.17	-0.01	0.25	0.26	-0.01
		10	0.08	0.10	-0.02	0.11	0.14	-0.02
0.6	100	100	0.86	0.86	0.00	0.93	0.93	0.00
		50	0.72	0.72	0.01	0.86	0.86	0.01
		25	0.51	0.50	0.00	0.71	0.71	0.00
		10	0.25	0.26	-0.01	0.41	0.42	-0.01
	50	100	0.57	0.56	0.01	0.67	0.66	0.01
		50	0.42	0.43	0.00	0.56	0.56	0.00
		25	0.27	0.28	0.00	0.42	0.42	0.00
		10	0.14	0.15	-0.02	0.23	0.24	-0.01
	25	100	0.30	0.30	0.00	0.38	0.38	0.00
		50	0.22	0.23	-0.01	0.29	0.30	0.00
		25	0.15	0.16	-0.01	0.22	0.22	0.00
		10	0.08	0.10	-0.02	0.13	0.14	-0.01

Table 14A (continued)

0.8	100	100	0.79	0.79	0.00	0.87	0.87	0.00
		50	0.67	0.67	0.00	0.80	0.80	0.00
		25	0.47	0.48	0.00	0.65	0.65	0.00
		10	0.24	0.25	-0.02	0.41	0.42	-0.01
	50	100	0.49	0.49	0.00	0.58	0.58	0.00
		50	0.38	0.38	0.00	0.50	0.50	0.00
		25	0.26	0.26	-0.01	0.38	0.38	0.00
		10	0.13	0.15	-0.01	0.21	0.23	-0.02
	25	100	0.26	0.27	0.00	0.31	0.31	0.00
		50	0.21	0.21	0.00	0.27	0.27	0.00
		25	0.14	0.15	-0.01	0.20	0.20	-0.01
		10	0.08	0.09	-0.02	0.12	0.13	-0.01

Note. $ME=0.1$ and $\rho=0.1$

Analytic Model with Three-level Partially Nested Data

Table 15A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in a Three-one Partially Nested Design

				Rejection Rate	Predicted Power	Difference
ω	n_3	n_2	n_1	$ME = 0.1$		
0.2	25	25	25	0.89	0.88	0.01
	10	25	25	0.45	0.44	0.01
	10	10	25	0.39	0.38	0.01
	10	10	10	0.32	0.32	0.00
0.4	25	25	25	0.64	0.63	0.01
	10	25	25	0.26	0.26	0.00
	10	10	25	0.24	0.24	0.00
	10	10	10	0.21	0.22	-0.01
0.8	25	25	25	0.36	0.36	0.00
	10	25	25	0.15	0.15	0.00
	10	10	25	0.15	0.15	0.00
	10	10	10	0.13	0.14	0.00
$ME = 0.0$						
0.4	25	25	25	0.04	0.05	-0.02
	10	25	25	0.02	0.05	-0.04
	10	10	25	0.02	0.05	-0.04
	10	10	10	0.02	0.05	-0.03

Note. All results are for a balanced design with $R^2 = 0.4$, $\sigma_{m^{(t)}}^2 = 1$, $\sigma_{y^{(t)}}^2 = 0.8$, $\tau_{00^{(t)}}^2 = 0.1$, $\phi_{00^{(t)}}^2 = 0.1$,

$\sigma_{y^{(c)}}^2 = 1$, and $\sigma_{m^{(c)}}^2 = 1$

Table 16A. Comparison of Formula Based Statistical Power and Monte Carlo Simulation Results for a Continuous Moderator in a Three-two Partially Nested Design

				Rejection Rate	Predicted Power	Difference
ω	n_3	n_2	n_1	$ME = 0.1$		
0.2	25	25	25	0.48	0.46	0.02
	10	25	25	0.18	0.18	0.00
	10	10	25	0.16	0.18	-0.01
	10	10	10	0.15	0.16	-0.02
0.4	25	25	25	0.27	0.27	0.01
	10	25	25	0.11	0.12	-0.01
	10	10	25	0.10	0.11	-0.02
	10	10	10	0.10	0.11	-0.01
0.8	25	25	25	0.15	0.16	0.00
	10	25	25	0.06	0.08	-0.02
	10	10	25	0.07	0.08	-0.01
	10	10	10	0.06	0.08	-0.02
				$ME = 0.0$		
0.4	25	25	25	0.03	0.05	-0.02
	10	25	25	0.01	0.05	-0.04
	10	10	25	0.01	0.05	-0.04
	10	10	10	0.01	0.05	-0.04

Note. All results are for a balanced design with $R^2 = 0.4$, $\sigma_{m^{(t)}}^2 = 1$, $\sigma_{y^{(t)}}^2 = 0.8$, $\tau_{00^{(t)}}^2 = 0.1$, $\phi_{00^{(t)}}^2 = 0.1$,

$\sigma_{y^{(c)}}^2 = 0.9$, $\phi_{00^{(c)}}^2 = 0.1$, and $\sigma_{m^{(c)}}^2 = 1$