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COVARIANCE STRUCTURE AND STRUCTURAL EQUATION MODELLING IN RESEARCH: A CONCEPTUAL OVERVIEW OF LISREL MODELLING

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Synopsis

The principal objective of this paper is to demonstrate conceptually the relationship between various modelling techniques commonly employed in data analysis in social and behavioral research. The paper focuses on the commonalities between such multivariate techniques as path analysis, regression analysis, panel models, longitudinal models, common factor analysis, higher order factor analysis, factorial models (eg multitrait-multimethod models), test score models, error structure analysis models and the ANCOVA model.

It shows how the covariance structure model which underlies the LISREL model can be employed to reconceptualise and parameterise each of the above models in terms of a more general framework. In particular, these models can be conceptualised as a specific configuration of the sub-models which comprise the LISREL model. The measurement and structural models of the general covariance model are employed as the basic building blocks to reconceptualise the specific models on which each of the various techniques are

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based. The LISREL model has been chosen as the vehicle for demonstrating these conceptual commonalities because it is the most widely used general covariance structure model. Although other models such as COSAN (McDonald, 1978), EQS (Bentler, 1982) and LACCI (Muthen, 1983) are similar to the LISREL model and will thus also allow for the parameterisation of the models discussed in this paper, the associated computer programs for estimating them are not as yet as widely used as the LISREL program.

Significant advances have been incorporated into the recently released version V of the LISREL program. In particular, the program now includes a procedure which automatically estimates a set of initial start values for the iteration process in the maximum liklihood method of estimation. The provision of these start values by the user had been a major obstacle to the use of the program in previous versions. Version V also provides a wider range of statistics for judging the fit of the model, in addition to an option for estimating relationships between 'discrete' variables and another for using a least squares estimation procedure where the assumptions underlying the maximum liklihood model are not met by the data and model under investigation.

Further, the LISREL program is soon to be interfaced to one of the most widely used packages available in the social, behavioral and medical sciences. It is expected that this will mean a much wider availability and use of the program than has been the case hitherto.

The paper is written and presented in a schematic and didactic style suited to novice modellers in social and behavioral research. The only requirement is that readers have an idea of and some previous experience with at least one or two of the multivariate techniques mentioned in the opening paragraph above. They are not required to have an understanding of matrix algebra or statistics in general. Path diagrams are employed as visual representations of the conceptual models.

1.00 Introduction

This article aims to fill a major lacuna in the social science literature covering the general linear model upon which several types of statistical analysis are based. Statistical models such as factor analysis, test theory models, regression models, analysis of covariance etc. can be shown to be specialisations of a more general mathematical and statistical framework. By explicating these submodels in terms of a more general model it is possible to gain greater insight into the particular similarities and differences between individual submodels.

The general model is not new, but its development has received much greater attention over the last decade and it has been used in a wide range of applications in education, psychology, economics, sociology, and related sub-disciplines. However, due to the technical sophistication required to use the computer program and to parameterise the models its application has remained an elite speciality. Although various forms of the general model have been advanced in the literature (McDonald, 1978; Bentler, 1982; Weeks, 1978; Joreskog and Sorbom, 1977, 1978, 1981), I focus on the particular formulation which appears to have gained the widest currency. This model, now commonly referred to as the LISREL model, is accompanied by a burgeoning technical literature reporting the statistical and mathematical theory on which it is based and 'state of the art' applications of it. This literature is accessible to only a small subgroup of specialists who are familiar with the statistical and mathematical theory underlying advanced multivariate statistical analyses, however, an elementary overview for the less technically minded researcher is not readily available in the published literature. It is this lacuna in the literature which I aim to fill. Reviews of the applied literature are to be found in Bentler (1980) and Bielby and Hauser (1977) while general expositions of the statistical model can be found in Joreskog (1973, 1974, 1977, 1981), Joreskog and Sorbom (1977, 1978). Carmines and McIver (1981) and Rindskopf (1981) provide the most readable general introduction to covariance structure models while Bentler (1980) provides a lucid discussion of methodological issues. Woofle (1982) and Lomax (1982) Joreskog and Sorbom (1981) provide introductions to the application of LISREL.

My aim is to provide an didactic introductory overview of the general model and some of its specialisations, so that a much wider group of researchers may appreciate the scope and nature of the method within a context of the more familiar multivariate data analysis methods found in the social sciences.

Although multi-equation linear models have been used widely in social science research over the last two decades or so, their parentage and development in other disciplines now spans more than half a century. The biologist Sewell Wright, is usually credited with the first substantive use of such models in a paper published in 1925. The US Government sponsored Cowles Commission set up in the late 1930's recognised the potential for their application in economics and early work in educational psychology also produced variants of such models. During the 1950's and 1960's the sub-disciplines of econometrics and psychometrics were fostered and developed by a surge of interest in the application of these models in economics and psychology, respectively. Their application in education and sociology also commenced during the 1960's, however, it was not until late in the decade that researchers in these latter disciplines generally became aware of the extent to which similar models had been developed and applied in psychology and economics. The seminal article by Goldberger (1971) explicated the relationship between the variants of the model employed in economics and psychology and indicated how an approach based on the merging of these could provide models of more general interest. Today the most sophisticated development of the model is known through the generic terms of covariance structure analysis and structural equation modelling. The work of Joreskog and Sorbom (1976, 1977, 1978, 1981) has largely been responsible for solving the complex mathematical and statistical problems associated with the model so that it is now possible to use it in a wide variety of general applications.

The computer program LISREL V has been specifically designed for the estimation of these models. Other similar programs have been developed by McDonald (COSAN), Bentler and Weeks (EQS) and Muthen (LACCI) but they are not yet as well documented and developed as LISREL, nor are they as widely available. Each program has slightly different features and capabilities but LISREL is the most widely used in the research reported in current academic journals.

2.00 Structural Modelling: The General Framework

The topics to be discussed here can be summarised in the following distinctions:

(1) exploratory v's confirmatory analysis,

(2) fallible v's assumed infallible data,

(3) latent (unobserved) variables v's observed variables,

(4) linear v's non-linear models,

(5) model fitting v's parameter estimation,

(6) over-identified v's just-identified models.

These six distinctions will be employed to ellucidate differences and similarities between the more general covariance structure model and those which underlie the more familiar multivariate methods in common use.

(1) Exploratory v's Confirmatory Analysis

When the level of knowledge available about a given problem is relatively underdeveloped one has to proceed to analyse and interpret that problem with elementary methods (Wold, 1966, 1969). The methods are elementary only in the sense that they embody little accumulated knowledge about the phenomena of interest, ie they are not based on an elaborated formal theory of the phenomena. Such methods can be described as exploratory ,since they take as their only criterion, the standard of 'whether or not they provide a means to make some sort of sense of the data'. When the method employed produces a non-interpretable or unexpected finding, it is either discarded or given a <u>post hoc</u> interpretation. Since discarding a finding leads to the process of searching or exploring the data for another, more appropriate, interpretation, this procedure also results in a <u>post hoc</u> explanation of the patterns existing in the data. The problem with such explanations is that there is, in principle, an unlimited number of them which describe the data equally well (on statistical criteria) for any given situation. In the limiting situation where we start from a position of 'no theory', not even a common sense one, no formal statistical procedure will choose between the logical or explanatory validity of competing <u>post hoc</u> explanations on the basis of statistical analyses of the data. Hence we need to have recourse to a method of confirmation, or in the parlance of statistics, a method of 'testing' the various competing explanations.

Confirmatory analysis is the procedure used in such a situation. In order to conduct a confirmatory analysis it is necessary to come to the data armed with a 'theory' of what may have generated the grid of relationships which exist in that data. This theory may come from many sources; scholarly, common sense, exploratory analysis of other data, etc, and may be more, or less, well developed. The one requirement which the theory must be able to satisfy, however, is that is must be amenable to formulation in a testable form. This may be based on a weak specification - eg variables X1.......X5 are related to each other in a non-random way or, on a strong specification - eg X1 causes X3 but not X2, or some combination of options in such a range of specifications. Once the theory is formulated in a testable specification it is possible to conduct such tests as appropriate to confirm or reject the particular hypotheses. A collection of hypotheses which describe the overall theory in greater detail can also be tested simultaneously. In such a case we say that the overall model which describes the theory is being tested. Since model testing is usually based on several individual hypotheses which are simultaneously tested, it provides a greater degree of rejection or confirmation of the theory than independent tests of individual hypotheses. Structural modelling is specificaly aimed at such confirmatory analysis of theories.

However, the distinction between exploratory and confirmatory analysis is not as marked as I have presented it so far. We probably always have at least one rudimentary theory in mind when we plan a study. At the simplist level, a theory of measurement is required just to collect data on a particular variable. If we put a little effort into constructing a theory on the basis of what is already known about the phenomena of interest, we will usually be able to provide one or more potential explanations on the basis of this prior knowledge. These hypothesised models can then be tested against the data to see if they could have generated the patterns therein.

If a model provides an acceptable fit to the data, then we have an independent confirmation of the theory, although it is possible that other models (possibly unknown) could also fit the data equally well. Where the model is not confirmed we can test alternative models in a similar fashion, and in the event that none of them fits the data, explore the data by making modifications to these rejected models. Exploratory analyses aimed at providing refinements and changes to the models may suggest appropriate respecifications which can then be tested against new data, or validated against a sample of the data held back for such purposes.

