MULTIPLE LINEAR REGRESSION VIEWPOINTS

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A METHOD FOR ESTIMATING INDIRECT EFFECTS IN PATH ANALYSIS

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In an earlier paper, Wolfle (1980) considered four kinds of causal models: recursive, block, block-recursive, and nonrecursive. By applying the first law of path analysis, he decomposed zero-order correlations among variables in causal models, and discussed the circumstances under which the components of the decompositions could be interpreted as direct, indirect, and spurious causal effects, plus a component he called joint associations. Since the publication of that paper, a number of people have inquired about the availability of a computer program to compute the components of decompositions explicated in the original paper. There is no computer program to calculate these components, but there is a means by which direct and indirect effects may be calculated with a minimum of effort. (Earlier papers by Griliches and Mason [1972] and Alwin and Hauser [1975] inform this discussion.)

Since joint associations, which involve components of a decomposition that include correlations among exogenous variables, and spurious effects may be considered to be noncausal components of a correlation between variables in a causal model, let us call the sum of direct and indirect effects the "total effect." The purpose of this paper is to demonstrate algebraically that total effects may be obtained through reduced-form regression equations, and the indirect effects may be calculated by taking the difference between the reduced-form regression coefficients and the direct effect. Following the algebraic proof, an empirical illustration will aid in understanding how the method works in practice.

To begin, consider a four-variable, fully recursive path model in which:

$$x_3 = p_{34}x_4 + p_{3u}u \quad (1),$$

$$x_2 = p_{23}x_3 + p_{24}x_4 + p_{2v}v \quad (2),$$

$$x_1 = p_{12}x_2 + p_{13}x_3 + p_{14}x_4 + p_{1w}w \quad (3),$$

in which x_i ($i = 1,2,3,4$) are standardized variables; p_{ij} are standardized regression (path) coefficients from x_j to x_i; and u, v, and w are unmeasured disturbance terms assumed to be independent of the x_i on the same side of the equality. Thus,

$$E(x_4u) = E(x_3v) = E(x_4v) = E(x_2w) = E(x_3w) = E(x_4w) = 0 \quad (4).$$

That the x_i are assumed to be standardized is a convenience which simplifies the algebra to follow. The conclusions to be drawn from the following presentation apply without loss of generalization to metric regression coefficients.

If one multiplies eq. 1 by x_4, and takes expectations, one obtains:

$$E(x_3x_4) = p_{34}E(x_4^2) + p_{3u}E(x_4u) \quad (5),$$

in which $E(x_3x_4) = \rho_{34}$, and $E(x_4^2) = 1$, since these are standardized variables, and $E(x_4u) = 0$ by the assumptions in eq. 4. Thus,

$$E(x_3x_4) = \rho_{34} = p_{34} \quad (6).$$

In analytic terms, eq. 6 indicates that the direct effect, p_{34}, of x_4 on x_3 is measured by the correlation, ρ_{34}.

Now consider eq. 2, but instead of estimating eq. 2 as is, consider:

$$x_2 = p'_{24}x_4 + p'_{2v}v \qquad (7),$$

which is merely the regression of x_2 on x_4. If one multiplies eq. 7 by x_4, and takes expectations, one obtains:

$$E(x_2x_4) = p'_{24}E(x_4^2) + p'_{2v}E(x_4v) \qquad (8),$$

which reduces to:

$$E(x_2x_4) = p'_{24} \qquad (9).$$

Substituting eq. 2 into eq. 9 yields:

$$p'_{24} = E[(p_{23}x_3 + p_{24}x_4 + p_{2v}v)\,x_4] \qquad (10);$$

multiplying the parenthetical expression by x_4 yields:

$$p'_{24} = p_{23}E(x_3x_4) + p_{24}E(x_4^2) + p_{2v}E(x_4v) \qquad (11),$$

which is equal to:

$$p'_{24} = p_{23}p_{34} + p_{24} \qquad (12),$$

because $E(x_3x_4) = p_{34}$, $E(x_4^2) = 1$, and $E(x_4v) = 0$.

