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DISCRIMINANT FUNCTION ANALYSIS AS A POST - HOC OPERATIONS RESEARCH DETERMINANT OF CRITERION STRENGTH AND BINARY DECISIONING RELIABILITY

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INTRODUCTION TO THE STUDY

This investigation sought to evaluate the utility of discriminant functions and their related statistics, in providing a practical post-hoc determinant of criterion strength and decisioning (sic) reliability for decision-making in the multiple alternatives environment (Wholeben, 1980a). Past experience with the use of binary integer programming (operations research) models in the selection of elementary school sites for closure during severe enrollment decline had demonstrated, that discriminant functions could provide a useful tool to the decision modeler -- not only to assist an evaluation of the model's reliability in constructing various solution set vectors (i.e. the schools to be closed versus those to remain open) in the form:

$$[\,1\;0\;0\;1\;1\;1\;0\;0\;0\;\ldots\;0\,]$$

where 1=open and 0=close; but also to provide an accountability framework for the public's understanding of the methodology utilized and the reasonableness of

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the results (solutions) according to the criterion references employed. This current paper seeks to expand upon that 1979 investigation, and provide additional data supporting the use of discriminant functions as an effective post-hoc technique for evaluating not only decisioning reliability but also the relative impact which each of the applied criterion references provided to the construction of the resulting decision (solution set vector formulation).

This paper will proceed to first acquaint the reader briefly with the idea of multiple alternatives modeling (MAM), and present a strong rationale for evaluating and simulating potential alternative decisions via an easily constructable criterion-referenced methodology. Secondly, the reader will be introduced to the "tools" of the MAM evaluator, and the rudiments of a nomenclature which will be utilized within the body of this report. Next, the findings of the 1979 school closure model (SCHCLO) will be summarized as an indication of the utility of discriminant functions in assessing decisioning model reliability for the "complete" matrix model case -- that is, a criterion model with no empty cells due to missing or incomplete (irrelevant) data entries. Finally, the use of discriminant functions for assessing modeling reliability and individual criterion strength associated with each decision will be studied, utilizing the 1981 fiscal deallocation model (ROLBAK) for evaluating budgeting unit alternatives for deallocation during funding roll-backs; and emphasizing the "scant" matrix model case.

The objective of this paper remains to demonstrate the utility of discriminant functions in assessing the relationship between those criterion references designated as providing the rationale underlying the decisions made; that is, to correlate decision sets (solution vectors) with the criteria, and thus measure the relationship of criterion variance in the prediction of solution vector membership. Furthermore as an auxillary objective, the use of discriminant functions will also provide a useful 'at-hand' technique for understanding the weighted value (or strength) for each of the criterion referenced variables entered into the discriminant function formulation. Finally, these results will demonstrate the utility of discriminant functions in the assessment of decisioning reliability and criterion strength for both the "complete" and "scant" criterion matrix of values.

CRITERION STRENGTH AND DECISIONING RELIABILITY

Evaluation and all decision-making resulting therewith, demand a high degree of accountability, visibility and responsibility. Today's complex issues require equally complex methodologies to assess both content and process of such issues, and to provide an understandable environment within which to simulate potential decisions and measure resulting effect or impact. As important moreover, is the secondary demand for providing a means for post-hoc evaluating not only the results of the simulated decisions, but also the influence (singularly as well as collectively) which the criterion references lend in making the original decisions. The clear need for the criterion-referenced decision-maker therefore is to satisfy the following five objectives:

[1] to validate the sophisticated decisioning methodologies which are so necessary for addressing today's complex problems -- yet so often ignored, discounted or feared;

[2] to study criterion effect upon the decisions made, and the ~~impact~~ which the system receives via those decisions; and thereby understand differential criterion weighting and influence -- "what" made a difference in constructing the decisions, and the varying impact resulting;

[3] to provide a high degree of visibility, and therefore accountability, to the public interests served and affected via those decisions -- generating a milieu of trust within which the decisions, no matter how unexpected, can be trusted and accepted;

[4] to simulate the variable impact upon the decisions made by introducing additional criterion influences into the model, and thereby perform a path analysis from solution to solution as different criteria are utilized to construct each decision or solution -- satisfying the innate need of some individuals who must always ask, "... but, what if ...?"; and

[5] to permit easy and quick decisioning replication within an ever changing environment -- knowing the relationships between past successful decisions and the criteria used to construct those solutions, in order to understand the potential of future decisions based upon the new values of more current criterion measures.

This paper demonstrates the superlative ability of a parametrically-based, statistical technique to satisfy each of the five objectives stated above. Relying upon multivariate, linear regression techniques, DISCRIMINANT FUNCTIONS, constructed to relate criterion vectors to a singular 'solution set vector' containing either a binary (1,0) decision representation or the composite entries of a 'selection tally vector' (0,1,2,3,...), provide the basis upon which the required measures of criterion strength and decisioning reliability will be constructed.

Generally, the notion of criterion strength refers to the identification of those measures which in effect constructed the final decision or solution to the modeled problem; and furthermore provide a 'factor' measure of ordinal value or weight within that same group of 'solution-formation' variable measures. Specifically, criterion strength will address three fundamental questions existent within all decisioning evaluation:

[1] which criterion references most clearly defend the decisions made?

[2] to what extent are the criteria individually representative of the decisions made?

[3] how do the most discriminating criteria within this decision setting relate to each other in terms of importance and influence?.

This paper will illustrate the utility of discriminant function(s) formulation for answering these questions of criterion strength, respectively, by evaluating the following rudiments of discriminant analysis:

[1] criteria included within the formation of discriminant functions -- that is, which references were 'entered' into the composition of the prepared functions;

[2] order-of-entry of each of the variables which discriminate the final solution vector; and

[3] weight (or factor strength) relationship between the standardized canonical discriminant coefficients.

Generally, the notion of decisioning reliability refers to the degree of trust which is implicit to the decision model (in this case, the "multiple alternatives model" - MAM); implicit in the sense, that the decision-maker can accept the results of such a criterion-referenced technology, both in terms of content (viz., effect of the criterion references within the model) as well as process (viz., effect of the model upon the criterion references). Specifically, decisioning reliability will address two fundamental questions existent within all decisioning evaluation:

[1] to what extent are the criteria collectively representative of the decisions made?

[2] to what extent can the defined matrix of criterion references re-predict the original binary (include v. exclude) solution?.

This paper will illustrate the utility of discriminant function(s) formulation for answering these questions of decisioning reliability, respectively, by evaluating the following charactistics of discriminant analysis:

[1] canonical correlation coefficients which offer a measure of relationship between the 'set' of discriminating criterion references and the 'set' of dummy variables which are used to represent the solution vector; and

[2] the frequency of mis-inclusions and/or mis-exclusions (or over-estimations and/or under-estimations) discovered when the classification coefficients constructed to predict a solution with the known relationships among the discriminating criteria variables, are utilized to re-predict the original dependent variable (original solution).