(2) Fallible v's Assumed Infallible Data

The foregoing has assumed that we can actually measure the theoretical variables of interest in order to test the theory. We still have a problem to face, even if such variables are directly measurable. When we measure an item or variable of interest two questions immediately arise: (a) Is the measurement valid? (b) Is it reliable? These two issues are studied under the rubric of 'measurement error'. Any sophisticated mathematical or statistical analyses will want to be able to take account of such error, so that we can model the pattern of relationships between the underlying true scores for each variable in the data. The problem of measurement error has been studied in educational psychology in the development of 'test score theory' and in economics as the 'errors in variables' problem. The essential component of both approaches is an attempt to distill out the 'true score' of the variable from the measurement error. The sub-model underlying this aspect of the LISREL model is illustrated in Figure 2.1. It states merely that the observed(measured) variable(X) consists of two parts: a true score represented as a latent variable(ξ), plus an error of measurement(δ). (See Figure 4.1 for a description of the symbols employed in the figures.)

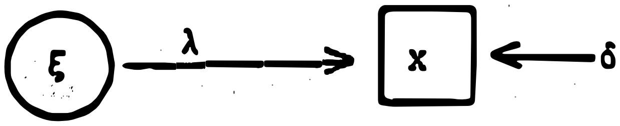

Figure 2.1 LISREL Measurement Error Model

$$x = \lambda\xi + \delta$$

Here measurement error may be interpreted as a residual covering a range of influences, other than that of the true score, which produce the observed score. Influences such as questionnaire method effects, respondents self image, etc, which vary across individuals and influence their responses, are included. The measurement model thus posits the existence of a true score for the particular theoretical variables of a theory, however, the effects of measurement error mean that it is not possible to directly observe these true scores. The notion of a latent (unmeasured) variable is introduced to represent the true score. The variable ξ in Figure 2.1 is of this type.

(3) Latent (unobserved) Variables v's Observed Variables

The preceeding section has indicated one situation in which a latent variable is introduced into the model so as to provide a more informative analysis of theoretical models against (observed) data.

Another situation in which it is necessary to introduce latent variables is when the theory employs concepts which are, in principle, not directly observable, eg social class, anxiety, intelligence. In order to obtain a measurement on these variables it is necessary to measure observable variables which are designated as indicators of them. Essentially, this amounts to saying that the measurement theory of these variables is more complex than for directly observable variables. For instance, simplistic measurement of the variable social class may assume that a single observed variable, fathers occupation, say, is a suitable measure of it. More complex measurement of social class may take account of other aspects of the concept , also. In this case, a wider range of variables which are held to be more representative of the total 'conceptual domain' of the concept (eg relationship to the mode of production, mothers' and fathers' education,family wealth,etc.) may be employed as complementary indicators of the theoretical variable. In such a situation it is necessary to expand Figure 2.1 to include more than one observed measure (indicator) of the latent variable, so that the theoretical variable is represented by the common variation across the set of designated indicators of it. This is represented in Figure 2.2 and the latent variable in such cases is referred to as a common factor.

We note that the form of the relationship between the parts of the model in Figure 2.2 is formally the same as that in Figure 2.1, thus they are represented in the same way in the LISREL model. Whether the variable ξ is to be interpreted as a latent variable or common factor is dictated by its particular theoretical purpose and not by statistical or mathematical considerations. When it is interpreted as a common factor (as in factor analysis),the error terms($\delta_1 - \delta_3$) may be interpreted as the unique part of each observed variable and the common factor itself as the shared part or common construct underlying the set of them. The term latent construct will be employed to refer to both common factors and latent variables when there is no particular need to distinguish between them. This usage helps us to keep in mind the fact that such variables are formally 'constructed' measures of theoretical variables, as opposed to direct measurements on the variable. Direct measurements on a variable assume that it takes the same scale of measurement as the measuring instrument, however, indirect measurement means that the scale of measurement for the constructed variable depends on the 'weight' of each particular observed indicator in the constructed variable, in addition to the scale of measurement for each indicator. Thus, constructed variables will often have 'arbitrary' scales of measurement and these may change, depending on the membership of the particular observed variable set employed as indicators of the construct.

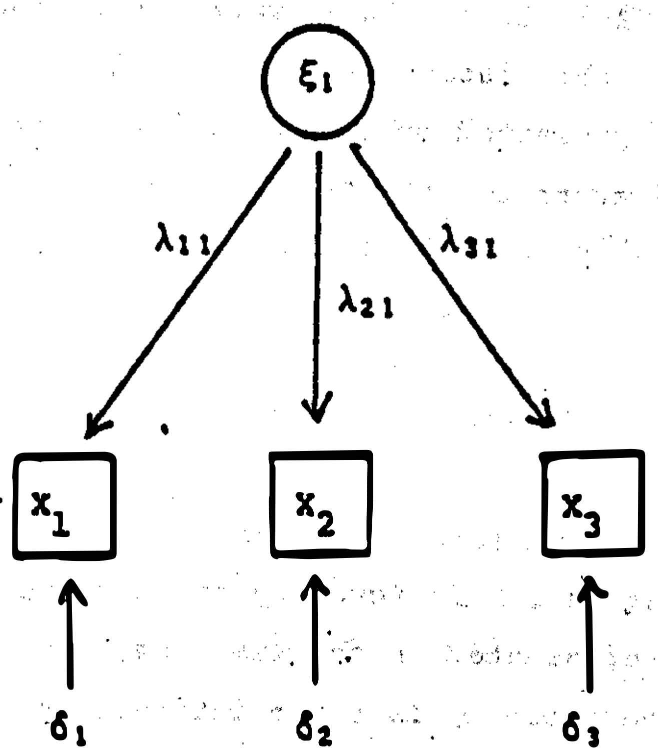

Figure 2.2 A Single Latent Factor with Fallible Observed Variables

$$x_1 = \lambda_{11}\xi_1 + \delta_1$$
$$x_2 = \lambda_{21}\xi_1 + \delta_2$$
$$x_3 = \lambda_{31}\xi_1 + \delta_3$$

(4) Linear v's Non-Linear Models

Linear models have become popular for analysing the relationships between variables because they afford a tractable solution to the numerical and statistical problems in such analyses. The true relationships between variables may not be linear, thus the use of linear models necessarily implies a more or less approximate fit to reality. Ad hoc procedures are available for linearising the relationships between variables through the application of non-linear mathematical transformations to the data when these are well behaved relationships, but parametric techniques for linearising non-well behaved relationships are more complex (see Kendell and Stuart, 1973; Box and Cox, 1964). The property of 'continuity' in measurement scales is an important aspect of linear relationships. A measurement scale for a variable must be based on an assumed underlying continuum if it is to have the potential of a linear relationship with other variables.

This aspect of measurement has been singled out for particular attention because it is one of the stronger assumptions made in linear models. Variables which are of a categorical nature are by definition not continuous. If the underlying theoretical variable is, however, continuous in nature then the observed variable is often treated as if it were continuous. The LISREL V program includes an option to implement this type of analysis. Essentially, it rescales the measuring instrument on the assumption that the underlying theoretical variable is continuous and normally distributed in the population. These rescaled variables are then used to calculate the correlations, etc, for the analysis of the relationships between variables. Where the theoretical variable is not continuous in character, then the assumption of linearity is inappropriate. For such categorical theoretical variables, the correct procedure is to analyse the data for each categorical group separately and then compare model estimates between groups or to include a (binary) dummy variable in the model to represent membership/non-membership in a group. In the latter case it is assumed that the groups defined by the dummy variable share the same structure of relationships (ie same slopes for their 'regression lines') but that their regression lines may be non-coincident.

When the data to be analysed contain a mixture of interval and ordinal (discrete) measures the LISREL approach can be used to estimate the relationships between the variables in the data. The LISREL program calculates different types of 'correlation coefficient' between the different types of variable pairs (interval, discrete polytomous, discrete dichotomous) depending on the particular pairing of variable types involved in each instance. The matrix of these mixed 'correlation coefficients' is then used to estimate the relationships between the variables in a model. However, where all, or almost all, of the variables in a data set are of a dichotomous categorical type it may be more appropriate to use other methods specifically developed for this case (eg log linear models, latent structure models). Muthen (1983) and Bock and Aitkin (1981) have also developed programs which can estimate structural models for this case. The Muthen program is based on a framework similar to the LISREL model. Examples of the application of these methods can be found in Muthen(1981) and, Aitkin,Anderson and Hinde(1981). In the case where the data contain a mixture of the variable types noted above, the use of these latter methods may result in a considerable loss of information if all the variables are collapsed into dichotomies. It is possible to avoid this loss of information by creating dummy variables to represent each of the categories in the polytomous and interval variables in the data. This procedure, however, results in n-1 dummy variables for n categories in a measurement scale, thus it can result in a very large number of variables in the raw data set which forms the input into these programs. Also, formal inequality constraints have to be included in the structure of the model if these sets of transformed variables are to retain the information contained in the ordering of categories in the original measurement scale. The LISREL approach can thus be seen as a compromise solution for analyses in data where the variables are measured with mixed types of measurement scales. This compromise is achieved at the cost of assuming that the distribution of the true score for the observed variable is normally distributed in the population.

