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THE USE OF FULL MLR MODEL TO CONDUCT MULTIPLE COMPARISONS IN A REPEATED MEASURES DESIGN: AN INDUSTRIAL APPLICATION

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Abstract

This paper presents the MLR models used to analyze the impact of the aging process on the ductility of steel tubing. The discussion includes the procedure by which multiple comparisons can be made in a repeated measures design by using only one full MLR model.

[1]The company's name will not be used as requested by the company.

Introduction

The management of a firm that manufactures steel tubing was faced with the problem of determining what impact the aging processes had on the ductility of the steel tubing.[1] The purpose of this paper is to present the multiple linear regression (MLR) models developed to analyze the impact that the aging process had on the ductility of the steel tubing. Specifically, the discussion centers on the joint utilization of item vectors and multiple comparisons in the MLR models, as discussed by Williams (1980). Included in this discussion, however, is the outline of a procedure through which only one full regression model is required to make multiple comparisons.

Hypothesis

The question being addressed by the company was: Does aging affect the ductility of non-aluminum kilned steel tubing? If this question was answered in the affirmative, the ductility of steel tubing stored for periods of time in inventories could fall below a buyer's minimum standards. Such a result could cause inventory policy to change.

Since the ductility of the tubing was measured by elongation values and management was interested in three specific time periods, the null hypothesis corresponding to the question of interest was as follows:

H_0: There is no difference between the three time periods with respect to the tube elongation values.

The analysis of this hypothesis was best accomplished by the use of MLR models. The MLR models can best be understood after reviewing the sampling procedure and the method by which the aging process was simulated.

Sample and Treatments

Sample items for this project were selected from production lots of tubing of various sizes of non-aluminum kilned steel. Since the tubing had to be exposed to three treatments to reflect the aging process and the collection of the dependent variable readings caused the tubing to be destroyed, three specimens were used from each sample tube. Extreme care was taken to insure that the three specimens machined from each selected tubing sample were alike. All three specimens were cut from the sample tube in the same orientation and from the same side of the tube. Therefore, it was assumed that the three specimens from each sample of tubing were matched prior to the aging process.

The aging process was simulated by exposing the specimens to various heat treatments. One specimen from each tubing sample was not heat treated or allowed to age prior to testing. Those specimens not aged were considered exposed to Treatment A. The second specimen was heated to 300°F for 23 minutes to simulate one year of normal aging. Those specimens aged one year were considered to be exposed to Treatment B. The third specimen from each sample tube was heated at 300°F for 60 minutes to simulate 2 1/2 years of aging. This treatment was labeled Treatment C.

Twenty-seven separate samples of various sizes of both square and rectangular structural tubing were selected for use in the study. As previously mentioned, each sample tubing was divided into three sections and each section was exposed to either Treatment A, B, or C. Thus, the total sample size was 81 with 27 tubing specimens being exposed to each treatment.

Variables and Regression Models

Dependent Variable

Elongation values were used as indicators of the ductility of the tube. The elongation values were obtained by administering a standard strip-tensile test to each tubing specimen after it had been exposed to the appropriate heat treatment. Each specimen of tubing was notched in the middle. Marks were placed two inches apart in the notched section of the tubing and the tubing was stretched until it broke. The pieces were placed back together and the percentage increase in length between the two marks served as the elongation value.

Independent Variables

To insure that the regression models developed to test the null hypothesis were accurate reflections of the situation, i.e., a Type VI error would be avoided (Newman et al., 1976), four independent variables were required. The four independent variables were as follows:

X_1 = Treatment A (no aging). X_1 was equal to 1 if the tubing specimen was exposed to Treatment A; 0 otherwise.

X_2 = Treatment B (aged 1 year). X_2 was equal to 1 if the tubing specimen was exposed to Treatment B; 0 otherwise.

X_3 = Treatment C (2 1/2 years of aging). X_3 was equal to 1 if the tubing specimen was exposed to Treatment C; 0 otherwise.

X_4 = Vector to represent the item vectors. X_4 was equal to the average elongation value (Y) for the three specimens obtained from a given piece of tubing.

The dummy variables X_1, X_2, and X_3 represented the treatments that simulated the aging process. Since the regression models were analyzed through a matrix inversion process, only two of the treatment vectors could be entered into a model at one time.

Variable X_4 was a key variable to include in the model and a variable that required that caution be taken in the interpretation of the computer printouts. Since the three specimens taken from each sample tube were considered matched, it was necessary that the regression models reflect that fact. The matching characteristic would necessitate the use of 26 item vectors, as discussed by McNeil, Kelly, and McNeil (1975). Pedhazur (1977) and Williams (1977), however, outlined a procedure by which the impact of the matching can more easily be represented by one vector. This vector, represented by X_4 in this study, was formed such that the entries for the three specimens for a given tube were equal to the average elongation values (Y) for those three specimens.