DESIGN OF THE MULTIPLE ALTERNATIVES MODELING (MAM) FORMULATION

The complex issue of multiple alternatives decision-making is no stranger to the educational analyst. The selection of some number of schools from a relatively large pool of potential candidates for closure is a MAM problem. Each school site represents varying measures of effectiveness, efficiency, satisfaction and expenditure for each of a number of criterion references (e.g. capacity of building, heating requirements, building age, projected enrollment change over future years, safety factors of neighborhood, and proximity of other schools and their ability to absorb transferees in the event of the first school's closure). Some of these measures will be adjudged satisfactory (or nonsatisfactory) to varying degrees, and will be comparable with other schools across the district. However, to include one site for closure as opposed to another site means, that "good" aspects of a 'to-be-closed' school must be sacrificed in order to keep the other school operational, even though the 'to-be-kept-open' school may have certain unsatisfactory measures on the same criterion variables which the now closed school exhibited as satisfactory. Such modeling of this decisioning situation is known as interactive effects modeling (Wholeben, 1980a), and represents the necessity of constructing solutions sets which will invariably include some form of 'controlled' preference/trade-off mechanics as the various alternatives are evaluated. The issue of complexity is also represented in the statement of the problem: to select some number of schools for closure in order to promote certain defined goals of the district; and thus to determine how many schools will be closed and which ones. Obviously, such a model must in effect be simultaneously performing these two inter-related decisions: "how many?" and "which ones?".

The determination of which program unit budgets will be decisioned for continued funding (versus deallocation) is another example of the multiple alternatives framework, and its superior contribution to the realm of accountable and criterion-referenced evaluation and decision-making (Wholeben and Sullivan, 1981). In the fiscal deallocation model, criteria represent the projected expenditures within each object cost code for each of the units under evaluation; and in addition contain perceptual measures of administrative level of expendability. Once again of course, exists the dual responsibilities for determining how many program budgets will be discontinued, and which ones -- based upon the interactive modeling effects of the various criterion weights across unit alternatives.

The multiple alternatives model is simply a system of simultaneous linear inequalities and equalities which collectively represents the problem to be solved. Such an algebraic linear system is portrayed in ‹Figure 1›. Note how each linear combination represents a vector of values (<u>viz.,</u> coefficients) which identifies the total, measureable impact to a system of the alternatives being modeled. Thus there exists a unique (normally) combination of coefficients for each of the criterion references used as input to the decisioning process. The alternatives themselves are further defined as binary variables (that is, taking on the value of either 0 or 1 (to be excluded in the final solution set, or to be included, respectively). Vector formulation for each criterion reference,

$$[\ a_{i1}x_1 \quad a_{i2}x_2 \quad a_{i3}x_3 \quad \cdots \quad a_{ij}x_j\ ]$$

portraying $\underline{i}$ criterion references across $\underline{j}$ alternatives, will then provide a basis for measuring total impact to the system as a whole attributable to the solution set constructed. Bounds (or limits) to what is allowable as a total impact to the system are expressed as vector entries within the conditional vector (or normally named, RHS, the right-hand-side). The RHS-values are the constants of the equations and inequalities modeling the system. ‹Figure 2› presents a listing of the four generic types of criteria to which each model should address content validity; and ‹Figure 3› depicts these criterion entries as members of the modeling framework previously illustrated within Figure 1.

The remainder of the modeling process concerns the use of an additional vector to assist in determining from the potentially hundreds (or millions, in some exercises) of possible alternatives, that one, best mix for which the best, possible solution exists. This process is called the search for optimality, and the vector is known as the objective function (or sometimes, the cost vector). Geometrically, the objective function is a n-1 dimensional figure passing through the n-tuple space (convex) which is feasible (that is, includes all of the constraints postulated through the use of the linear equalities and inequalities) and which seeks a minimum point within the feasible region (if the goal is to minimize the impact of the objective function's values upon the system) or a maximum point within the feasible region (if the goal is to maximize the defined objective function's impact to the system as a whole).

Simply stated, the multiple alternatives model is a technique which seeks to construct a solution set (a vector of 1's and 0's), such that this same solution vector represents the solution of the simultaneous system, constrained by a series of competing criterion measures (vectors), and based upon the optimality demands of the objective function.

Figure 1. Representation of the Augmented Decision Matrix Model as the "Multiple Alternatives Model" (MAM).

(Decision Variables)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8		(RHS)
Constraint #01	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}		b_1
Constraint #02	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	⋮	b_2
Constraint #03	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	$\leq$ = $\geq$	b_3
Constraint #04	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	⋮	b_4
Constraint #05	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}		b_5

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8

Cost Vector Coefficients

$$\text{Optimize: } \sum_{j=1}^{8} c_j x_j \quad \text{st: } \sum_{j=1}^{8} a_{ij} x_j \ (\leq, =, \geq)\ b_j \quad x \geq 0$$

(If MILP, x is integer; if decisonal, x=0,1 only.)

Figure 2. Representation of a Generic-Criterion Decisioning Model for Analyzing Multiple Competing Alternatives.

Criterion	Foci	Multiple Alternatives A_1 A_2 A_3 A_4 . . . A_n
(Effectiveness Criteria)		a x n sub-matrix
$CRIT_1$	EFFEC-1	effectiveness measures across alternatives
$CRIT_2$	EFFEC-2	
.	.	
.	.	
$CRIT_a$	EFFEC-a	
(Efficiency Criteria)		b x n sub-matrix
$CRIT_{a+1}$	EFFIC-1	effectiveness measures across alternatives
$CRIT_{a+2}$	EFFIC-2	
.	.	
.	.	
$CRIT_{a+b}$	EFFIC-b	
(Satisfaction Criteria)		c x n sub-matrix
$CRIT_{a+b+1}$	SATIS-1	satisfaction measures across alternatives
$CRIT_{a+b+2}$	SATIS-2	
.	.	
.	.	
$CRIT_{a+b+c}$	SATIS-C	
(Expenditure Criteria)		d x n sub-matrix
$CRIT_{a+b+c+1}$	EXPEN-1	expenditure increases across alternatives
$CRIT_{a+b+c+2}$	EXPEN-2	
.	.	
.	.	
$CRIT_{a+b+c+d}$	EXPEN-d	

Figure 3. Fiscal Allocations as a Multiple Alternative Problem, Utilizing the Decision Matrix Framework.