(5) Model Fitting v's Parameter Estimation

Prior to the development of the general LISREL model it was not possible to routinely test how well the estimates of the parameters in a structural model had reproduced the observed correlation or covariance matrix for the data. Typically the researcher was interested in estimating the parameters of each equation in the model independently of constraints across equations, contrary to what is usually suggested by the substantive theory underlying social science models. Relationships between variables were usually viewed as being all of the same causal or relational status. For example, parental education and students' educational aspirations as determinents of achievement would be entered into the same regression equation. However, most theories would suggest that one variable should have causal priority over the other.

In general, it is always possible to estimate parameters for relationships between variables when an unconstrained, ie saturated, model is posited. For instance, any set of variables can be factor analysed to derive an estimate of their common factor variation. In more complex, ie formally constrained, models the estimation process attempts to provide the best set of estimates given the theory derived constraints placed on the relationships between constructed (latent) variables in the model. In an unconstrained situation, of which the first order common factor model is a simple case, the model estimates will exactly reproduce the pattern of variances and covariances in the data. However, in a constrained model (ie one in which certain parameters are fixed in value, say, set to zero) it is problematic as to how closely the estimated parameters reproduce the relationships in the data. The fit of the model is a measure of how well the parameter estimates of a particular constrained model reproduce the relationships in the data as represented by the correlation or covariance matrix. If a particular theory is a poor description of the processes which produced the data, then the constrained model which represents that theory will result in a poor fit between the model and the data.

The issues of parameter estimation and model fit are conceptually quite distinct. Model fit indicates how well a particular set of parameter estimates describe the first and second order moments (correlations or variances and covariances) in the data, while parameter estimation relates to the estimation of the magnitude of individual parameters such as factor loadings or regression coefficients. The maximum liklihood method of estimation is based on an iterative process which employs a criterion of how well a particular set of estimates fit the data at each iteration in order to derive a solution which fits better at the next step, given the constraints imposed on the model. The capacity to test for fit is an advance over methods which provide only the opportunity to estimate parameters. For instance, none of the multivariate statistical procedures in the widely used SPSS computer package provide for testing the fit of models based on their parameter estimates. In part this is justified, since the procedures therin are all designed for estimating unconstrained (saturated) models, hence the models will all have an exact fit to the data. However, the methodology of using such models often calls for modifications to the estimated solution, eg a standard recommendation is that variables with a factor loading of less than 0.3 should be deleted from the particular factor when using them to build a composite measure from the factor. Such rules of thumb raise the question of how well these constrained composites then fit the data. The rule of thumb amounts to the placing of ad hoc restrictions on the multiple factor model, which is then, by definition, a constrained model. Thus the composites will not necessarily fit the data perfectly, as would the unconstrained model on which they were based.

(6) Over-Identified v's Just-Identified Models

The idea of identification can be demonstrated by an analogy with the conditions for solving a set of algebraic equations. The discussion of parameter estimation and model fit above assumed that the parameters of the model were identified. A model is said to be identified if the parameters can, in principle, be estimated as unique values.By unique I mean that the set of equations can be satisfied by just the one 'unique' set of values for the unknowns in them. For instance the variable Y is said to be just-identified in the following equation set:

$$Y = X + 2$$

$$Y = 3X - 2$$

By substituting the expression for Y from the first equation into the second equation we obtain an expression which is in terms of the 'X' variable only.

$$X + 2 = 3X - 2$$

On simplification this reduces to X=2 and by resubstitution into the first equation we solve for Y as Y=4.Now,this solution (Y=4,X=2) is the only set of values which will solve both of these equations simultaneously,hence they are said to be unique.

In structural modelling the equations express the relationships between the known variances and covariances which we calculate directly from the raw data and the unknown 'parameters' which represent the model specific effects of one variable on another.If we have more unknown parameters in the equations the number of variances and covariances for the variables employed in a model, then the model is said to be under-identified, that is,there is insufficient known information in the system to satisfy the requirement that each unknown can be uniquely estimated.What this means in practice is that each parameter can at most only be solved as a function of the other parameters in the model.

If we have exactly the same number of unknown parameters as ther are known variances and covariances (or correlations) in the model,then it should,in principle, be possible to uniquely estimate each of the parameters in the model.I say 'in principle' because it sometimes occurs that a set of the variances and covariances in the model are an exact linear combination of some other subset of variances and covariances and this means that the total amount of known information is reduced, because a linear combination of variances and covariances does not add to the information already contained among the uncombined set of variances and covariances. That is, there is less information available in sets of variables with exactly collinear variances and covariances than there is in two subsets each of which is indepedent of each other. Where there is more known information contained in the set of variances and covariances than there are parameters to be estimated, then the model is said to be over-identified

If the equations of the model were exactly deterministic,that is, if each outcome could was specified as an exact combination of the other variables in the model,then all pairs of equations would give the same solution as any other pair of identified equations for the model.In practice the models used in the social and behavioral sciences are not exactly deterministic but 'stochastic' in nature.That is,they specify that the endogenous(outcome) variable is only partially determined by the other variables included in the model and partially determined by a 'stochastic' residual or error term.This stochastic residual then,in effect,represents the combined influences of all the variables which have been left out of the model(eg because we do not have data on them). The result of combining the stochastic element of modelling with the condition of over-identification is to arrive at a situation where the different pairs of equations may give slightly different solutions for the parameters. Rather than arbitrarily accepting one set of parameter estimates as preferable to the other possible sets in an over-identified stochastic model we can use the additional information available from the solutions for different subsets of the equations to find a solution which in some sense is the best 'average' estimate of the parameters ,given the model and data at hand.In practice this job is done by a computer algorithm which minimises the difference between the set of variances and covariances implied by a given set of parameter estimates for the model and the set of population variances and covariances which are estimated from the raw sample data, by assessing which estimates give the best fit to the data.

In complex models the number of parameters to be estimated is often too large for manual checks on the identifiability of each to be undertaken, since the procedure is logical or mathematical, as opposed to statistical, in nature and standard numerical or statistical computation techniques are not suitable. The LISREL program does undertake a check on model identification when the option for maximum liklihood estimation is used but not when the unweighted least squares method is employed.

3.00 The LISREL Model

As noted earlier, the structural models of concern in this paper are those which can be formally specified on the basis of a consistent theory. The term 'theory' is used here to denote any set of consistent notions about the relationships between the set of variables of interest. The model derived from the theory may be more or less elaborated, depending on the state of development of the theory. The particular feature of models in the LISREL framework is that they make explicit provision for the estimation of relationships between the underlying theoretical variables of a theory. Heuristically this amounts to the estimation of the regression and correlational relationships between the latent constructs in the structural model. This aspect of the LISREL model is depicted by Figure 3.1.

The curved lines in the figures presented in this paper represent correlations or covariances, and straight lines represent a special form of regression type relationships, known as structural relationships, ie they are causal, (asymetric) relationships. The single arrow head indicates the direction of the relationship.

The Greek letter ξ (Ksi) indicates endogenous (otherwise known as predetermined or independent) observed constructs and the letter η (Eta) indicates endogenous (dependent) constructs (ie determined by other constructs in the model). The convention used in referring to LISREL models is that an exogenous construct is one which is not caused by any other constructs in the model. In the terminology of experimental design some constructs which are designated to be dependent in same equations in the model may appear in other equations as an independent construct. For example, in Figure 3.2, the construct η_1 is both dependent and independent, similarly for η_2 depending on which part of the model is being considered. It is for this reason that the term exogenous is reserved for referring to constructs which are causally independent of all others in the model. Any construct which is caused by any other construct in the model is then referred to as an endogenous construct.

Since not all of the variation in an exogenous construct is necessarily determined by the other constructs included in the model, these constructs are specified to have a residual term denoted by ζ (Zeta). The Greek letters are employed so as to distinguish between unobserved and observed variables in the full model. Unobserved variables (latent constructs) are denoted by Greek letters and directly observed variables are denoted by Roman