Thus, regressing x_2 on x_4 yields a coefficient which is equal to the sum of the direct (p_{24}) and indirect effects ($p_{23}p_{34}$). By using a normal regression routine, one can regress x_2 on x_4, and thereby obtain the total effect from x_4 to x_2. The regression of x_2 on both x_3 and x_4 yields the direct effects of x_3 and x_4 on x_2 (p_{23} and p_{24}, respectively). The difference between p'_{24} and p_{24} ($p'_{24} - p_{24} = p_{23}p_{34}$) therefore gives the indirect effect of x_4 on x_2 through x_3. In other words, while the indirect effect of x_4 on x_2 may not be calculated directly, the product, $p_{23}p_{34}$, is obtainable by first regressing x_2 on x_4, then regressing x_2 on both x_3 and x_4, and calculating the difference between the two coefficients for x_4.

Now consider eq. 3, but instead of estimating eq. 3 as is, one estimates:

$$x_1 = p'_{14}x_4 + p'_{1w}w \qquad (13).$$

Multiplying eq. 13 by x_4, and taking expectations, yields:

$$E(x_1x_4) = p'_{14}E(x_4^2) + p'_{1w}E(x_4w) \qquad (14),$$

which is equal to:

$$E(x_1x_4) = p'_{14} \qquad (15).$$

Substituting eq. 3 into eq. 15 yields:

$$p'_{14} = E[(p_{12}x_2 + p_{13}x_3 + p_{14}x_4 + p_{1w}w)\, x_4] \qquad (16);$$

multiplying the parenthetical expression by x_4 yields:

$$p'_{14} = p_{12}E(x_2x_4) + p_{13}E(x_3x_4) + p_{14}E(x_4^2) + p_{1w}E(x_4w) \qquad (17).$$

Because $E(x_4^2) = 1$, and $E(x_4w) = 0$, one obtains:

$$p'_{14} = p_{12}E(x_2x_4) + p_{13}E(x_3x_4) + p_{14} \qquad (18).$$

By substituting eq. 12 and eq. 6 for $E(x_2x_4)$ and $E(x_3x_4)$, respectively, one obtains:

$$p'_{14} = p_{12}(p_{23}p_{34} + p_{24}) + p_{13}p_{34} + p_{14} \qquad (19).$$

Thus, were one to obtain p'_{14} by regressing x_1 on x_4, and then obtain p_{14} by regressing x_1 on x_2, x_3 and x_4, the difference would equal:

$$p'_{14} - p_{14} = p_{12}p_{23}p_{34} + p_{12}p_{24} + p_{13}p_{34} \qquad (20),$$

which is the sum of all the indirect effects through x_2 and x_3.

Now consider the regression of x_1 on x_3 and x_4:

$$x_1 = p''_{13}x_3 + p''_{14}x_4 + p''_{1w}w \qquad (21).$$

Multiplying eq. 21 by x_4, and taking expectations, yields:

$$E(x_1x_4) = p''_{13}E(x_3x_4) + p''_{14}E(x_4^2) + p''_{1w}E(x_4w) \qquad (22).$$

With a slight rearrangement of terms, eq. 22 reduces to:

$$p''_{14} = E(x_1x_4) - p''_{13}E(x_3x_4) \qquad (23).$$

Substituting eq. 6 for $E(x_3x_4)$, and eq. 3 for x_1, yields:

$$p''_{14} = E[(p_{12}x_2 + p_{13}x_3 + p_{14}x_4 + p_{1w}w)\,x_4] - p''_{13}p_{34} \qquad (24),$$

which becomes:

$$p''_{14} = p_{12}E(x_2x_4) + p_{13}E(x_3x_4) + p_{14}E(x_4^2) + p_{1w}E(x_4w)$$
$$- p''_{13}p_{34} \qquad (25).$$