Multiple Alternatives

Criteria		Prog1	Prog2	Prog3	. . .	Progn	
Positive Impact	1.__	+11	+12	+13	. . .	+1n	
	2.__	+21	+22	+23	. . .	+2n	Maximize
	3.__	+31	+32	+33	. . .	+3n	
Negative Impact	1.__	-11	-12	-13	. . .	-1n	
	2.__	-21	-22	-23	. . .	-2n	Minimize
	3.__	-31	-32	-33	. . .	-3n	
Specific Costs	1.__	$11	$12	$13	. . .	$1n	
	2.__	$21	$22	$23	. . .	$2n	Sum $\leq$ total budget available
	3.__	$31	$32	$33	. . .	$3n	

TOOLS OF THE MULTIPLE ALTERNATIVES MODELING MAM FORMULATION

To construct discriminant functions from the relationships between the model just discussed above and the resulting solutions formulated, require the use of linear vectors and combinations of vectors (matrix). Only those vector and matrix formulations most germane to this paper will be discussed below. The reader is invited to be patient until the scheduled publication of the manuscript, "Multiple Alternatives Analysis for Educational Evaluation and Decision-Making" in late summer of 1982, for a detailed illustration of all vectors and matrice pertinent to MAM.

Solution Set Vector. In order to distinguish between alternatives included or excluded as members of the final solution to the system modeled, a vector of binary-decision representations is required, in the form:

$$[\,1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ \ldots\ 1\,]$$

where '1' means that the criterion values associated with that particular x(j) will be computed to measure resulting system impact; and '0' means that the underlying criterion values will have no impact upon the system.

Selection Tally Vector. To observe the effect of each criterion reference upon construction of the system solution, a method called cyclic optimization (Wholeben, 1980a; Wholeben and Sullivan, 1981) is used. Under this regimen, the model is executed once for each unique criterion being used to constrain the model, where each unique criterion is cycled through the model as the objective function. For example, during one execution in the case of the school closure model, the intent may be be prepare a solution set whereby existing capacity of the remaining schools will be maximized; in another cycle, the model will be executed such that the schools remaining open within the district will minimize the amount of energy expended for facility heating requirements. The selection tally vector is basically a frequency summation vector, compiling the number of times each alternative was chosen as part of the solution vector, across all cyclic optimizations. Such a vector will be represented as:

showing that the first alternative was selected as solution a total of 3 times, the second alternative a total of 7 times, and so forth. This vector is extremely important when the MAM procedure requires a step-wise decisioning process such as the school closure model -- evaluating a revised database after closing a single school such that the effects of closing each individual site is summarily incorporated into the next decision for determining additional site closures.

Discriminant Criterion Inclusion Vector. This vector simply represents another binary entry vector of 1's and 0's, signifying which particular criterion references were utilized via discriminant functions to develop the canonical classification coefficients, and the standardized canonical discriminant function coefficients.

Discriminant Criterion Entry Vector. This vector contains 1,2,...,k entries, where k criteria were utilized in the development of the discriminant functions, and the 1,2,...,k entries represent their order of entry into the discriminant formulation. Criterion variables not entered into the function(s) receive a value of '0', by convention.

Discriminant Weighting Summary Vector. Applying discriminant procedures to the binary solution vectors will result in the computation of standardized canonical discriminant function coefficients. These coefficients will reflect the utility of entered criterion vectors if those vectors contain standardized measures in lieu of the normal raw scores. By dividing each of the standardized canonical coefficients by the smallest of the standardized canonicals, the quotient will provide a factor of importance for each of the criteria as relative to the other criterion entered in the discriminant formulation. The discriminant weighting summary vector is a linear representation of these factors (quotients), where the minimum entry value is always '1.00' (smallest standardized coefficient divided by itself). Non-entered criterion locations receive a value of '0.00' by convention.

Other 'tools' have been referenced in the proceeding section of this paper: criterion constraint matrix, condition limits vector (RHS), objective function vector, and the cyclic optimization tracking matrix. Other formulations are currently under study by the author (e.g. the optimality weighting matrix) to investigate new relationships which may allow greater accountability and useful reliability of the multiple alternatives modeling framework.

THE "COMPLETE" MATRIX CASE: THE SCHOOL CLOSURE MODEL (SCHCLO)

A total of 32 elementary school sites were measured across 24 relatively independent criteria, resulting from previous factor analyses of an original set of 64 criterion references. The criteria chosen were utilized by the multiple alternatives model for school closures (SCHCLO; Wholeben, 1980a) to evaluate the population of sites for some set of defined closures based upon the characteristics of the data; and the needs of the school district involved. Because the criteria utilized portrayed different value orientations (i.e., positive effects to be maximized; or negative effects to be minimized), the model consisted of a total of 18 cyclic MAXIMIZATIONS, and 6 cyclic MINIMIZATIONS -- for the total 24 optimizations required. The strategy was to operationalize the cyclic model, evaluate the full N=32 sites, analyze the selection tally results, choose a single site for closure, update the database to signify the closure, and then re-evaluate the now reduced N=31 site model for an additional closure. This step-wise closure strategy was considered consistent with the pragmatic reality of deciding school closures due to severe enrollment declines.

<Figure 4> displays the results ("tracking matrix") of the N=32 cyclic optimization; and in addition, the selection tally vector entries (right column vector). The asterisked (*) vector entries signify those sites considered having the most potential for closure, due to the selection tally entries. These 4 sites were simulated 'closed' (i.e. included as '0' in the solution set vector); and a stepwise discriminant function analysis performed to analyze the relationship between the 24-vector criterion matrix which purportedly constructed the solution set, and the solution set thus constructed.

<Figure 5> displays the results of the N=32 discriminant analysis. The single discriminant function constructed required a total of 8 criterion vectors to adequately explained the variance found within the binary solution set of 4-0's and 28-1's. The group-correlative relationship between these 8 criteria and the dummy variables formed by the solution set vector, was a canonical of .8512, explaining 72.5 percent of the variance between the criterion and solution sets. Based upon the re-classification coefficients formed, the discriminant function

CRITERION SOURCE(S) FOR SCALING OF THE OPTIMIZATION (COST) VECTOR (1=MAXIMIZATION, 2=MINIMIZATION)

(N=32) SITE	(1) AREA	(1) ENROL	(1) SURVIVE	(1) POTENT	(1) STUD PROX	(2) STUD QL	(2) ZONE QL	(2) SITE AGE	(1) INTER 13	(2) INTER 01	(1) THERM EFF	(2) ENER WAST	(1) CLASS RM	(1) MINOR ITY	(2) SITE AGE	(1) AREA CAPC	(1) AREA HEAT	(1) AREA ELEC	(1) AREA MAIN	(1) AREA REPR	(1) AREA UTIL	(1) ENR HEAT	(1) ENR ELEC	(1) ENR MAIN	(DECISION=CLOSE) FREQUENCY
01			X																						1
02																									--
03						X								X											2
04												X			X										2
05						X										X					X				3
06									X						X					X					3
07											X	X											X		3
08						X				X						X		X							4
09																								X	1
10	X						X	X					X												4
11	X					X			X				X												4
12						X																			1
13																			X						1
14		X									X	X										X	X	X	6 *
15																X									1
16									X	X							X		X		X				5 *
17		X								X	X	X			X							X		X	7 *
18										X					X										2
19										X			X									X	X		4
20			X		X							X								X					4
21																									--
22																	X	X	X		X				4
23					X												X								2
24							X	X							X										3
25					X																				1
26				X										X											2
27			X																						1
28				X			X	X												X					4
29	X																								1
30		X		X			X	X						X											5 *
31																		X							1
32							X	X																	2
Frequency Total: (Site Selection Per Criterion Focus)	3	3	3	3	3	5	5	5	3	5	3	5	3	3	5	3	3	3	3	3	3	3	3	3	(*=MIP-4A-03 membership)

Frequency of Site Selection for MIP-4A Modeled Closure (N=32) Based Upon Varying Optimality Factors

Figure 4.