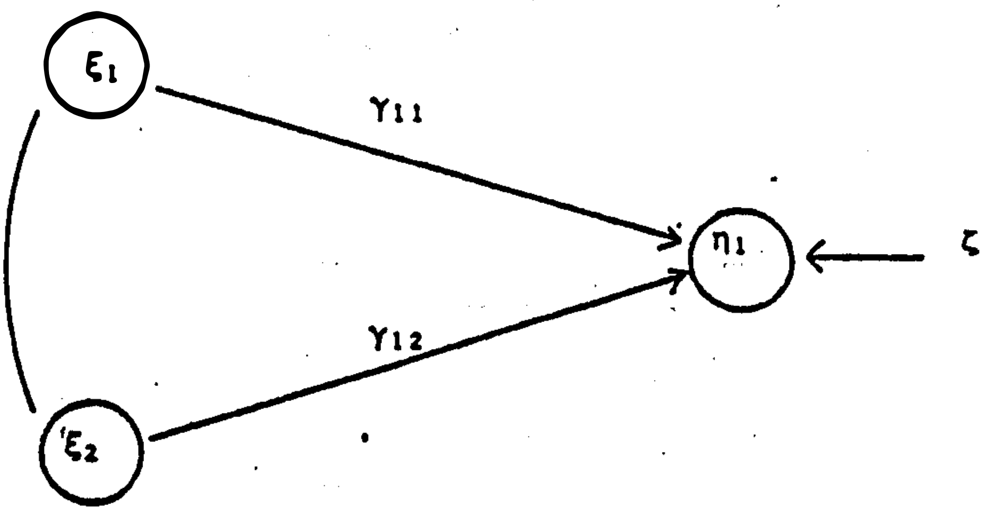

Figure 3.1 A Simple Structural Model

$$\eta_1 = \gamma_{11}\,\xi_1 + \gamma_{12}\,\xi_2 + \zeta$$

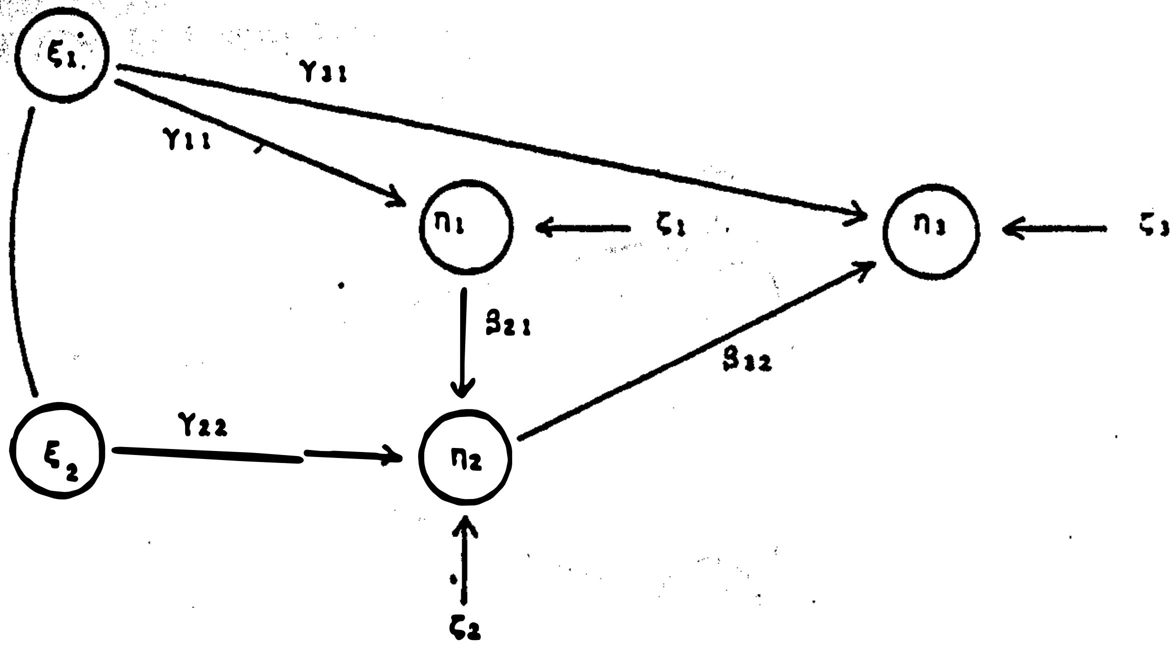

Figure 3.2 A More Complex Structural Model

$$\eta_1 = \gamma_{11}\xi_1 + \zeta_1$$

$$\eta_2 = \gamma_{22}\xi_2 + \beta_{21}\eta_1 + \zeta_2$$

$$\eta_3 = \gamma_{31}\xi_1 + \beta_{32}\eta_2 + \zeta_3$$

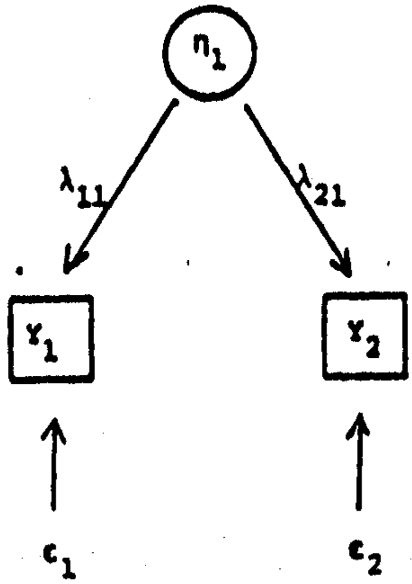

Figure 3.3 Measurement Model for a Single Latent Factor

$$Y_1 = \lambda_{11}\eta_1 + \epsilon_1$$

$$Y_2 = \lambda_{21}\eta_1 + \epsilon_2$$

letters and enclosed in square boxes. Circles denote that a latent construct is meant to represent a specific theoretical variable (ξ, η); in contrast to those of an omnibus composition represented by residuals (ζ) or error terms (δ, ε). A more complex structural model is depicted in Figure 3.2. In this diagram the constructs ξ_1, ξ_2 are exogenous and $\eta_1 - \eta_3$ are endogenous constructs. We note that the presence of three endogenous constructs means that the model attempts to explain a theory in which the exogenous theoretical variables simultaneously cause these different outcome variables.

The equations in Figure 3.2 can be interpreted as multiple regression equations for the particular structural relationships specified between the latent constructs. The constructs are unobserved (latent), thus we cannot actually estimate the equations as they stand at present. Estimation of the relationships between the latent constructs requires that we have estimates of the values of the latent variables based on measurements on the observed variables, thus it is necessary to add a measurement model to the structural model of Figures 3.1 and 3.2.

The measurement model provides the required link between the observed and unobserved variables of the model. The measurement model is used to describe the latent constructs in terms of the reliabilty and validity of the measurements on the observed variables. Figure 3.3 presents a measurement model for a single latent construct. The same form of the relationship between observed variables and unobserved (latent) constructs applies for both exogenous and endogenous constructs. The Roman letter 'X' is used to denote observed variables which are indicators for exogenous constructs and the letter 'Y' is used to denote those which are indicators for endogenous constructs. In the LISREL model, unlike in standard multiple regression analysis, the implied paths of causation do no travel between observed X's and the Y's, because these are 'impure' measures of the theoretical variables being modelled. The variables of a particular theory and, hence, of the structural equations, are those represented as the 'true' or latent constructs of the structural model. We note that the measurement model of Figure 3.3 is of the same form as that presented earlier in Figures 2.1 and 2.2.

Figure 3.4 takes the simple structural model of Figure 3.1 and adds measurement models to each latent construct. For purposes of illustration the number of observed measures has been varied for each construct. ξ_1 has only one observed measure (X1), ξ_2 has three observed measures (X2,X3,X4) and η_1 has two observed measures (Y1,Y2). We note that errors of measurement in the Y variables are denoted by the Greek letter Epsilon (ε) and measurement errors in the X variables are denoted by the Greek letter Delta (δ). Figure 3.4 represents the completed model and the relationships marked by curved and straight lines can be estimated with the LISREL computer program.

In order to use the LISREL program it is necessary to translate the formulation of a model into matrix notation and to satisfy certain teohnioal conditions. These aspeots of the model are discussed briefly at the end of this paper. The next section shows how the model developed so far can be employed to describe several sub-models of the general LISREL model. The discussion continues through the expository use of path diagrams for such models. The equations for each model are presented with the diagrams but may be passed over by the introductory reader without much loss of information.

