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AN ESTIMATE OF POWER FOR INTACT GROUPS AND FOR INDIVIDUAL SUBJECTS: A NOTE

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The problem of intact groups has been clearly delineated in a variety of texts and papers (Campbell and Stanley, 1962; Kerlinger, 1973; Newman and Newman, 1978; Poynor, 1977; etc.). Cohen (1977) has popularized the concept and importance of power for the applied practitioner. In calculating power, one has to consider four parameters: alpha (α), N size (N), effect size (f^2), and power (P).

ALPHA

Alpha is the probability of making a Type I error. It is generally set of .05, .01, or .001. If one has no other reason, traditionally in educational research, alpha is set at .05 for a two-tailed test.

EFFECT SIZE

Effect size can be thought of conceptually as how far the means of two groups are apart in terms of standard deviation units (i.e., one standard deviation, 1/2 standard deviation, etc.). Another way of looking at it is in terms of r^2

(proportion of variance accounted for). For example, is 5% being accounted for, is 30% being accounted for, etc. Cohen (1977) uses f^2 to represent effect size. He subjectively defines three effect sizes: large (.35), medium (.15), and small (.02).

Effect size in reality is subjectively set depending on how well you know your area and what you are looking for. Effect size that is large in one instance may be small in another. Cohen's guidelines are subjective examples.

POWER

Power is defined as (1 - probability of making a Type II error). Another way of saying this is that power is the probability of detecting a difference when a difference exists. For example, if the power of a test is .76, this means that 76 times out of 100 the statistical procedures will be capable of detecting the relationship if it exists.

N SIZE

N is the total number of subjects used in the study.

CALCULATING POWER FOR INDIVIDUALS

The following formulas are used to calculate power:

$$L = f^2 v$$

where: $v = df_2 = (N - m_1)$

$u = df_1 = (m_1 - m_2)$

u is needed to enter the table. The two values needed to enter the table are L and u. Alpha helps you determine which table to enter.

The following is an example.

Let's assume we have one hundred subjects ($N = 100$). We want to be able to detect a medium size effect ($f^2 = .15$). Let's assume we have ten linearly independent variables that includes the unit vector. We're interested in asking the following question. Do these ten variables account for a significant amount of variance in the criterion over and above no information? Let's assume our alpha level is set at .01. We now can determine power.

The first step is to solve for L. $l = f^2 v$. Since $v = df_2$, which is $(N - m_1)$, or $(100 - 10)$, then $v = 90$.. And $F^2 = .15$. Then, $L = (.15)(90) = 13.5$. Alpha is equal to .01. u is equal to df_1 or $(m_1 - m_2) = (10 - 1) = 9$*. Therefore, we have:

$L = 13.5$

$u = 9$

$\alpha = .01$

Since alpha = .01, we would use Cohen's Table 9.3.1. We enter it at a u of 9. We look for an L value of 13.5. This would fall between L values of 12 and 14, or an estimated power of 49.5

*Newman, I. and Thomas, J., "A Note on the Calculation of Degrees of Freedom for Power Analysis using Multiple Linear Regression Models." Multiple Linear Regression Viewpoints, 1979, 9, 53-58.

If we are interested in doing this same problem at an alpha of .05, we would use Cohen's Table 9.3.2. Look at u = 9, L = between 12 and 14, we have an estimated power of 72. We can see that as alpha becomes less stringent, the power increases.

SOLVING FOR N

Given the same research question, in this case we are interested in determining N size. Given the following, then:

alpha = .01

f^2 = .02 (small effect size)

We subjectively set power equal to .80. (Cohen recommends a power of .80 if no other information is given. This is comparable rationale to setting alpha equal to .05).

The formula we now use is:

$$N = \frac{L}{f^2} + u + m_2$$

To determine the L size for an alpha of .05, we use Cohen's Table 9.4.2. We enter the table for a given power and a particular u. Since power is set at .80 and u is 9, our L = 15.65. Using the above formula, we solve for N.

$$N = \frac{15.65}{.02} + 9 + 1$$

$$N = 792.5$$

The suggestion is, that whenever you are solving for N and you get a decimal, you always round upwards, so N would be equal to 1082.