Figure 5.

Summary of Discriminant Function Analysis

Based Upon MIP-4A Results (N=32)

STEP	CRITERION ENTERED	CRITERION REMOVED	WILKS' LAMBDA	SIGNIFICANCE
1	ENROL		.7068	.0014
2	AREAUTIL		.5904	.0005
3	CLASSRM		.4684	.0001
4	AREAREPR		.4124	.0001
5	ENRMAIN		.3628	.0000
6	INTERO1		.3183	.0000
7	SURVIVE		.2921	.0000
8	POTENT		.2755	.0001

Eigenvalue = 2.63000   Canonical Correlation = .8512

Classification Results:

Actual Group	Cases	Close	No Close
Close	4	4 (100.0%)	--
No Close	28	--	28 (100.0%)

Percent of "grouped" cases correctly classified: 100.0

Figure 6.

Summary of Discriminant Function Analysis

Based Upon MIP-4A Sum Results (N=32)

STEP	CRITERION ENTERED	CRITERION REMOVED	WILKS' LAMBDA	SIGNIFICANCE
1	INTERO1		.5745	.0096
2	STUDOL		.3864	.0044
3	AREAREPR		.2320	.0008
4	ENERWAST		.1502	.0003
5	POTENT		.0972	.0001
6	MINORITY		.0666	.0001
7	AREAELEC		.0457	.0001
8	SITEAGE		.0310	.0001
9	THERMEFF		.0208	.0001
10	ENRELEC		.0142	.001
11	ENRHEAT		.0075	.000
12	AREAHEAT		.0036	.000
13	ENRMAIN		.0021	.000
14		THERMEFF	.0027	.000
15	AREACAPC		.0017	.000
16	CLASSRM		.0011	.000
17	ENROL		.0007	.000
18	AREAUTIL		.0005	.000

FUNCTION	EIGENVALUE	PERCENT OF UNIQUE VARIANCE EXPLAINED	CANONICAL CORRELATION
1	24.994	71.67	.9806
2	5.481	15.72	.9200
3	2.300	6.60	.8348
4	1.500	4.30	.7745
5	.600	1.72	.6124

Figure 6. (continued)

Classification Results:

(Predicted Group Membership)

ACTUAL GROUP	CASES	FREQ=0	FREQ=1	FREQ=2	FREQ=3	FREQ=4	FREQ=5
FREQ=0	2	2 (100.0%)	--	--	--	--	--
FREQ=1	9	--	9 (100.0%)	--	--	--	--
FREQ=2	6	--	--	6 (100.0%)	--	--	--
FREQ=3	4	--	--	--	4 (100.0%)	--	--
FREQ=4	7	--	--	--	--	7 (100.0%)	--
FREQ=5	4	--	--	--	--	--	4 (100.0%)

Percent of "grouped" cases correctly classified: 100.0

was able to re-predict group membership for the solution vector (inclusion v. exclusion) with 100.0 percent accuracy.

<Figure 6> illustrates the results of the discriminant analysis to evaluate the compositional relation between the selection tally vector and the full criterion database. For convenience, any selection frequency ≥ 5 was entered into the discriminant model as a frequency = 5. This was considered necessary in order to provide some control over problems associated with singular frequency tally entries, and a loss therefore of variance potential. To explain the variance existent within the selection tally vector (0,1,...,5), a total of 16 criterion vectors were entered into the final construction of 5 independent discriminant functions. Re-prediction of the original vector entries proceeded with 100.0 accuracy.

Upon the choice of a single school site for closure (j=17, since tally entry = 7), the database was updated to reflect a N=31 base, and the net effect of the student transfers from the closed site. The model was re-executed, and a new tracking matrix constructed, as displayed in <Figure 7>. A total of 4 new sites were now simulated as closed (with tally entries ≥ 4); and the discriminant model re-run.

<Figure 8> displays the discriminant results of analyzing the N=31 solution set. A total of 10 criteria were required to explained the independent variance -- two more than the N=32 analysis. The canonical correlation existed at .8392, or 70.5 percent explained (independent) variance. Re-classification resulted in a 100.0 percent accuracy level. As before, the selection tally vector for the N=31 case was analyzed by discriminant functions; and these results are illustrated in <Figure 9>. A total of 4 functions were constructed; and a re-prediction of 87.1 percent accuracy achieved. Within the re-classification, 7 occurrences of 'over-estimation' resulted (viz., an 'expected' tally entry greater than the original 'observed' value); and 1 occurrence of 'under-estimation' (viz., an 'expected' tally entry lesser than the original 'observed' value). Thus, it would seem that reclassification errored on the non-conservative side.

CRITERION SOURCE(S) FOR SCALING OF THE OPTIMIZATION (COST) VECTOR (1- MAXIMIZATION) (2-MINIMIZATION)

(N=31) SITE	(1) AREA	(1) ENROL	(1) SURVIVE	(1) POTENT	(1) STUD PROX	(2) STUD OL	(2) ZONE OL	(2) SITE OL	(1) INTER 01	(2) INTER 13	(1) THERM EFF	(2) ENER WAST	(1) CLASS RM	(1) MINOR ITY	(2) SITE AGE	(1) AREA CAPC	(1) AREA HEAT	(1) AREA ELEC	(1) AREA MAIN	(1) AREA REPR	(1) AREA UTIL	(1) ENR HEAT	(1) ENR ELEC	(1) ENR MAIN	(DECISION=CLOSE) FREQUENCY
01			X																						1
02							X																		1
03						X																			1
04								X				X			X										3
05																X					X				2
06															X										1
07											X	X													2
08						X			X	X						X									4 *
09																								X	1
10																									--
11	X								X	X			X												4 *
12						X																			1
13																			X						1
14		X									X	X										X	X	X	6 *
15																									--
16																	X				X				2
18								X							X										2
19													X									X	X		3
20					X							X								X					3
21																									--
22																	X	X	X						3
23					X																				1
24															X										1
25																									--
26				X										X											2
27			X																						1
28							X													X					2
29	X							X																	2
30		X		X			X							X											4 *
31																		X							1
32							X	X																	2
Frequency Total: (Site selection per criterion focus)	2	2	2	2	2	3	4	4	2	2	2	4	2	2	4	2	2	2	2	2	2	2	2	2	

Frequency of Site Selection for HIP-4B Modeled Closure (N=31)

Based Upon Varying Optimality Factors

Figure 7.