To illustrate the data coding procedure for the independent variables consider the data for the first 9 of the 81 specimens as listed in Table 1. The vector values indicate that the first specimen had an elongation value (Y) of 22. It was exposed to Treatment A ($X_1 = 1$, $X_2 = 0$, $X_3 = 0$), i.e., it was not aged. Finally, the value of 20 for X_4 was obtained by averaging the elongation values of 22, 19, and 19 recorded for the three specimens obtained from the same piece of tubing. Thus, the value of 20 recorded for Specimens #1, #2, and #3 indicated that they were obtained from the same piece of tubing.

Table 1

Vectors Used in Full Regression Model

Specimen	Vectors				
	Y	X_1	X_2	X_3	X_4
#1	22.0	1	0	0	20.0
#2	19.0	0	1	0	20.0
#3	19.0	0	0	1	20.0
#4	24.1	1	0	0	22.7
#5	22.0	0	1	0	22.7
#6	22.0	0	0	1	22.7
#7	23.1	1	0	0	20.7
#8	20.0	0	1	0	20.7
#9	19.0	0	0	1	20.7

Regression Models

The full regression model that reflected the research hypotheses used in this study was:

$$Y = a_0U + b_1X_1 + b_2X_2 + b_4X_4 + E_1 \qquad \text{(Model 1)}$$

The restriction placed on Model 1, which was required to test the impact of aging, was $b_1 = b_2 = 0$. The resulting restricted model was:

$$Y = a_0U + b_4X_4 + E_2 \qquad \text{(Model 2)}$$

The results of the computer analysis of the MLR models are contained in Table 2. The R^2 values for the full regression model (Model 1) and the restricted regression model (Model 2) were .9197 and .6789, respectively. To determine whether the decrease in the R^2 value was due to sampling error or the influence of aging, an F test was conducted.

Table 2

Analysis of Model 1 and Model 2

Model 1:

$$Y = a_0U + b_1X_1 + b_2X_2 + b_4X_4 + E$$

(-1.55) (4.22) (.148) (1.00)

$$R^2 = .9197$$

Model 2:

$$Y = a_0U + b_4X_4 + E_2$$

(-.097) (1.00)

$$R^2 = .6789$$

Note. The values contained in the parentheses are the regression coefficient values.

The formula for calculating the required F test was as follows:

$$F = \frac{(R_F^2 - R_R^2)/df_n}{(1-R_F^2)df_d}$$

where:

R_F^2 = the R^2 value for Model 1

R_R^2 = the R^2 value for Model 2

df_n = the number of restrictions placed on the full model to obtain the restricted model

df_d = total sample size minus the number of intercepts and independent variables

The F value was calculated as follows:

$$F = \frac{(.9197 - .6789)/2}{(1 - .9197)/52} = 77.97$$

It is important to note that the degrees of freedom for the denominator was equal to 52. Since the sample size was 81 and it appears that there are three independent variables in the full regression model (Model 1), one might think that the df_d should be equal to [81-(3+1)] or 77. One should remember, however, that variable X_4 is a surrogate for 26 item vectors. Therefore, the full regression model "contains" 28 independent variables. The correct df_d would be equal to [81-(28+1)] or 52.

The F value was statistically significant at the predetermined alpha level of .01. Thus, the researchers concluded that aging did have an impact on the elongation values of the tubing. To gain further insight into what impact aging had on the elongation values, the researchers conducted multiple comparison tests.

Multiple Comparisons

The use of multiple comparisons tests would allow the researchers to make specific statements concerning the impact of the aging process on elongation values. Since the regression coefficients for the treatment variables represent the differences between the means of the groups (Treatment A and B) and the group contained in the constant term (Treatment C), the differences between the means of the three treatments could be obtained as follows:

$$\bar{Y}_A - \bar{Y}_C = b_1 = 4.222$$
$$\bar{Y}_B - \bar{Y}_C = b_2 = .148$$
$$\bar{Y}_A - \bar{Y}_B = b_1 - b_2 = 4.222 - .148 = 4.074$$

Williams (1980) outlined a procedure by which the t values of the regression coefficients could be used to test each comparison through Tukey's Honestly Significant Difference (HSD) test. As noted by Williams, the t values had to be adjusted due to the fact that a variable (X_4) was used as a surrogate for the item vectors. That is, the standard error of the coefficient values was calculated based on the "apparent" denominator degrees of freedom of the full model rather than the "correct" number. In this study the standard error of the coefficients wes based on 52 rather than 77.

The procedure discussed by Williams demonstrated that corrected t values can be obtained by multiplying each t value by a constant term. The constant term (C) was defined to be:

$$C = \sqrt{\frac{MS_W \text{ (incorrect df)}}{MS_W \text{ (correct df)}}}$$

where:

MS_W (incorrect df) is equal to the mean square within value obtained from the full regression model (Model 1) that contains the surrogate variable (X_4).

MS_W (correct df) is equal to the mean square within value obtained from the full regression model that does not use the surrogate variable but uses the actual dummy item vectors. Such a model would be based on the correct degrees of freedom.