CALCULATING POWER FOR INTACT GROUPS

If one wanted to use the intact group, on the other hand, as the unit of analysis instead of the individual subject, a problem arises.

When the researcher has a group of five, the N is 1 for intact group analysis. When the number in the group is ten, the N is 1, and then the number in the group is twenty-five, the N is 1. A problem occurs because stability of scores varies from a group of five to a group of twenty-five. This has implications for power analysis.

A key underlying consideration in any analysis is the determination of the independent unit of analysis. When one analyzes individuals, the assumption is that each subject is performing independently of any other subject. The unit of analysis is the subject. However, when one analyzes subjects in a classroom setting or therapy groups, it is unlikely in most situations that the individual performance is independent of others in that group. The unit of analysis in this case, then, is the group.

In considering power analysis, it is important to be aware of the conditional effects on the decision to analyze subjects or groups. The research design will usually determine the unit of analysis. The power analysis, then, must be consistent with that unit of analysis.

Under certain conditions, analyzing individual subjects may result in more power due to the larger N for subjects than groups. In other conditions, analyzing groups instead of individuals results in a smaller N, but may give greater power due to the decrease in variability. A study by Malinke (1980) illustrates this difference. Data was analyzed both as from separate subjects and as from intact groups. More statistical significances were found when the intact group was the unit of analysis.

Barcikowski (1980) is one of the few researchers who has addressed the analyses and estimations of power for group means for different numbers of subjects in groups and different effect sizes. When the researcher uses the suggested effect sizes of .02 (small effect size), .15 (medium effect size), and .35 (large effect size) as suggested by Cohen (1977); also discussed by Newman and Benz, 1979) one can use the tables developed by Barcikowski and estimate what the power will be for alpha levels of .01, .05, .10 when the number of subjects in each group is either 1, 10, 15, 20, 25, 30, 35, or 40, and when the population interclass correlation is .01 and .05.

As one can see from the considerable increase in power analysis emphasis in the last few years, and the value to such insight, it is evident that power analysis for intact groups must be an important consideration for researchers.

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TABLE 9.3.1

Power as a Function of L and u at a = .01

	L											
u	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00	25.00	30.00
1	12	28	45	60	72	81	88	92	95	97	99	*
2	08	20	35	49	61	72	80	87	91	94	98	99
3	07	16	29	42	54	65	74	82	87	91	97	99
4	06	14	25	37	49	60	69	77	84	89	96	98
5	05	12	22	33	44	55	65	74	80	86	94	98
6	05	11	19	30	41	51	61	70	77	83	93	97
7	04	10	18	27	37	48	58	67	74	81	91	96
8	04	09	16	25	35	45	55	64	72	78	90	96
9	04	08	15	23	33	42	52	61	69	76	88	95
10	03	08	14	22	31	40	49	58	66	74	87	94
11	03	07	13	20	29	38	47	56	64	71	85	93
12	03	07	12	19	27	36	45	54	62	69	83	92
13	03	06	12	18	26	34	43	52	60	67	82	91
14	03	06	11	17	25	33	41	50	58	65	80	90
15	03	06	10	16	23	31	40	48	56	64	79	89
16	03	06	10	16	22	30	38	46	54	62	77	89
20	02	05	08	13	19	26	33	41	48	56	72	84
24	02	04	07	12	17	22	29	36	43	51	67	80
28	02	04	07	10	15	20	26	32	39	46	62	76
32	02	04	06	09	13	18	22	29	32	42	58	72
40	02	03	05	08	11	15	20	25	30	36	51	65
50	02	03	05	07	09	13	16	21	25	31	44	58
60	02	03	04	06	08	11	14	18	22	26	39	52
80	02	02	03	05	06	09	11	14	17	21	31	43
100	01	02	03	04	06	07	09	11	14	17	26	36

* Power greater than .995.