Figure 8.

Summary of Discriminant Function Analysis

Based Upon MIP-4B Results (N=31)

STEP	CRITERION ENTERED	CRITERION REMOVED	WILKS' LAMBDA	SIGNIFICANCE
1	ENROL		.8883	.0661
2	INTER13		.8164	.0585
3	MINORITY		.7238	.0309
4	STUDPROX		.6504	.0206
5		ENROL	.6590	.0095
6	AREAMAIN		.5944	.0073
7	CLASSRM		.5659	.0102
8	ENROL		.5336	.0126
9	ENRHEAT		.4957	.0132
10	ENERWAST		.4330	.0081
11	THERMEFF		.3387	.0020
12	ENRELEC		.2954	.0015

Eigenvalue = 2.38541   Canonical Correlation = .8394

Classification Results:

Actual Group	Cases	Close	No Close
Close	4	4 (100.0%)	--
No Close	27	--	27 (100.0%)

Percent of "grouped" cases correctly classified: 100.0

Figure 9.

Summary of Discriminant Function Analysis

Based Upon MIP-4B-Sum Results (N=31)

STEP	CRITERION ENTERED	CRITERION REMOVED	WILKS' LAMBDA	SIGNIFICANCE
1	INTERO1		.6912	.0412
2	INTER13		.5064	.0213
3	MINORITY		.3596	.0091
4	POTENT		.2703	.0070
5	STUDPROX		.2067	.0064
6	SITEOL		.1517	.0047
7	AREAUTIL		.1193	.0057
8	AREAMAIN		.0822	.0034
9	AREA		.0635	.0044

FUNCTION	EIGENVALUE	PERCENT OF UNIQUE VARIANCE EXPLAINED	CANONICAL CORRELATION
1	1.840	42.30	.8049
2	1.522	34.99	.7769
3	.680	15.63	.6363
4	.308	7.08	.4852

Figure 9. (continued)

Classification Results:

ACTUAL GROUP	CASES	FREQ=0	FREQ=1	FREQ=2	FREQ=3	FREQ=4
FREQ=0	4	4 (100.0%)	--	--	--	--
FREQ=1	11	--	10 (90.9%)	1 (9.1%)	--	--
FREQ=2	8	--	1 (12.5%)	5 (62.5%)	1 (12.5%)	1 (12.5%)
FREQ=3	4	--	--	--	4 (100.0%)	4 (100.0%)
FREQ=4	4	--	--	--	--	4 (100.0%)

Percent of "grouped" cases correctly classified: 87.10

THE "SCANT" MATRIX CASE: THE FISCAL DEALLOCATION MODEL (ROLBAK)

A total of 31 program budgeting (unit) alternatives were evaluated for defunding across a total of 10 competing criterion references. In lieu of a step-wise procedure as represented in the school closure modeling framework, the model is further constrained to choose those programs for refunding such that the new operating district budget is not less than 675,000 dollars, but not more than 700,000 dollars for the particular programs under scrutiny. To study the effect of the model's solution generation process, the feasibility region as defined by the constraint matrix and the RHS-values is constructed in two distinct patterns: a highly restricted region in which very stringent controls are defined for the modeling procedure; and a relatively relaxed region in which less stringent controls are modeled. In addition, the ROLBAK formulation is executed both for cyclic maximization of the objective functions, and for cyclic minimization of the objective functions. Thus, a total of 4 tracking matrice containing 10 potential solution sets (each) result.

This particular modeling application represents the "scant" matrix case, in that a high proportion (48.7 percent) of criterion matrix cells contained a 'zero' entry, signifying no cost for that particular alternative within a specific object-expenditure category. For the SCHCLO model, the criterion matrix was "complete" -- all cells contained a value greater than zero.

Under the 'restricted' formulation, the 17 resulting solution sets signify only 2 distinct solution vectors. In contrast under the 'relaxed' formulation, a total of 17 distinct solution vectors result. Under both restricted and relaxed limitations, 3 objective functions were unable to declare optimality due to the inability to find an initial integer-feasible solution.

‹Figure 10› and ‹Figure 11› display the solution sets resulting from optimization within the restricted region environments. The selection tally vector is noted, as well as the impact upon the total budget based upon the simulated cuts (i.e., where X=funded). As can be easily seen, the solutions resulting from optimization within the restricted environment present only two distinct alternatives for later discriminant analyses.

Effect Upon Budget Deallocation Decisions Based Upon the Variable Flows of a Cyclic Objective Function, and the Interaction of a "Maximized, Restricted" Constraint Iterative Problem.

Objective = Maximization   Constraints: Restricted (EXP=16; PERC = 500)

Budget Alternatives	01 CERT	02 CLAS	03 BENE	04 SUPL	05 INST	06 CONT	07 TRAV	08 CAPI	09 PERC	10 COMP	SELECTION TALLY	BUDGET AMOUNT
01	X	X	X	X	X	X	X	X	X	X	10	87.5
02	X	X	X	X	X	X	X	X	X	X	10	44.5
03	X	X		X			X	X			5	34.5
04	X	X	X	X	X	X	X	X	X	X	10	71.5
05	X	X	X	X	X	X	X	X	X	X	10	70.5
06											--	32.5
07	X	X	X	X	X	X	X	X	X	X	10	51.5
08											--	1.5
09	X	X		X			X	X			5	43.0
10											--	4.0
11	X	X	X	X	X	X	X	X	X	X	10	54.0
12											--	1.0
13											--	5.5
14											--	4.0
15	X	X	X	X	X	X	X	X	X	X	10	116.0
16			X		X	X			X	X	5	23.0
17	X	X	X	X	X	X	X	X	X	X	10	107.0
18											--	13.0
19											--	2.0
20											--	1.0
21											--	16.0
22											--	10.5
23			X		X	X			X	X	5	55.0
24											--	4.5
25											--	2.5
26											--	19.0
27											--	1.0
28											--	1.0
29											--	2.0
30											--	12.0
31											--	2.5
	10	10	10	10	10	10	10	10	10	10		
O.F. Value:	340.7	274.5	217.9	433.9	330.0	362.1	50.0	534.6	496.2	680.5		
Iteration at Optimality:	36	69	76	115	228	27	114	51	5000+	369		
Time (secs):	.266	.298	.288	.325	.384	.264	.383	.274	4.498	.850		
Roll-Back Savings:	680.0	680.0	680.5	680.0	680.5	680.5	680.0	680.0	680.5	680.5		
(- Cut)	(-213.5)	(-213.5)	(-213.0)	(-213.5)	(-213.0)	(-213.0)	(-213.5)	(-213.5)	(-213.0)	(-213.0)		

Note: Total Initial Budget = 893.5 ($1000's)

Figure 10.