A closer examination of the computation of the constant term C, however, reveals that it is nothing more than the square root of the ratio of the "correct" degrees of freedom to the "incorrect" degrees of freedom. Thus, the computation of the value of C would be as follows:

$$C = \sqrt{\frac{\text{correct } df_d \text{ for full model}}{\text{incorrect } df_d \text{ for full model}}}$$

where:

correct df_d for full model = (n+1 - # of subjects - # of groups).

incorrect df_d for full model = n - (# of independent variables + # of intercepts). Note: no consideration is given to the fact that one variable represents numerous item vectors.

According to Williams' procedure the t values for b_1 and b_2 could be corrected by multiplying their respective t values by C. This procedure, however, would require that an additional model be analyzed to obtain the t value for the coefficient which represented the comparison between Treatment A and Treatment B. The following model (Model 3) would provide the necessary regression coefficient (b_5) and corresponding t value:

$$Y = a_0U + b_5X_1 + b_3X_3 + b_4X_4 + E \qquad \text{(Model 3)}$$

<u>One Model Procedure</u>

It is possible, however, through the use of a simple calculation to avoid the necessity of analyzing a third regression model when correcting the t values. The corrected t values could be obtained as follows:

$$t_c = \frac{b_i}{\frac{S_{b_i}}{c}}$$

where:

t_c = corrected t value

Sb_i = standard error of the coefficient

$$c = \sqrt{\frac{\text{correct } df_d \text{ for full model}}{\text{incorrect } df_d \text{ for full model}}}$$

correct df_d for full model = (n + 1 - # of subjects - # of groups)

incorrect df_d for the full model = n - (# of independent variables + # of intercepts)

The computation of c for this study was

$$c = \sqrt{\frac{52}{77}} = .821$$

Since the standard error of the coefficient values (S_{b_i}) will be the same for all treatment variables, the values for $\frac{S_{b_i}}{c}$ will be the same for all the treatment coefficients. For this study $\frac{S_{b_i}}{c} = \frac{.317}{.821} = .386$.

Dividing the differences between the means by the value obtained for $\frac{S_{b_i}}{c}$ would produce the corrected t values. The differences in the means were obtained from the regression coefficients of the treatment variables found in the only full regression model (Model 1) utilized in the analysis. The differences in the means between Treatment A and Treatment C, between Treatment B and Treatment C, and between Treatment A and Treatment B were equal to b_1, b_2, and b_{1-2}, respectively. See Table 3 for the calculations of the corrected t values.

Table 3

Calculations of the Corrected t Values

Comparison	Regression Coefficient	$\frac{S_{b_i}}{c}$	Corrected t Value
$\bar{Y}_A - \bar{Y}_C$	$b_1 = 4.22$	.386	4.22/.386 = 10.93
$\bar{Y}_B - \bar{Y}_C$	$b_2 = .148$	.386	.148/.386 = .38
$\bar{Y}_A - \bar{Y}_B$	$b_1 - b_2 = .4074$	.386	4.074/.386 = 10.55

The corrected t values were compared to the critical value of $\frac{q}{\sqrt{2}}$ where q was obtained from a table of Studentized Range Values.[2]

The critical value for this study at the .01 was:

$$= \frac{4.33}{\sqrt{2}} = 3.06$$

The df_w and J used to locate the value in the table were:

df_w = the correct number of degrees of freedom for Model 1 (df_w = 52)

J = the # of groups (J = 3)

If a corrected t value exceeds the critical value of $\frac{q}{\sqrt{2}}$, the difference between the means was judged to be significant.

Impact of Aging on Ductility

A comparison of the corrected t values to the critical value of 3.06 indicated that the mean for Treatment A was higher than both the means of Treatment B and Treatment C. The difference between the means of Treatment B and Treatment C, however, was not statistically significant. Therefore, the ductility of the steel tubing, as measured by elongation values, decreased as the tubing aged. However, the loss in ductility occurred for the most part during the first year.

The α/N Method

The multiple comparisons could also have been conducted by using the α/N method, where N is equal to the number of comparisons. Since α was set at .01 and three comparisons were made, α/N was equal to .003. The corresponding t value obtained from the t table was approximately equal to 3.36. Thus, if a corrected t value recorded for any comparison exceeded the absolute t value of 3.36, the difference between the groups

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[2]Williams (1980) provides tables (pp. 82, 83) in which the Studentized value (q) is already divided by $\sqrt{2}$.

was statistically significant. Such a comparison made for the three comparisons revealed that the mean elongation value for Treatment A was significantly higher than the means of either Treatment B or Treatment C, and there was no difference between the means of Treatment B and Treatment C. It should be noted that these results were the same as the results obtained through the use of Tukey's HSD method.

Conclusion

A question facing an industrial firm could easily be analyzed by utilizing MLR models. The MLR models, however, required the inclusion of two major concepts previously discussed in the literature. First, the MLR models incorporated the use of a variable that served as a surrogate variable to numerous item vectors as outlined by Pedhazur (1977) and Williams (1977). Second, the MLR models were used to make multiple comparisons in a repeated measures design (Williams, 1980). Unlike the procedure outlined by Williams, which requires the use of multiple full models, a procedure that allows multiple comparisons to be conducted by using only one full model was developed and effectively implemented in this study.

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