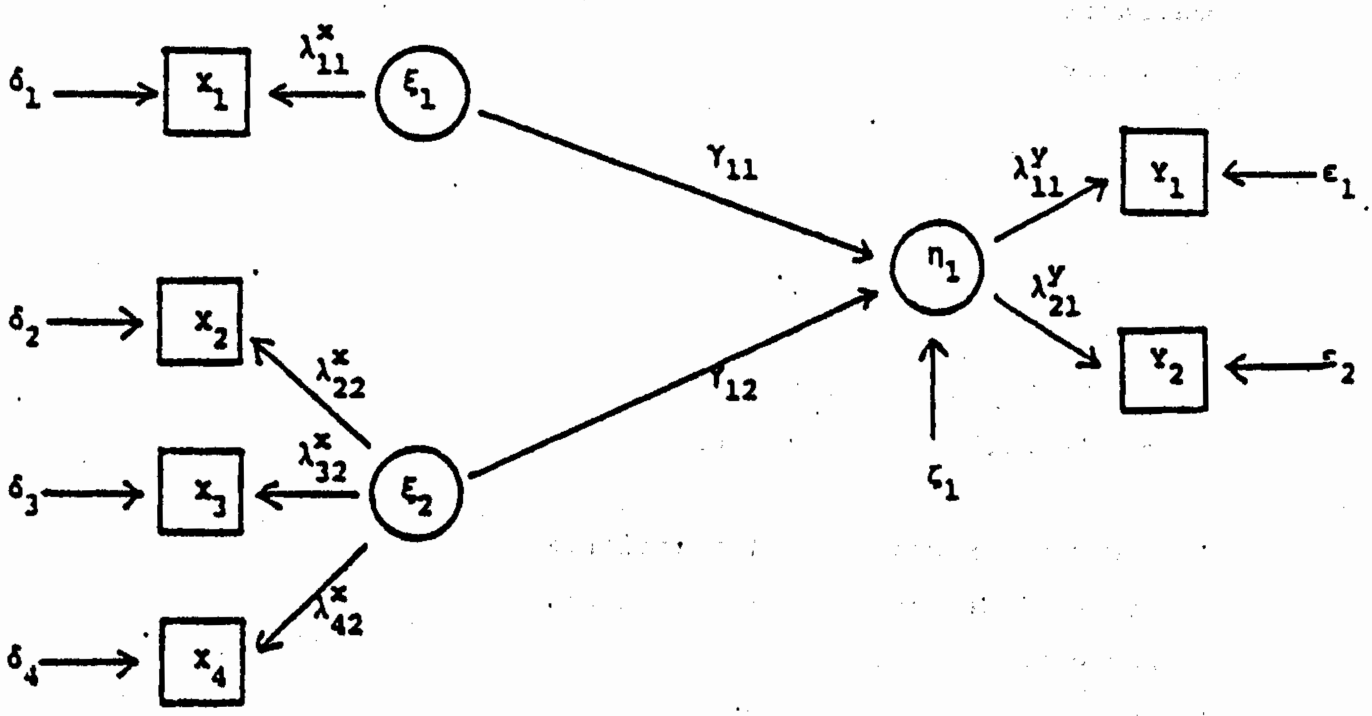

Figure 3.4 A LISREL Model: Structural Model plus Measurement Model

Measurement Models

$$x_1 = \lambda^x_{11}\xi_1 + \delta_1$$

$$x_2 = \lambda^x_{22}\xi_2 + \delta_2$$

$$x_3 = \lambda^x_{32}\xi_2 + \delta_3$$

$$x_4 = \lambda^x_{42}\xi_2 + \delta_4$$

$$y_1 = \lambda^y_{11}\eta_1 + \epsilon_1$$

$$y_2 = \lambda^y_{21}\eta_1 + \epsilon_2$$

Structural Model

$$\eta_1 = \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \zeta_1$$

The system assumptions made by LISREL are: (1) that there is zero covariance between the latent constructs and the residuals on the latent constructs, ie $E(\xi\zeta)=0$, $E(\eta\zeta)=0$, (2) there is zero covariance between the measurement errors on the observed exogenous and those on the observed endogenous variables, ie $E(\delta\varepsilon)=0$ and, (3) there is zero covariance between the latent constructs and the errors of measurement on the observed variables, ie $E(\xi\varepsilon)=0$, $E(\xi\delta)=0$, $E(\eta\varepsilon)$, $E(\eta\delta)$. These assumptions apply to the full model as depicted in Figure 4.1. (In some situations models can be reparameterised in alternative ways which allow for a relaxation in the restrictions implied by these assumptions.) The following sub-models will now be illustrated:

(1) first order common factor analysis model,
(2) higher order factor analysis model,
(3) regression model,
(4) recursive path model,
(5) reciprocal effects path model,
(6) test score model,
(7) multitrait multimethod model,
(8) error structure analysis model,
(9) analysis of covariance (ANCOVA) model.

(1) First Order Common Factor Analysis Model

The basic structure for a factor analysis model with a single common factor is presented in Figure 4.2a. In this model all three observed variables (X1,X2,X3) are assumed to be measures which reflect the underlying latent factor (ξ). The error terms (δ_1- δ_3) indicate that the observed variables may be measured with error. A basic assumption of the Classical factor analysis model is that these error terms are uncorrelated, ie $E(\delta_i \delta_j) = 0, i \neq j$. However, such an assumption may not always be warranted. If common variation exist between a subset, but not all, of the observed variables, then it may not be captured by the factor which represents influences common to all the observed variables.

4.00 Sub-Models of the General LISREL Model

The general LISREL model can be summarised by the diagram in Figure 4.1

The sections within the broken lines represent the measurement models for the exogenous and endogenous constructs, and the section of the model linking them represents the structural model. The individual symbols do not have subscripts, which indicates that they represent all the constructs of their type in this representation of the general model. Note that the observed exogenous variables (X's) are not directly linked to observed endogenous variables (Y's). The structural model which lies between the measurement models mediates the causal influence of the exogenous variables on the endogenous variables. Although not shown in Figure 4.1, correlational links are allowed for between the individual exogenous constructs and regression type (causal) relationships are allowed for between those within the endogenous construct set.

There are two types of assumptions which go with the LISREL model: (1) System assumptions - these are the general statistical assumptions necessary in order to estimate all models in the LISREL framework and ,(2) Model assumptions - the additional assumptions which are specific to a particular model, eg the Classical factor analysis model specifies that the errors of measurement on the observed variables are uncorrelated. In effect these latter assumptions are the restrictions which we place on the general model to define the sub-models as particular specialisations of the general model. Hence, these assumptions are, in principle, testable within the LISREL framework, whereas the former set of assumptions are not, since they are an integral (ie necessary) part of that framework.

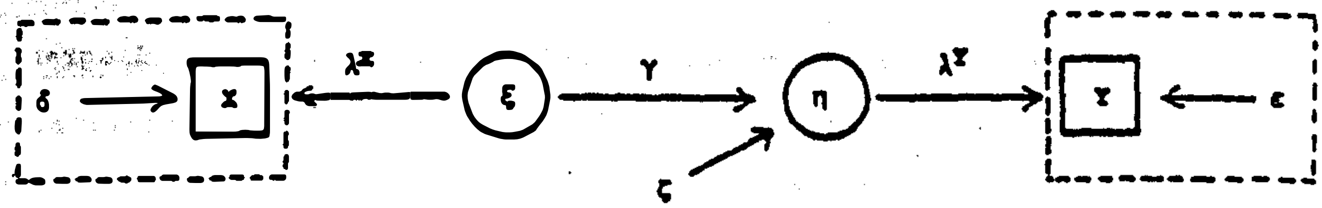

Figure 4.1 Schematic Representation of the General LISREL Model

X	-	observed exogenous variables
ξ (ksi)	-	latent exogenous constructs
η (eta)	-	latent endogenous constructs
Y	-	observed endogenous variables
δ (delta)	-	measurement error in observed exogenous variables
ε (epsilon)	-	measurement error in observed endogenous variables
λ^x (lambda X)	-	construct loadings (validities/reliability) for observed exogenous variables
λ^y (lambda Y)	-	construct loadings (validities/reliability) for observed endogenous variables
γ (gamma)	-	regression type coefficients of latent exogenous construct on latent endogenous constructs
ζ (zeta)	-	stochastic residual term for each latent endogenous construct

The above notation differs from that found in the literature for the general model. I have employed lower case Greek letters to indicate the parameters of the model - as in the other models in this paper. Usually, lower case Greek letters denote that the system is specified in equation form while upper case Greek letters denote the corresponding specification of these equations in matrix algebra format. The literature usually discusses the general model in terms of its matrix format.

The model has three other sets of parameters in addition to those shown in the above diagram:

θ^δ (theta δ)	-	the covariance matrix for the errors on the observed exogenous variables
θ^ε (theta ε)	-	the covariance matrix for the errors on the observed endogenous variables
β (beta)	-	the matrix of structural coefficients between the latent endogenous constructs
Φ (phi)	-	the matrix of correlations between the exogenous latent constructs