Effect Upon Budget Deallocation Decisions Based Upon the Variable Forms of a Cyclic Objective Function, and the Interaction of a "Minimized, Restricted" Constraint Interative Problem.

Objective = Minimization   Constraints = Restricted (8xP=16; PERC=500)

Budget Alterna-tives	01 CERT	02 CLAS	03 BENE	04 SUPL	05 INST	06 CONT	07 TRAV	08 CAPI	09 PERC	10 COMP	SELECTION TALLY	BUDGET AMOUNT
01		X	X	X	X	X		X	X		7	87.5
02		X	X	X	X	X		X	X		7	44.5
03			X		X	X			X		4	34.5
04		X	X	X	X	X		X	X		7	71.5
05		X	X	X	X	X		X	X		7	70.5
06											--	32.5
07		X	X	X	X	X		X	X		7	51.5
08											--	1.5
09			X		X	X		x	X		4	43.0
10											--	4.0
11		X	X	X	X	X		X	X		7	54.0
12											--	1.0
13											--	5.5
14											--	4.0
15		X	X	X	X	X		X	X		7	116.0
16		X		X				X			3	23.0
17		X	X	X	X	X		X	X		7	107.0
18											--	13.0
19											--	2.0
20											--	1.0
21											--	16.0
22											--	10.5
23		X		X				X			3	55.0
24											--	4.5
25											--	2.5
26											--	19.0
27											--	1.0
28											--	1.0
29											--	2.0
30											--	12.0
31											--	2.5
	10	10	10	10	10	10	10	10	10	10		
O.F. Value:	--	234.5	197.0	366.8	314.8	313.0	--	482.6	489.0	--		
Iteration at Optimality:	--	5000+	686	200	85	902	--	203	53	--		
Time (sec):	--	4.581	.933	.563	.304	1.193	--	.407	.256	--		
Roll-Back Savings:	--	680.5	680.0	680.5	680.0	680.0	--	680.5	680.0	--		
(-Cut)	--	(-213.0)	(-213.5)	(-213.0)	(-213.5)	(-213.5)	--	(-213.0)	(-213.5)	--		

Note: Total Initial Budget = 893.5 ($1000's)

Figure 11.

‹Figure 12› and ‹Figure 13› display those solution sets resulting from the optimizations within a relaxed environment. A total of 17 distinct solution set vectors are formed; and thus the selection tally matrix demonstrates greater variability than existent within the restricted orientation.

Discriminant functions were computed for the relaxed modeling setting first, requiring a separate discriminant execution for each of the distinct solution vectors resulting from the MAM analysis. As noted in an earlier section to this paper, criterion strength was evaluated utilizing the three composites vectors:

DISCRIMINANT CRITERION INCLUSION VECTOR

DISCRIMINANT CRITERION ENTRY VECTOR

DISCRIMINANT WEIGHTING SUMMARY VECTOR.

The first vector is composed of binary (1,0) entries signifying whether a specific criterion was entered into the discriminant analysis for explaining the variance within the solution set. The second vector contains entries of 1,2,3,... , such that the order-of-entry for the discriminant criteria is represented. Finally, the third vector contains a factor-weight entry for each of the 'entered' vectors, to measure the relative importance of each of the discriminating criterion references.

The notion of decisioning reliability was evaluated utilizing two techniques:

CANONICAL CORRELATION

RE-CLASSIFICATION ANALYSIS.

‹Figure 14› contains the discriminant results for solutions accountable to maximization within a relaxed region. The first ten columns contain the information from the discriminant analyses for each of the ten simulated solution sets. The ordinal numerals represent order-of-entry, while the bracketed entries

Effect Upon Budget Oeallocation Decisions Based Upon the Variable Flows of a Cyclic Objective Function, and the Interaction of a "Maximized, Relaxed" Constraint Iterative Problem.

Objective = Maximization Constraints: Relaxed (EXP=10; PERC = 600)

Budget Alterna-tives	01 CERT	02 CLAS	03 BENE	04 SUPL	05 INST	06 CONT	07 TRAV	08 CAPI	09 PERC	10 COMP	SELECTION TALLY	BUDGET AMOUNT
01	X	X	X	X	X	X	X	X	X	X	10	87.5
02	X	X	X	X	X	X		X			7	44.5
03	X	X	X				X	X			5	34.5
04	X	X	X	X	X	X	X	X	X	X	10	71.5
05	X		X	X	X	X	X		X	X	8	70.5
06	X			X	X		X	X	X		6	32.5
07	X	X		X	X	X	X	X	X	X	9	51.5
08											--	1.5
09	X	X	X	X	X	X	X	X	X	X	10	43.0
10						X					1	4.0
11		X	X	X		X		X		X	6	54.0
12											--	1.0
13					X		X				2	5.5
14									X		1	4.0
15	X	X	X	X	X	X	X	X	X	X	10	116.0
16						X		X	X	X	4	23.0
17	X	X	X	X	X	X	X	X	X	X	10	107.0
18				X					X		2	13.0
19		X									1	2.0
20											--	1.0
21	X		X								2	16.0
22								X			1	10.5
23		X	X		X		X		X	X	6	55.0
24											--	4.5
25											--	2.5
26	X	X					X			X	4	19.0
27				X							1	1.0
28				X					X		2	1.0
29										X	1	2.0
30						X					1	12.0
31											--	2.5

	01 CERT	02 CLAS	03 BENE	04 SUPL	05 INST	06 CONT	07 TRAV	08 CAPI	09 PERC	10 COMP
	12	12	11	13	11	12	12	12	13	12
O.F. Value	485.4	425.5	316.1	615.9	476.6	477.7	100.0	659.04	600.0	700.0
Starting at Optimality:	20	60	202	16	43	52	163	65	5000+	457
Time (sec):	.246	.359	.416	.227	.297	.337	.589	.310	6.022	1.166
Roll-Back Savings:	685.5	685.5	699.5	693.0	684.5	684.5	693.5	675.5	675.5	700.0
	(-208.0)	(-208.0)	(-194.0)	(-200.5)	(-209.0)	(-209.0)	(-200.0)	(-218.0)	(-218.0)	(-193.5)

Note: Total Initial Budget = 893.5 ($1000's)

Figure 12.

Effect Upon Budget Deallocation Decisions Based Upon the Variable Flows of a Cyclic Objective. Function, and the Interaction of a "Minimized, Relaxed" Constraint Iterative Problem.