It can be shown that $p''_{13} = p_{13} + p_{12}p_{23}$; also $E(x_4w) = 0$, and $E(x_4^2) = 1$; substituting these quantities, and eq. 12 for $E(x_2x_4)$, and eq. 6 for $E(x_3x_4)$, yields:

$$p''_{14} = p_{12}(p_{24} + p_{23}p_{34}) + p_{13}p_{34} + p_{14}$$
$$- (p_{13} + p_{12}p_{23})p_{34} \qquad (26),$$

which reduces to:

$$p''_{14} = p_{12}p_{24} + p_{14} \qquad (27).$$

Remember that p'_{14} is obtained by regressing x_1 on the exogenous variable, x_4; p''_{14} is obtained by regressing x_1 on the exogenous variable, x_4, and the first endogenous variable, x_3; p_{14} (the direct effect of x_4 on x_1) is obtained by regressing x_1 on all of its antecedent causes. With estimates of these coefficients, taking the differences among them yields the estimates of the indirect effects. Thus,

$$p'_{14} - p_{14} = p_{12}p_{23}p_{34} + p_{12}p_{24} + p_{13}p_{34} \qquad (28),$$

which is the sum of all the indirect effects from x_4 to x_1 through x_2 and x_3 together, through x_2, and through x_3, respectively;

$$p''_{14} - p_{14} = p_{12}p_{24} \qquad (29),$$

which is the indirect effect from x_4 to x_1 through x_2; and

$$p'_{14} - p''_{14} = p_{13}p_{34} + p_{12}p_{23}p_{34} \qquad (30),$$

which are the indirect effects from x_4 to x_1 through x_3, and through x_3 and x_2 together.

These results are not model specific; they are applicable to any hierarchical causal model. To obtain the total effect of any variable, x_j, in a causal model on any subsequent variable, x_i, in the model, simply regress x_i on x_j and all other variables that precede x_j, or occur causally in the same block with x_j (e.g., the set of exogenous variables). To obtain the direct effect of x_j on x_i, regress x_i on all of its causal antecedents. To obtain the sum of the indirect effects from x_j to x_i, take the difference between the total effect and the direct effect.

AN ILLUSTRATION

To illustrate these algebraic principles in practice, consider the block-recursive path model shown in Figure 1. This is the most general of the hierarchical models considered by Wolfle (1980), and was taken originally from Heyns (1974). She was interested in the degree to which stratification within schools mediates the effect of socioeconomic background on educational outcomes of students. The model shown in Figure 1 indicates that the exogenous variables, PaEduc, PaOcc, and SIBS are correlated for reasons unanalyzed in the present model. A measure of verbal ability is considered to be dependent upon the three exogenous variables, plus an error term assumed to be uncorrelated with the independent variables. Grades and curriculum track membership are thought to be dependent upon the four preceding manifest variables, but no causal nexus is assumed between grades and curriculum. Their disturbance terms, however, are assumed to be correlated with each other, but not with the four preceding manifest variables. Finally, educational aspirations is dependent upon the six causally antecedent variables. In algebraic terms, the regression equations implied by Figure 1 are:

$$x_1 = p_{12}x_2 + p_{13}x_3 + p_{14}x_4 + p_{15}x_5 + p_{16}x_6 + p_{17}x_7 + p_{1w}w \quad (31),$$