TABLE 9.3.2

Power as a Function of L and u at a = .05

	L											
u	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00	25.00	30.00
1	29	52	69	81	89	93	96	98	99	99	*	*
2	23	42	58	72	82	88	93	96	97	99	*	*
3	19	36	52	65	76	84	90	93	96	98	99	*
4	17	32	47	60	72	80	87	91	94	96	99	*
5	16	29	43	56	68	77	84	89	93	95	98	*
6	15	27	40	53	64	74	81	87	91	94	98	99
7	14	25	38	50	61	71	79	85	89	93	97	99
8	13	24	36	48	59	68	77	83	88	92	97	99
9	13	23	34	45	56	66	74	81	86	90	96	99
10	12	21	32	43	54	64	72	79	85	89	96	98
11	12	21	31	42	52	64	70	78	83	88	95	98
12	11	20	30	40	50	60	69	76	82	87	94	98
13	11	19	29	39	49	58	67	74	80	85	93	97
14	11	18	28	37	47	57	65	73	79	84	93	97
15	11	18	27	36	46	55	64	71	78	83	92	97
16	10	17	26	35	45	54	62	70	76	82	91	96
20	10	16	23	31	40	49	57	65	72	78	88	94
24	09	15	21	29	37	45	53	60	67	74	85	92
28	09	14	20	27	34	42	49	57	64	70	82	91
32	08	13	18	25	32	39	46	53	60	67	80	88
40	08	12	17	22	28	40	41	48	55	61	74	84
50	08	11	15	20	25	31	37	43	49	55	69	80
60	07	10	14	18	23	28	33	39	45	50	64	75
80	07	09	12	16	20	24	28	33	38	43	56	67
100	07	09	11	14	18	21	25	29	34	38	50	61

* Power greater than .995.

Note. From Statistical Power Analysis for the Behavioral Sciences by Jacob Cohen. New York: Academic Press Inc., 1977. Reprinted by permission.

TABLE 9.3.3

POWER AS A FUNCTION OF L AND u AT a = .10

u	L 2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00	18.00	20.00	25.00	30.00
1	41	64	79	83	94	97	98	99	*	*	*	*
2	33	54	70	81	89	93	96	98	99	99	*	*
3	30	48	64	76	85	90	94	97	98	99	*	*
4	27	44	60	72	81	88	92	95	97	98	*	*
5	25	41	56	69	78	85	90	94	96	98	99	*
6	24	39	53	66	75	83	89	92	95	97	99	*
7	23	37	51	63	73	81	87	91	94	96	99	*
8	22	35	49	61	71	79	85	90	93	95	98	*
9	21	34	47	58	69	77	84	88	92	95	98	99
10	20	33	45	57	67	75	82	87	91	94	98	99
11	20	31	43	55	65	74	80	86	90	93	97	99
12	19	31	42	53	63	72	79	85	89	92	97	99
13	19	30	41	52	62	70	78	84	88	92	97	99
14	19	29	40	50	60	69	76	82	87	91	96	99
15	18	28	39	49	59	68	75	81	86	90	96	98
16	18	28	38	48	58	66	74	80	85	89	95	98
20	17	26	35	44	53	62	70	76	82	86	93	97
24	16	24	32	41	50	58	66	72	78	83	92	96
28	16	23	31	39	47	55	62	69	75	80	90	95
32	15	22	29	37	45	52	60	66	72	78	88	94
40	15	20	27	34	41	48	55	61	67	73	84	91
50	14	19	25	31	37	44	50	56	62	68	79	88
60	14	18	23	29	34	40	46	52	58	64	75	84
80	13	17	21	26	31	36	41	46	52	57	69	78
100	13	16	20	24	28	32	37	42	47	52	63	73

* Power greater than .995.