Objective = Minimization Constraints: Relaxed (EXP=10; PERC = 600)

Budget Alternatives	01 CERT	02 CLAS	03 BENE	04 SUPL	05 INST	06 CONT	07 TRAV	08 CAPI	09 PERC	10 COMP	SELECTION TALLY	BUDGET AMOUNT
01	X		X	X	X	X	X		X		7	87.5
02	X			X	X	X			X		5	44.5
03				X	X	X					3	34.5
04	X		X	X	X	X	X		X		7	71.5
05			X	X	X		X		X		5	70.5
06			X			X	X		X		4	32.5
07	X		X	X		X	X		X		6	51.5
08											--	1.5
09	X		X		X	X	X		X		5	43.0
10											--	4.0
11	X		X		X		X		X		5	54.0
12											--	1.0
13			X								1	5.5
14											--	4.0
15	X		X	X	X	X	X		X		7	116.0
16	X		X	X	X		X				5	23.0
17	X		X	X	X	X	X		X		7	107.0
18	X		X		X	X					4	13.0
19											--	2.0
20											--	1.0
21							X				1	16.0
22	X										1	10.5
23	X			X		X					3	55.0
24											--	4.5
25											--	2.5
26				X	X	X	X				4	19.0
27											--	1.0
28			X								1	1.0
29											--	2.0
30											--	12.0
31				X	X						2	2.5
	12	--	13	12	13	12	12	--	10	--		
Optimal Value	238.8	--	110.0	304.7	265.9	198.3	50.0	--	482.3	--		
Iteration at Optimality:	869	--	5000+	197	625	1030	5000+	--	34	--		
Time (sec):	1.665	--	4.143	.548	.731	1.823	4.010	--	.249	--		
Roll-Back Savings:	676.5	--	676.0	682.5	686.0	675.0	691.5	--	678.0	--		
(- Cut)	(-217.0)	--	(217.5)	(-211.0)	(-207.5)	(-218.5)	(-202.0)	--	(-215.5)	--		

Note: Total Initial Budget = 893.5 ($1000's)

Figure 13.

[x.xx] contain the factor-weights computed from dividing each of the standardized canonical discriminant coefficients by the smallest such coefficient for each discriminant analysis. For example in the first column signifying the results of discriminating the solution computed from maximizing 'certificated salaries', 5 criteria were required to explain available variance within the solution set. The criterion 'budgetary composites' was entered first, and represents a factor of 2.51 in its importance to the remaining 4 criterion discriminants. The criterion 'certificated salaries' was entered secondly, and represents a factor of 3.17 in its relative importance for discriminating the solution set being analyzed; and so forth. The selection tally vector is similarly analyzed via discriminant functions.

For understanding the dimension of decisioning reliability, computed canonical correlation coefficients existed as follows, for maximized-relaxed solutions:

Objective Function	Canonical Coefficient	Percent Variance Explained	Relative Rank
CERT	.9056	82.0	3
CLAS	.8633	74.5	6
BENE	.8729	76.2	4
SUPL	.9077	82.4	2
INST	.9339	87.2	1
CONT	.8679	75.3	5
TRAV	.8614	74.2	7
CAPI	.8419	70.9	8
PERC	.7870	61.9	9
COMP	.7281	53.0	10.

Thus it would seem, that a formalized objective of "maximizing" the expenditures associated with instructional materials in determining which programs to refund during a period of scant resourses, produced the highest correlation between the criterion matrix of 10 vectors and the proposed solution set vector constructed from the MAM analysis execution. Likewise, the maximization of

Summary of Criterion Vector Order-of-Entry, in Discriminating the Solution Set Vector for Each Cyclic MAXIMIZATION within a RELAXED Region. (Note: Source of Discriminant Criterion Inclusion Vector; Discriminant Criterion Entry Vector; and Discriminant Weighting Summary Vector)

⟨ VALUE OF OBJECTIVE FUNCTION DURING CYCLIC-OPTIMIZATION EVALUATIONS ⟩

Criterion Vector	CERT	CLAS	BENE	SUPL	INST	CONT	TRAV	CAPI	PERC	COMP	Selection Tally Vector	Discriminant Function #
Certificated Salaries	2 [3.17]	4 [1.00]	--	--	--	--	--	--	--	--	--	--
Classified Salaries	--	2 [3.08]	--	4 [1.57]	--	--	--	--	3 [2.04]	2 [1.00]	5	5
Employee Benefits	--	3 [1.48]	2 [2.74]	--	--	5 [1.00]	--	--	--	--	--	--
Supplies & Materials	5 [1.00]	--	--	1 [3.56]	--	3 [1.21]	--	2 [1.46]	--	--	--	--
Instructional Materials	--	--	5 [1.00]	--	1 [3.13]	--	2 [1.78]	3 [1.00]	4 [2.03]	--	2	2
Contractual Services	4 [2.16]	--	3 [1.15]	5 [1.00]	--	2 [1.87]	3 [2.19]	--	--	--	--	--
Travel Expenditures	--	--	--	--	4 [1.00]	--	4 [1.17]	--	5 [1.00]	--	--	--
Capital Outlay	3 [2.40]	--	4 [1.29]	--	--	4 [1.02]	5 [1.00]	1 [3.65]	--	--	3	3
Administrative Perception	--	--	--	3 [1.68]	3 [1.25]	--	--	--	2 [2.59]	--	4	4
Budgetary Composites	1 [2.51]	1 [2.29]	1 [2.91]	2 [3.46]	2 [3.03]	1 [1.71]	1 [2.63]	--	1 [3.04]	1 [3.67]	1	1
Number of Mis-inclusions	--	2	2	1	--	--	1	2	2	2	4	Number of Over-Estimates
Number of Mis-Exclusions	--	--	--	--	--	--	1	--	2	3	5	Number of Under-Estimates
Re-Prediction Accuracy (%)	100.0	93.6	93.6	96.8	100.0	100.0	93.6	93.6	87.1	83.9	71.0	Re-Prediction Accuracy (%)

* (No integer-feasible solution possible; Optimality not achieved)

'budgetary composites' produced the lowest correlation, explaining only 53.0 percent of independent variance within the MAM solution vector.

The second 'phase' of measuring decisioning reliability exists in the accuracy of re-predicting solution set membership based upon the classification function coefficients generated via the discriminant analysis. The bottom portion of igure 14 portrays these results for each of the 10 solution vectors formed by the varying criterion focus of the objective function. The results of e-classification for the selection tally vector are also displayed.