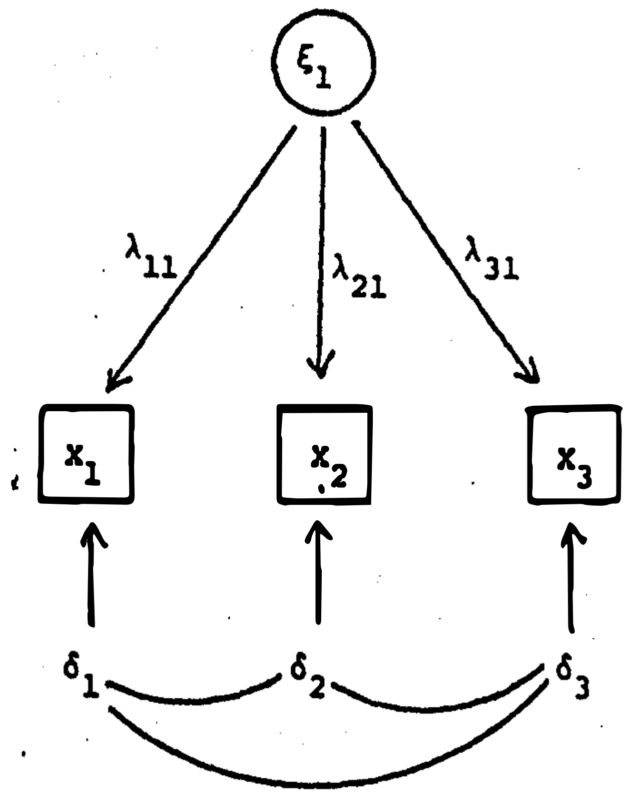

Figure 4.2a Factor Analysis Model

$$x_1 = \lambda_{11}\xi_1 + \delta_1$$

$$x_2 = \lambda_{21}\xi_1 + \delta_2$$

$$x_3 = \lambda_{31}\xi_1 + \delta_3$$

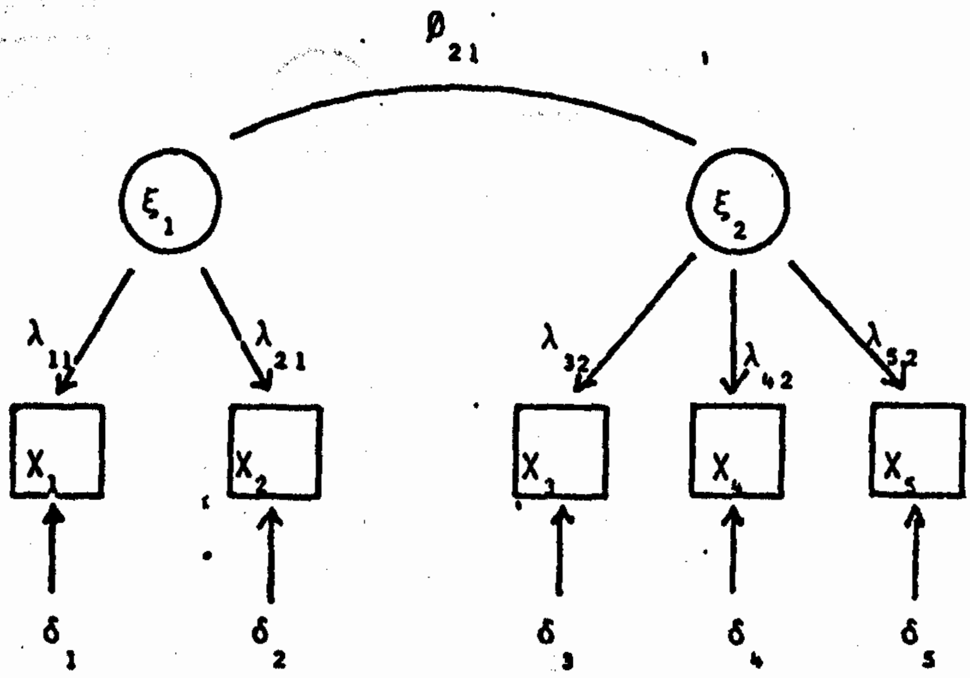

Figure 4.2b Multi-dimensional Factor Model

$$X_1 = \lambda_{11}\xi_1 + \delta_1$$

$$X_2 = \lambda_{21}\xi_1 + \delta_2$$

$$X_3 = \lambda_{32}\xi_2 + \delta_3$$

$$X_4 = \lambda_{42}\xi_2 + \delta_4$$

$$X_5 = \lambda_{52}\xi_2 + \delta_5$$

δ_{ij} (for $i \neq j$) not shown but all may be specified to be > 0.

For orthogonal model $\emptyset_{21} = 0$.

Variation common to a specific subset of the observed variables will be left unanalysed by a single common factor model such as that in Figure 4.2a. In such a model these unanalysed relationships will be represented by correlations among the error terms belonging to the specific subset of observed variables, thus the assumption of the Classical model will be inappropriate. Correlations between the error terms in this model are an indication that the observed variables are not unidimensional. The specification of the single factor (ξ_1) in Figure 4.2a implies that the model is unidimensional, given also that the errors are also specified to be uncorrelated.

A multi-dimensional factor structure is represented in Figure 4.2b. By specifying that the correlations between factors (ϕ_{21}) are zero we are able to test orthogonallity between the factors. A rotationally invariant solution is found by placing restrictions on the pattern of factor loadings (the λ_{ij}) for the observed variables. For example, in Figure 4.2b variables X_1 and X_2 are specified to load only on factor ξ_1, etc.

We note that the model in Figure 4.2a is of the same form as that in Figure 2.2, indicating that the separate measurement models of LISREL can each be viewed as simple common factor models. Further, note that the structure of the factor analysis model is the same as the LISREL model at each end of Figure 4.1. If we split the model of Figure 4.1 in two (ie delete the arrow denoted by the Greek letter Gamma- γ) we have two separate models. If the residual term ζ (Zeta) is specified to be zero then the left and right models are equivalent. Thus the simple factor analysis model can be parameterised in terms of either of one of two equivalent LISREL sub-models. This is a consequence of the fact that there is no designation of variables as endogenous or exogenous, since there are no 'causal' relationships between the latent constructs, in a factor analysis model.

The righthand model in Figure 4.1 does, however, allow for a non-zero residual (Zeta-ζ) on the latent factors (η) and is known as the ordinary factor analysis model, whereas that on the left is known as the Classical factor analysis model, due to its specification that the latent factor (Ksi-ξ) has no residual term.

(2) Higher Order Factor Analysis Model

The higher order factor model allows for the specification of a more detailed analysis of the structure among subsets of the observed variables. This is shown in Figure 4.3. The LISREL framework does not allow for directional (regression type) relationships between latent constructs on the left side of Figure 4.1, thus it is necessary to specify a second order model in terms of the ordinary factor analysis model on the right of Figure 4.1. The factors η_1 and η_2 in Figure 4.3 are first order factors while η_3 is a second order factor. We note that while the first order factor model in Figure 4.2a appeared to consist of a measurement model only, it can now be seen from Figure 4.3 to represent a trivial structural model in which the other factors are null. Since the first order factors will be fully determined by the second order factor only in the special case that the different factors are perfectly correlated, eg. $r(\eta_1\ \eta_3)=1$, a residual term (ζ) is specified for each first order factor in the general form of the model. Weeks (1980) gives examples of the use of such models.

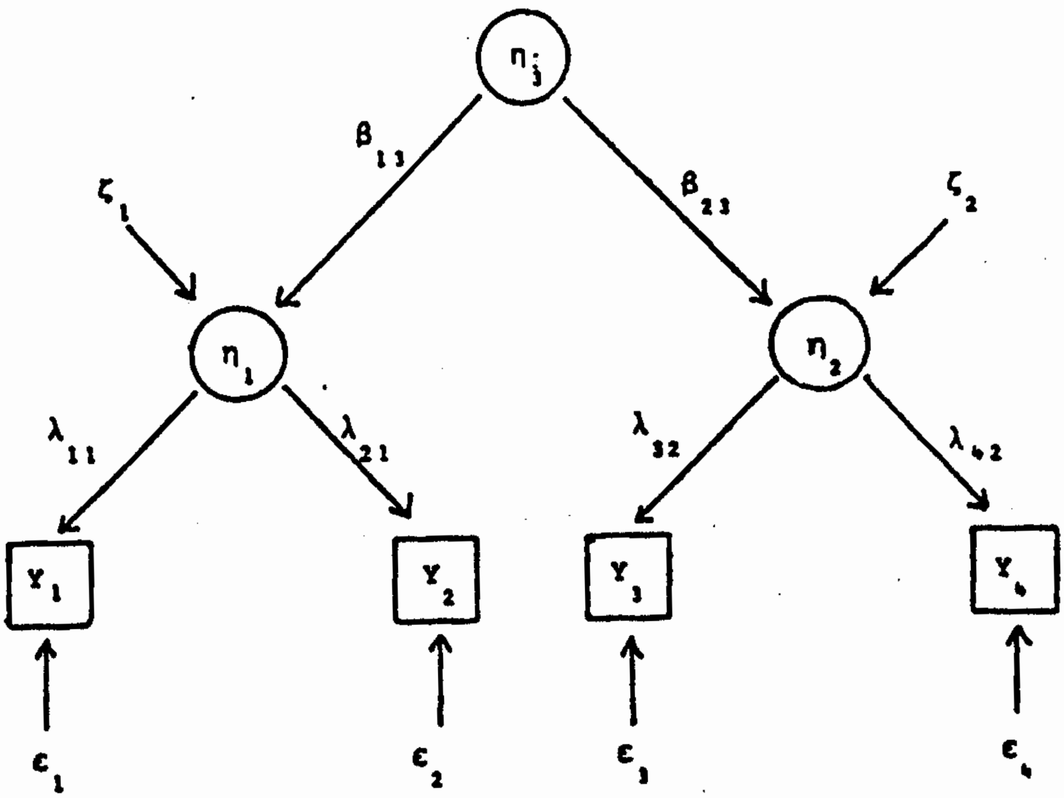

Figure 4.3 Second Order Factor Analysis Model

Structural Model

$$\eta_1 = \beta_{13}\eta_3 + \zeta_1$$

$$\eta_2 = \beta_{23}\eta_3 + \zeta_2$$

Measurement Model

$$Y_1 = \lambda_{11}\eta_1 + \varepsilon_1$$

$$Y_2 = \lambda_{21}\eta_1 + \varepsilon_2$$

$$Y_3 = \lambda_{32}\eta_2 + \varepsilon_3$$

$$Y_4 = \lambda_{42}\eta_2 + \varepsilon_4$$

variables as fixed variables. ('Fixed variable' means that the values of the variable found in the data are the only values which that variable can take, ie that these values are not sampled from a probability distribution of values for that variable.) The regression estimates are then said to be 'conditional' upon the particular values of the 'X' variables in the data, as is the usual case with ordinary multivariate regression. If, however, the exogenous variables (X's) are observed with error, then the true values for the underlying theoretical variables will be dependent upon the probability distribution of the observed scores. In such cases the variable is said to be a random or stochastic variable. Measurement error is one source of a probability distribution for the estimates of the true values of a variable. By building in the stochastic nature of measurement error it is possible to parameterise a regression model in the LISREL framework such that the regression coefficients are directly disattenuated for the effects of measurement error. This development can be viewed as a special case of the path model introduced in the next section. Error is a pervasive feature of all measurement, but it presents particular problems where measuring instruments have not been universally standardised, as in much of the work in the social sciences. Hence, fixed variable regression is not a particularly appropriate modelling technique for much of the data available in the social sciences.