TABLE 9.4.1

L AS A FUNCTION OF POWER AND u AT a = .01

u	Power .25	.50	.60	2/3	.70	.75	.80	.85	.90	.95	.99
1	3.62	6.64	8.00	9.03	9.61	10.56	11.68	13.05	14.88	17.81	24.03
2	4.67	8.19	9.75	10.92	11.57	12.64	13.88	15.40	17.43	20.65	27.42
3	5.44	9.31	11.01	12.27	12.97	14.12	15.46	17.09	19.25	22.67	29.83
4	6.07	10.23	12.04	13.38	14.12	15.34	16.75	18.47	20.74	24.33	31.80
5	6.63	11.03	12.94	14.34	15.12	16.40	17.87	19.66	22.03	25.76	33.50
6	7.13	11.75	13.74	15.21	16.01	17.34	18.87	20.73	23.18	27.04	35.02
7	7.58	12.41	14.47	15.99	16.83	18.20	19.79	21.71	24.24	28.21	36.41
8	8.01	13.02	15.15	16.73	17.59	19.00	20.64	22.61	25.21	29.29	37.69
9	8.40	13.59	15.79	17.41	18.30	19.75	21.43	23.46	26.12	30.30	38.89
10	8.78	14.13	16.39	18.05	18.96	20.46	22.18	24.25	26.98	31.26	40.02
11	9.14	14.64	16.96	18.67	19.60	21.13	22.89	25.01	27.80	32.16	41.09
12	9.48	15.13	17.50	19.25	20.20	21.77	23.56	25.73	28.58	33.02	42.11
13	9.80	15.59	18.03	19.81	20.78	22.38	24.21	26.42	29.32	33.85	43.09
14	10.12	16.04	18.53	20.35	21.34	22.97	24.83	27.08	30.03	34.64	44.02
15	10.42	16.48	19.01	20.86	21.88	23.53	25.43	27.72	30.72	35.40	44.93
16	10.72	16.90	19.48	21.37	22.40	24.08	26.01	28.34	31.39	36.14	45.80
20	11.81	18.45	21.21	23.22	24.32	26.11	28.16	30.63	33.85	38.86	49.03
24	12.80	19.86	22.78	24.90	26.06	27.94	30.10	32.69	36.07	41.32	51.93
28	13.70	21.15	24.21	26.44	27.65	29.62	31.88	34.59	38.11	43.58	54.60
32	14.55	22.35	25.55	27.87	29.13	31.19	33.53	36.35	40.01	45.67	57.08
40	16.10	24.54	27.99	30.48	31.84	34.04	36.55	39.56	43.46	49.49	61.57
50	17.83	27.00	30.72	33.40	34.86	37.22	39.92	43.14	47.31	53.74	66.59
60	19.39	29.21	33.18	36.04	37.59	40.10	42.96	46.38	50.79	57.58	71.12
80	22.18	33.15	37.55	40.72	42.43	45.21	48.36	52.11	56.96	64.39	79.13
100	24.63	36.62	41.40	44.84	46.70	49.70	53.10	57.16	62.38	70.37	86.18

TABLE 9.4.2

L AS A FUNCTION OF POWER AND u AT a = .05

u	Power .25	.50	.60	2/3	.70	.75	.80	.85	.90	.95	.99
1	1.65	3.84	4.90	5.71	6.17	6.94	7.85	8.98	10.51	13.00	18.37
2	2.26	4.96	6.21	7.17	7.70	8.59	9.64	10.92	12.65	15.44	21.40
3	2.71	5.76	7.15	8.21	8.79	9.76	10.90	12.30	14.17	17.17	23.52
4	3.08	6.42	7.92	9.05	9.68	10.72	11.94	13.42	15.40	18.57	25.24
5	3.41	6.99	8.59	9.79	10.45	11.55	12.83	14.39	16.47	19.78	26.73
6	3.70	7.50	9.19	10.44	11.14	12.29	13.62	15.26	17.42	20.86	28.05
7	3.97	7.97	9.73	11.04	11.77	12.96	14.35	16.04	18.28	21.84	29.25
8	4.22	8.40	10.24	11.60	12.35	13.59	15.02	16.77	19.08	22.74	30.36
9	4.45	8.81	10.71	12.12	12.89	14.17	15.65	17.45	19.83	23.59	31.39
10	4.67	9.19	11.15	12.60	13.40	14.72	16.24	18.09	20.53	24.38	32.36
11	4.88	9.56	11.58	13.07	13.89	15.24	16.80	18.70	21.20	25.14	33.29
12	5.08	9.90	11.98	13.51	14.35	15.74	17.34	19.28	21.83	25.86	34.16
13	5.28	10.24	12.36	13.93	14.80	16.21	17.85	19.83	22.44	26.54	35.00
14	5.46	10.55	12.73	14.34	15.22	16.67	18.34	20.36	23.02	27.20	35.81
15	5.64	10.86	13.09	14.73	15.63	17.11	18.81	20.87	23.58	27.84	36.58
16	5.81	11.16	13.43	15.11	16.03	17.53	19.27	21.37	24.12	28.45	37.33
20	6.46	12.26	14.71	16.51	17.50	19.11	20.96	23.20	26.13	30.72	40.10
24	7.04	13.26	15.87	17.78	18.82	20.53	22.49	24.85	27.94	32.76	42.59
28	7.57	14.17	16.93	18.94	20.04	21.83	23.89	26.36	29.60	34.64	44.86
32	8.07	15.02	17.91	20.02	21.17	23.04	25.19	27.77	31.14	36.37	46.98
40	8.98	16.58	19.71	21.99	23.23	25.25	27.56	30.33	33.94	39.54	50.83
50	10.00	18.31	21.72	24.19	25.53	27.71	30.20	33.19	37.07	43.07	55.12
60	10.92	19.88	23.53	26.17	27.61	29.94	32.59	35.77	39.89	46.25	58.98
80	12.56	22.67	26.75	29.70	31.29	33.88	36.83	40.34	44.89	51.89	65.83
100	14.00	25.12	29.59	32.80	34.54	37.36	40.56	44.37	49.29	56.85	71.84