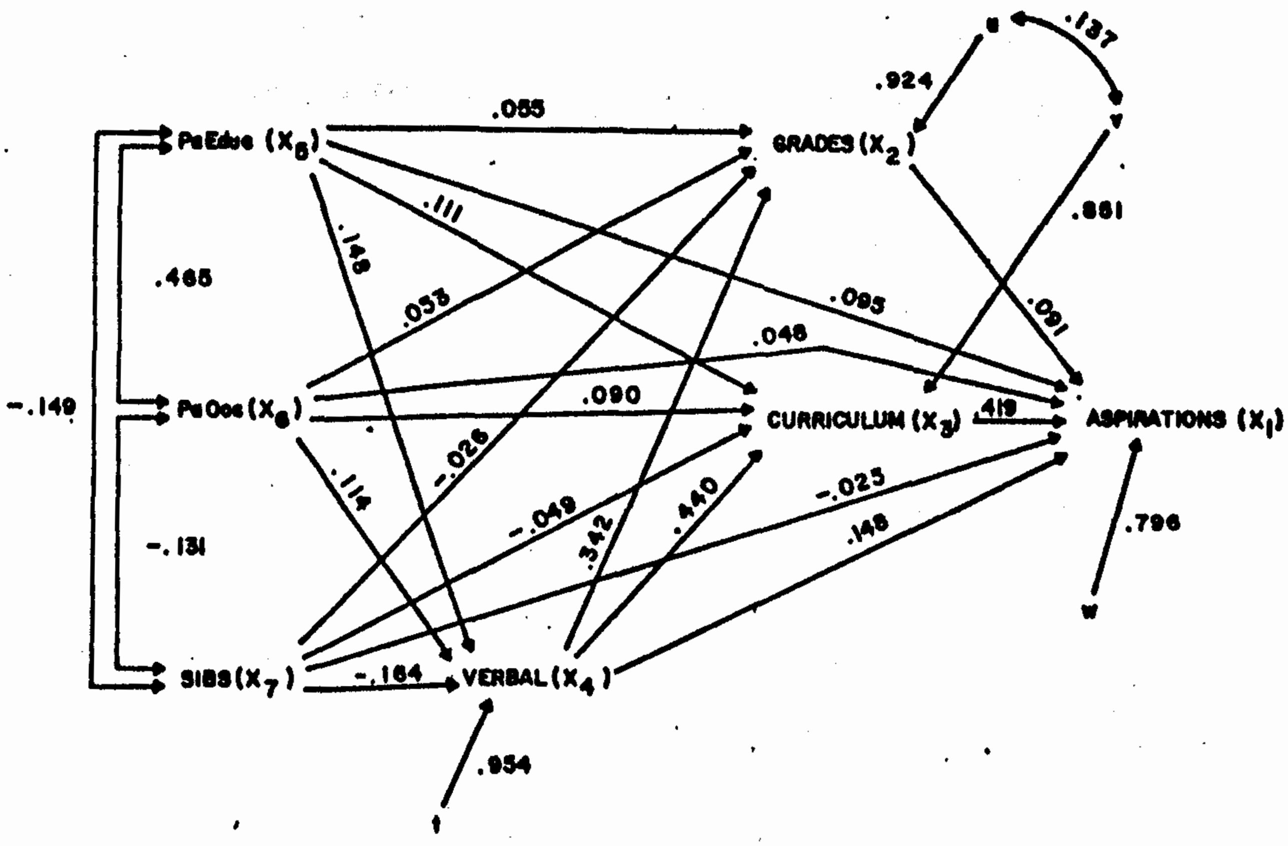

FIGURE 1. BLOCK-RECURSIVE EQUATION MODEL OF EDUCATIONAL ASPIRATIONS (SOURCE: HEYNS, 1974)

$$x_2 = p_{24}x_4 + p_{25}x_5 + p_{26}x_6 + p_{27}x_7 + p_{2u}u \qquad (32),$$

$$x_3 = p_{34}x_4 + p_{35}x_5 + p_{36}x_6 + p_{37}x_7 + p_{3v}v \qquad (33),$$

$$x_4 = p_{45}x_5 + p_{46}x_6 + p_{47}x_7 + p_{4t}t \qquad (34).$$

Estimating eq. 34 yields the total effects of x_5, x_6, and x_7 on x_4. These are equal to the direct effects, because no variables intervene between the exogenous variables and x_4; thus there can be no indirect effects.

The reduced-form regression of x_3 on x_5, x_6, and x_7 would yield the total effects of these exogenous variables on x_3; adding x_4 to the equation (i.e., eq. 33) would yield the direct effects, and the differences between the coefficients for x_5, x_6, and x_7 in the reduced-form equation and the fully specified equation yield the indirect effects of the respective exogenous variables on x_3 through the intervening variable, x_4.

Estimation of the remainder of the model would proceed accordingly; the numeric results for this model are shown in Table 1. The zero-order correlations for these data are available in Heyns (1974, p. 1441).