(4) Recursive Path Model

The recursive path model presented in Figure 4.5 reflects the dual development of complex structural models. ('Recursive' means that all the causal relationships between variables are unidirectional. Thus if X1 causes X2, X2 cannot cause X1, ie there are no feedback loops in a recursive model.) It can be viewed as a marrying of the psychometricians factor analysis model (the measurement model) and the econometricians regression model (the structural model). Furthermore, it is possible to allow for the added complexity of correlations between the errors of measurement

(3) Regression Model

Figure 3.1 introduced the way in which the relationships in the structural model could be viewed as being of the same form as a regression model. However, due to the fact that in the LISREL framework there is no direct link between the exogenous and endogenous observed variables, a regression model has to be reparameterised in terms of the structural model to perform a regression analysis between the observed variables. The appropriate parameterisation is that given in Figure 4.4. In this model the observed variables are each specified to be identical to a companion unobserved latent construct. Thus in the LISREL formulation the paths between each observed variable and its companion latent construct are specified to be unity, ie they are specified to be perfectly reliable measures of the underlying theoretical variable. This makes explicit one of the basic assumptions of ordinary multiple regression analysis: that the variables in the equation are identical with the theoretical variables of the theory being modelled.

If the residual term (ζ) in Figure 3.1 is specified to be zero then the endogenous variable is specified to be an exact linear function of the exogenous variables. Regression models usually allow the residual term to be non-zero, in which case it is an omnibus term representing the sum total of effects due to the exogenous variables not formally included in the model. Equivalently it can be interpreted as the variation in the endogenous variable which is not accounted for by the exogenous variables in the equation.

The assumption of perfect reliability for the exogenous variables is equivalent to a specific formulation of these

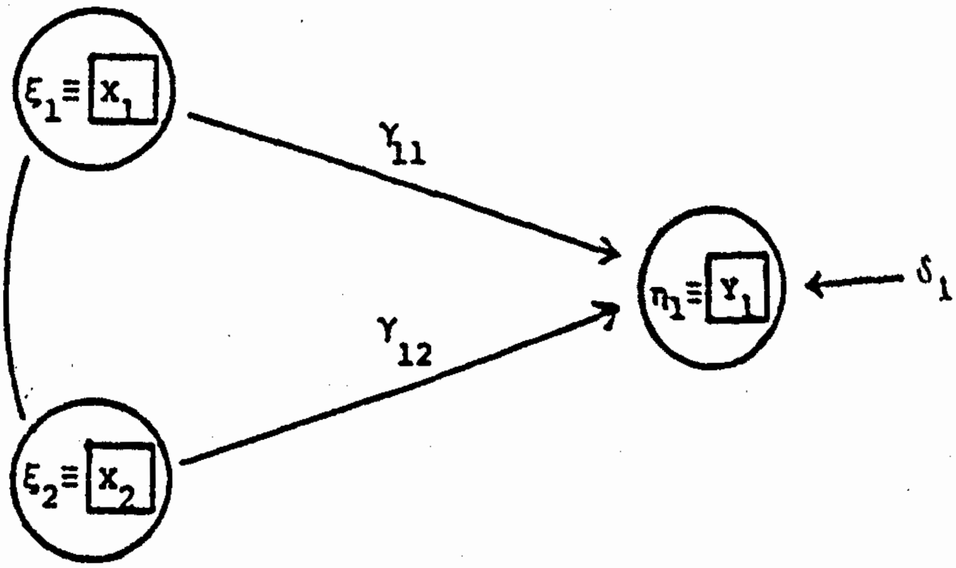

Figure 4.4 LISREL Specification of Ordinary Multiple Regression Model

Structural Model

$$\eta_1 = \gamma_{11}\xi_1 + \gamma_{12}\xi_2 + \delta_1$$

Measurement Model

$$x_1 = \xi_1$$

$$x_2 = \xi_2$$

$$Y_1 = \eta_1$$

The symbol $\equiv$ implies an identity relationship between the measured and unmeasured variables.

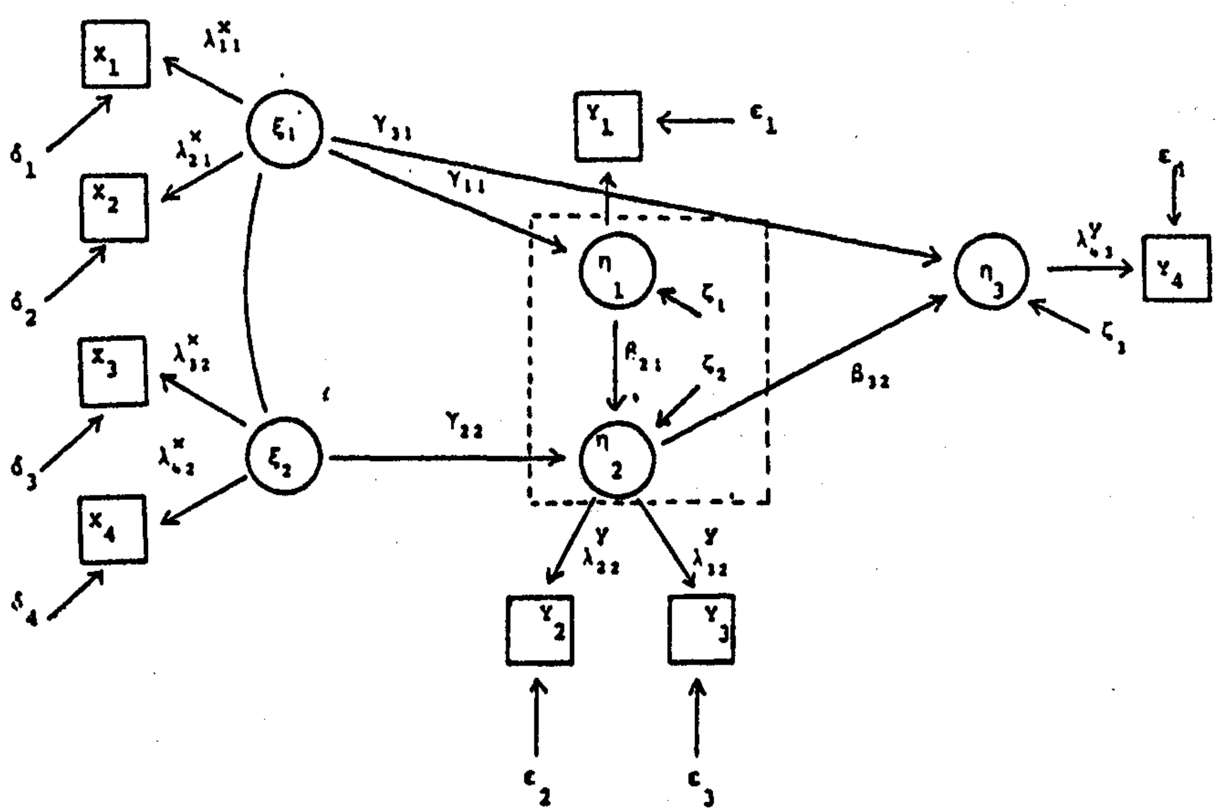

Figure 4.5 A Recursive Path Model

Measurement Models

$$x_1 = \lambda^x_{11}\,\xi_1 + \delta_1$$

$$x_2 = \lambda^x_{21}\,\xi_1 + \delta_1$$

$$x_3 = \lambda^x_{32}\,\xi_2 + \delta_3$$

$$x_4 = \lambda^x_{42}\,\xi_2 + \delta_4$$

$$Y_1 = \lambda^y_{11}\,\eta_1 + \epsilon_1$$

$$Y_2 = \lambda^y_{22}\,\eta_2 + \epsilon_1$$

$$Y_3 = \lambda^y_{32}\,\eta_2 + \epsilon_3$$

$$Y_4 = \lambda^y_{43}\,\eta_3 + \epsilon_4$$

on the observed variables across latent constructs. However, as in the Classical factor analysis model, the errors for different observed variables within a given construct may be assumed to be uncorrelated. Such an assumption is often made in order to identify the parameters of the model.

As indicated in the previous section, the random variable regression model can be parameterised in the general form of a recursive path model. However, path models have a particular advantage over regression models when the objective is to explain phenomena. Regression models do not incorporate the structure of relationships of relationships between the theoretical variables of a theory. A theory will usually imply a structure of relationships in which some of the theoretical variables partially determine others in a direct way and others indirectly. This means that a (mathematical) model of a theory which incorporates the form of this structure is a more appropriate representation of the theory. An exact correspondence in the form of the structure implied by both the mathematical theory and the model is said to describe an isomorphic relationship between the structures.