TABLE 9.4.3

L AS A FUNCTION OF POWER AND u AT a = .10

u	Power .25	.50	.60	2/3	.70	.75	.80	.85	.90	.95	.99
1	.91	2.70	3.60	4.30	4.70	5.38	6.18	7.19	8.56	10.82	15.77
2	1.27	3.56	4.65	5.50	5.97	6.77	7.71	8.88	10.46	13.02	18.56
3	1.55	4.18	5.41	6.36	6.88	7.76	8.20	10.08	11.80	14.57	20.51
4	1.78	4.69	6.04	7.06	7.63	8.57	9.68	11.05	12.88	15.83	22.09
5	1.98	5.14	6.58	7.66	8.27	9.27	10.45	11.89	13.82	16.91	23.44
6	2.16	5.53	7.06	8.20	8.84	9.90	11.13	12.64	14.65	17.87	24.65
7	2.33	5.90	7.50	8.70	9.36	10.47	11.75	13.32	15.41	18.75	25.74
8	2.49	6.24	7.91	9.16	9.85	10.99	12.32	13.95	16.11	19.55	26.76
9	2.63	6.55	8.29	9.58	10.30	11.48	12.86	14.54	16.77	20.31	27.70
10	2.77	6.85	8.65	9.99	10.73	11.95	13.37	15.10	17.39	21.02	28.58
11	2.90	7.13	8.99	10.37	11.13	12.39	13.85	15.62	17.97	21.69	29.42
12	3.03	7.40	9.31	10.73	11.52	12.81	14.30	16.12	18.53	22.33	30.22
13	3.15	7.66	9.62	11.08	11.89	13.21	14.74	16.60	19.06	22.94	30.99
14	3.26	7.91	9.92	11.42	12.24	13.59	15.16	17.06	19.57	23.53	31.72
15	3.37	8.15	10.21	11.74	12.58	13.96	15.56	17.50	20.06	24.09	32.42
16	3.48	8.38	10.49	12.05	12.91	14.32	15.95	17.93	20.54	24.64	33.10
20	3.88	9.24	11.53	13.21	14.14	15.65	17.40	19.51	22.30	26.66	35.62
24	4.25	10.02	12.46	14.25	15.24	16.85	18.70	20.94	23.88	28.48	37.88
28	4.58	10.73	13.32	15.21	16.25	17.95	19.90	22.25	25.33	30.14	39.95
32	4.89	11.39	14.11	16.10	17.19	18.97	21.01	23.46	26.68	31.69	41.87
40	5.46	12.60	15.57	17.73	18.90	20.83	23.03	25.69	29.13	34.50	45.37
50	6.10	13.95	17.19	19.54	20.82	22.91	25.29	28.15	31.87	37.64	49.27
60	6.68	15.18	18.66	21.18	22.55	24.78	27.33	30.38	34.34	40.47	52.78
80	7.71	17.34	21.26	24.09	25.62	28.11	30.95	34.34	38.72	45.43	58.99
100	8.61	19.26	23.55	26.65	28.32	31.04	34.13	37.81	42.58	49.90	64.45