Figure 15> illustrates the similar results from applying discriminant function nalyses to the solution vectors formed by minimization within a relaxed setting he three vectors for denoting criterion strength are easily distinguishable rom the 7 successful (columns) optimizations. The re-classification portion of easuring decisioning reliability is also shown.

he computed canonical correlation coefficients for minimized-relaxed solutions:

Objective Function	Canonical Coefficient	Percent Variance Explained	Relative Rank
CERT	.7721	59.6	6
CLAS	--	--	--
BENE	.7902	62.4	5
SUPL	.8194	67.1	2
INST	.7675	58.9	7
CONT	.8000	64.0	3
TRAV	.7928	62.9	4
CAPI	--	--	--
PERC	.9343	87.3	1
COMP	--	--	--

nstrated, that solution set formulated by minimizing the 'administrative eption' entries in determining a solution, to be the best fit with the

Summary of Criterion Vector Order-of-Entry, in Discriminating the Solution Set Vector for Each Cyclic MINIMIZATION within a RELAXED Region. (Note: Source of Discriminant Criterion Inclusion Vector; Discriminant Criterion Entry Vector; and Discriminant Weighting Summary Vector)

< VALUE OF OBJECTIVE FUNCTION DURING CYCLIC-OPTIMIZATION EVALUATIONS >

Criterion Vector	CERT	CLAS	BENE	SUPL	INST	CONT	TRAV	CAPI	PERC	COMP	Selection Tally Vector	Discriminant Function #
Certificated Salaries	2 [2.37]	*	—	3 [1.41]	—	3 [1.22]	3 [1.01]	*	—	*	—	—
Classified Salaries	—	*	2 [2.16]	—	5 [1.00]	4 [1.52]	—	*	—	*	—	—
Employee Benefits	—	*	—	—	—	5 [1.00]	4 [1.00]	*	—	*	2	6
Supplies & Materials	5 [1.00]	*	5 [1.00]	2 [1.97]	—	—	—	*	2 [1.37]	*	6	4
Instructional Materials	4 [1.38]	*	—	—	—	—	—	*	4 [1.00]	*	—	—
Contractual Services	—	*	—	5 [1.00]	4 [1.27]	2 [1.33]	—	*	—	*	3	5
Travel Expenditures	—	*	4 [1.45]	—	—	—	—	*	—	*	4	3
Capital Outlay	3 [1.99]	*	—	—	3 [1.90]	—	2 [1.63]	*	3 [1.17]	*	5	2
Administrative Perception	—	*	3 [1.34]	4 [1.58]	2 [2.74]	—	—	*	—	*	—	—
Budgetary Composites	1 [1.68]	*	1 [2.95]	1 [2.30]	1 [2.08]	1 [2.28]	1 [2.24]	*	1 [1.07]	*	1	1
Number of Mis-inclusions	2	*	1	1	1	1	2	*	—	*	3	Number of Over-Estimates
Number of Mis-Exclusions	1	*	3	1	2	1	1	*	—	*	2	Number of Under-Estimates
Re-Prediction Accuracy (%)	90.3	*	87.1	93.6	90.3	93.6	90.3	*	100.0	*	83.9	Re-Prediction Accuracy (%)

* (No integer-feasible solution possible; Optimality not achieved)

Figure 15.

overall criterion matrix; and the solution from minimizing 'instructional materials', the least 'best' fit.

Regarding the results of optimizing (both maximally and minimally) within the _restricted_ environment, ‹Figure 16› illustrates the discriminant function analysis framework. Similarly, the canonical coefficients were computed as:

Solution Vector	Canonical Coefficient	Percent Variance Explained	Relative Rank
#1	.8947	80.0	1
#2	.8628	74.4	2

Summary of Criterion Vector Order-of-Entry, in Discriminating the Two Distinct Solution Set Vectors Resulting from the Cyclic MAXIMIZATION and MINIMIZATION within a RESTRICTED Region. (Note: Source of Discriminant Criterion Inclusion Vector; Discriminant Criterion Entry Vector; and Discriminant Weighting Summary Vector)

Criterion Vector	Solution Set #1	Solution Set #2
Certificated Salaries	4 [1.05]	—
Classified Salaries	5 [1.00]	—
Employee Benefits	—	—
Supplies & Materials	2 [1.70]	—
Instructional Materials	—	3 [1.00]
Contractual Services	—	2 [1.65]
Travel Expenditures	—	—
Capital Outlay	3 [1.25]	—
Administrative Perception	—	—
Budgetary Composites	1 [2.11]	1 [3.24]
Number of Mis-inclusions	1	—
Number of Mis-exclusions	—	1
Re-Prediction Accurary (%)	96.8	96.8

Figure 16.

SUMMARY OF FINDINGS

The use of discriminant functions in providing a useful post-hoc evaluation strategy for multiple alternatives decision-making has been studied within two separate real-world settings: the closure of schools; and the deallocation of program unit budgetary items. Two generalized issues of content and process were the main foci: content, in as much as there is a need to relate criteria used to the decisions made; and process, in order to verify the reliability of the decisioning procedures based upon the criteria utilized.

The author maintains, that two related "abilities" are necessary for prudent and trustworthy decision-making. The first ability refers to that knowledge which clarifies (1) which criteria 'effected' the decisions, and to what extent; and (2) to what degree did this 'effect' vary across the results of the cyclical optimizations. The second ability relates the need to study (1) the relationship between the 'optimizing vector' (objective function) and the results of a discriminant analysis; and (2) the relationship between the extent of feasibility region constraint (relaxed v. restricted) and the results of a discriminant analysis. To accomplish these ends, the multiple linear regression technique, discriminant functions analysis, is utilized to measure the topics of criterion strength and decisioning reliability.

The results of these discriminant analyses illustrate the superior efficacy found in relating multiple correlational strategies to discovering relationships between solution vectors and the criterion vectors (matrice) supporting those decisions. Three measures of criterion strength and two measures of decisioning reliability are illustrated for the reader -- all measures normally products of discriminant function(s) formulation.

It is a fundamental by-product of this study though all to important not to note, that the formation of "classification coefficients" within the discriminant process provides an excellent way of projecting expected impact from a newly collected set of data variables. By utilizing the linear combinations of this new data, 'expected correlative' decisions can be computed which maintain the same variance relationship as the decisions utilized originally in the initial discriminant analyses.

In summary, the use of discriminant functions in addressing the issues of criterion strength and decisioning reliability has been illustrated to hold great promise for the decision-maker, evaluator and otherwise problem-solver. Increased accountability, visibility and responsibility are the maximized ends.

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Appendix I

Synthetic "True" Covariance Structure for Misspecification Categories I and III: The Covariance Structure used as the Input Matrix for the Simulation of Data Sets whith a Multivariate Normal Distribution.

	Y_1	Y_2	Y_3	X_1	X_2	X_3	X_4
Y_1	1.338						
Y_2	.781	1.1175					
Y_3	1.9525	2.54375	6.453975				
X_1	.84	.62	1.55	1.1			
X_2	.42	.31	.775	.5	.35		
X_3	.78	.55	1.375	.8	.4	1.1	
X_4	.234	.165	.4125	.24	.12	.3	.19

Appendix II

Synthetic "True" Covariance Structure for Misspecification Category II: The Covariance Structure used as the Input Matrix for the Simulation of Data Sets with a Multivariate Normal Distribution.

	Y_1	Y_2	Y_3	X_1	X_2	X_3	X_4
Y_1	1.538						
Y_2	1.18	1.5175					
Y_3	2.9675	3.0439	7.9094				
X_1	.84	.62	1.55	1.1			
X_2	.42	.31	.775	.53	.35		
X_3	.78	.55	1.375	.81	.41	1.1	
X_4	.234	.165	.4125	.13	.13	.32	.19