Table 1. Summary of Regression Analysis for Model of Educational Aspirations

Dependent Variables	Independent Variables: Pa Educ (x_5)	Pa Occ (x_6)	SIBS (x_7)	Verbal (x_4)	Grades (x_2)	Curric. (x_3)	α	R^2
			Standardized Coefficients					
Verbal (x_4)	.148	.114	-.164					
Grades (x_2)	.106	.092	-.083					
Grades (x_2)	.055	.053	-.026	.342				
Curric. (x_3)	.176	.140	-.121					
Curric. (x_3)	.111	.090	-.049	.440				
Aspir. (x_1)	.201	.132	-.108					
Aspir. (x_1)	.147	.091	-.048	.363				
Aspir. (x_1)	.095	.048	-.025	.148	.091	.419		
			Regression Coefficients*					
Verbal (x_4)	.561	.078	-.931				28.45	.090
	(.033)	(.006)	(.044)					
Grades (x_2)	.028	.004	-.033				2.94	.040
	(.002)	(.000)	(.003)					
Grades (x_2)	.015	.003	-.011	.024			2.25	.147
	(.002)	(.000)	(.003)	(.001)				
Curric. (x_3)	.026	.004	-.027				.089	.099
	(.001)	(.000)	(.002)					
Curric. (x_3)	.016	.002	-.011	.017			-.398	.275
	(.001)	(.000)	(.002)	(.000)				
Aspir. (x_1)	.131	.016	-.106				12.93	.104
	(.006)	(.001)	(.008)					
Aspir. (x_1)	.095	.011	-.047	.063			11.14	.225
	(.005)	(.001)	(.007)	(.001)				
Aspir. (x_1)	.062	.006	-.025	.026	.223	1.064	11.30	.367
	(.005)	(.001)	(.006)	(.001)	(.017)	(.034)		

* Standard errors are shown in parentheses.

If one happens to be interested in the extent to which two variables are causally related (total effect) in comparison to their total association (zero-order correlation), one compares the zero-order correlation with the reduced-form standardized coefficient. For example, the correlation of verbal ability, x_4, and educational aspirations, x_1, was .425 ; the reduced-form coefficient was .363 ; thus (.425 - .363)/.425 = .15 proportion of the correlation was due to spurious causal effects and joint associations among the exogenous variables.

Indirect effects may be calculated from the coefficients in Table 1. For example, the direct effect of father's education, x_5, on grades, x_2, is .055, and the indirect effect of x_5 on x_2 through verbal ability, x_4, is (.106 - .055) = .051 . Notice that these components could also be calculated from the metric regression coefficients, which enjoy a more substantively pleasing interpretation. Thus, a one-year increase in father's education produces an increase in grades of .028 units, .015 of which is a direct causal effect, and (.028 - .015) = .013 of which is an indirect effect through verbal ability. Notice that the ratios of direct and indirect effects are identical whether one uses standardized or metric coefficients. Thus, .055/.106 = .015/.028, within rounding error (see Wolfle, 1977, p. 47, for proof).

Consider the effects of father's education, x_5, on educational aspirations, x_1. The total effect is .201 ; the direct effect is .095 . The sum of all indirect effects is (.201 - .095) = .106 ; the indirect effects of x_5 on x_1 through verbal ability, x_4, are (.201 - .147) = .054 (note that this component includes <u>all</u> indirect effects through x_4, namely $p_{14}p_{45} + p_{12}p_{24}p_{45} + p_{13}p_{34}p_{45}$); and the indirect effects of x_5 on x_1 through grades, x_2, and curriculum, x_3, are (.147 - .095) = .052 .

CONCLUSION

The decomposition of causal components into direct and indirect effects may be substantively important, because the decomposition allows the consideration of how causal effects occur. For example, when indirect effects overwhelm direct effects, one has in essence described the social mechanism through which the causal relationship operates. For example, father's and son's occupational statuses are moderately correlated in samples of U.S. men. But the indirect effect of father's occupation on son's occupation through son's educational attainment is often greater in magnitude than the direct effect. In substantive terms, the reason father's and son's statuses are correlated is because sons acquire educational levels which lead to their acquiring occupational levels near those of their father's.

Causal models are useful analytic tools because they allow both the author and reader to understand explicitly the assumed order of effects. The interpretations of decompositions calculated as a part of the analysis depend on the assumed causal order of variables. Which associations are to be decomposed depends on the purpose of the analysis and the presentation of results. It would serve little purpose to use the methods explicated in this paper to calculate a wholesale collection of indirect effects; unless, of course, these were required by the research questions which motivated the analysis. The methods explicated herein should ease the burden of of analyzing causal models, but they are not substitutes for reflective analyses of social data.

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