Ordinary regression models do not usually provide estimtes of these fundamental structural parameters of a theory, because they do not model these particular relationships. Instead they estimate parameters measuring the effect of each observed exogenous variable on each observed endogenous variable, one at a time, given statistical control for the other exogenous variables in the model. Thus regression parameters are an omnibus measure of the direct and indirect effects between the endogenous variable and the other variables in a theory, ie regression models are based on a 'black box' approach to the structure of relationships which exist within the exogenous variable set: all exogenous variables are given the same 'causal' or 'relational' status with respect to the endogenous variable. Ordinary regression models may, however, be viewed as a first approximation to the representation of a theory if they include among their exogenous variables only those which are fully exogenous with respect to the theory, ie those which are not determined in any way within the theory. The ordinary regression model is then said to represent the reduced form of the structural model. The coefficients of the reduced form model indicate the total effects (the sum of the direct and indirect effects) of the exogenous variables on the endogenous variable in the structural model.

(5) Reciprocal Effects Path Model

The essential difference between a recursive and a reciprocal (nonrecursive) effects model is that the former specifies that causation between a pair of latent constructs is unidirectional whereas the latter implies that the two variables influence each other through a feedback mechanism. The recursive path model of Figure 4.5 can be made into a reciprocal effects model by the respecification of the section enclosed in broken lines. The relevant respecification is indicated by Figure 4.6a, otherwise the model has the same representation as Figure 4.5.

Although it is not the usual parameterisation of a nonrecursive model, the reformulation presented in Figure 4.6b also implies a particular form of reciprocal relationship between the endogenous constructs η_1 and η_2. In this case the effect of one construct on the other is indirect, operating via the variation captured by the residual terms (ζ_1, ζ_2) and the covariance between these residuals across equations. This indirect effect of one endogenous construct on another is a form of 'spurious' effect, since it implies that there are common variables (or at least, common variance) omitted from the equations for each of the endogenous constructs. Nonrecursive models usually suffer from problems in identifying the parameters which represent the reciprocal effect. The solutions to these problems usually lie in simplifying the assumptions about the nature of the effects (eg assuming that the effects are of the same magnitude in each direction) or in the use of instrumental variables techniques.

(6) Test Score Model

The test score model developed in educational psychology can be formulated as a special case of the factor analysis model. The model is presented in Figure 4.7. The interpretation in the test score model is that the common factor (ξ_1) is treated as the underlying true score for the observed variables (X1,X2,X3) and the error terms ($\delta_1 - \delta_3$) represent their unique variation. This unique variation is a composite measure of the test specific variation for each observed variable (item), and random error in measurement. Since the variation which is common across all the observed scores is accounted for by the true score factor, the uniquenesses are usually specified to be uncorrelated, ie $r(\delta_i \; \delta_j) = 0, \forall i \neq j$, in such models. This formulation then corresponds to the Classical Test Score Model. However the restriction that the uniquenesses must consist of non-systematic variation only, ie that they are uncorrelated, will not hold in the data unless the assumption of unidimensionallity between the observed scores is also satisfied (cf first order common factor analysis model). For much of the data in social and behavioral research, the uniquenesses can be thought of as consisting primarily of the error in measurement in the observed variables, although they may contain substantial amounts of 'method effects' on measurements also (Cuttance, 1983).

If the true score factor is standardised to have a variance of unity a parallel measures model is specified by restricting all of the loadings on the true score factor to be equal ($\lambda_1 = \lambda_2 = \lambda_3$) and, the variances of the uniquenesses terms to be equal to each other also ($\delta_1 = \delta_2 = \delta_3$). A tau-equivalent measures model is then obtained by

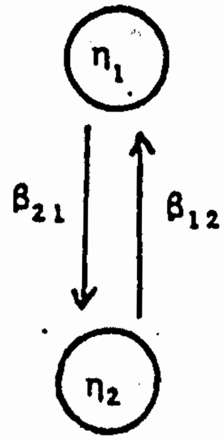

Figure 4.6a Respecification of Figure 4.5 to include direct reciprocal effects - (paths between η_1 and η_2 respecified, balance of model as in 4.5)

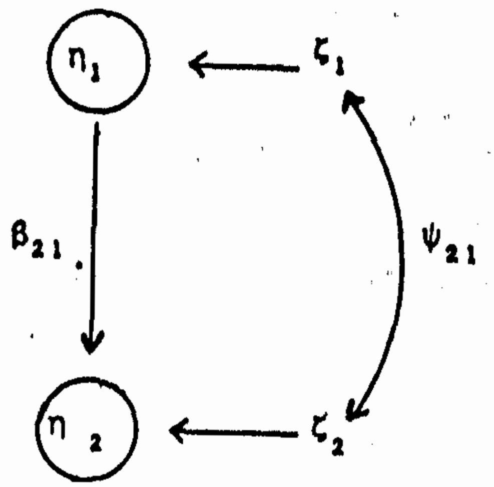

Figure 4.6b Respecification of Figure 4.5 to include indirect feedback (reciprocal) effects. ψ_{21} indicates a covariance between the construct residuals.

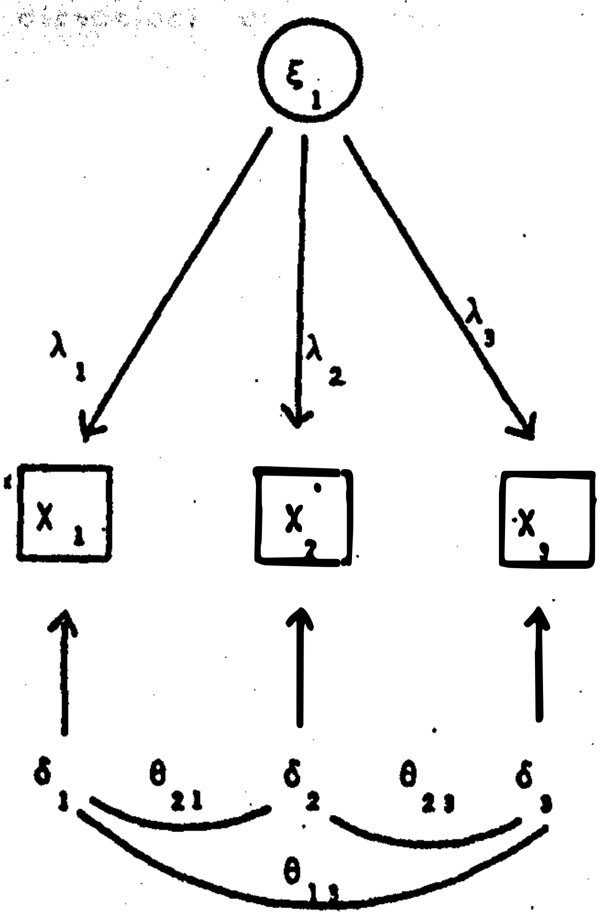

Figure 4.7 Test-score Model

$$X_1 = \lambda_{11}\xi_1 + \delta_1$$

$$X_2 = \lambda_{21}\xi_1 + \delta_2$$

$$X_3 = \lambda_{31}\xi_1 + \delta_3$$

$$\theta_{ij} = \mathrm{cov}(\delta_i, \delta_j) > 0$$

relaxing the equality restriction on the uniquene[illegible] congeneric measures model is obtained by the further rela[illegible] the equality restriction on the true score factor loadi[illegible] order to identify a congeneric measures model it is necess[illegible] have at least three observed measures of the true score factor[illegible]

It should be noted that standardisation may change the form o[illegible] the relationship between the components of variance in a measure. Parallel measures remain parallel after standardisation, since their variances are the same for different observed measures. Tau-equivalent measures, however, may become congeneric measures after standardisation. This is because the true score loadings are affected by the standardisation of each observed measure according to its own variance. Thus measures which have different variances but equal true score loadings before standardisation may have unequal true score loadings after standardisation. Congeneric measures generally remain congeneric after standardisation. An interesting special case for congeneric measures occurs if the alternative measures have equal reliabilities (ie equal ratios of true score variance to observed score variance) in their original metrics. Upon standardisation such measures become parallel measures. This information can sometimes be used to identify a model which is underidentified when the (unstandardised) covariance matrix is analysed. The model may be identified by restricting the measures to a parallel measures specification and analysing the correlation matrix. Identifying a model by such a device, however, means that the basis of the interpretation of some model parameters may change. Kim and Ferree (1981) provide a discussion of the issues of interpretation in structural models with standardisation of either variables and/or parameter coefficients.

(7) Multitrait-Multimethod Model

This model is a particular specification of a family of models known more generally as variance/covariance components models. In general, these are used for modelling quasi-experimental data from various factorial designs found in educational and psychological research.

The multitrait-multimethod model has been used in psychology to model data when more than one type of measuring instrument (method) has been employed to measure latent traits across individuals Further, Campbell and Fiske (1959) showed how the model can also be used to assess the validity of alternative measures of a latent construct and Cuttance (1982) has used it to study the method effects of different questionnaires in the context of a true score test theory model.

Figure 4.8 represents a model for the latter type of analysis. There are four true scores (ξ_1-ξ_4) each with two observed variables and there are two method effects (ξ_5, ξ_6), due to the two questionnaires employed to collect the data on the observed variables. If standardised data, ie correlations are analysed, then the true score factor loadings are the square root of the reliabilities and the squared method factor loadings indicate the proportion of the variance in the observed scores due to the method effect. There are corresponding relationships for the case where a covariance matrix, rather than a correlation matrix, is analysed. In most applications the covariance matrix is the more appropriate to analyse, since it retains information on the measurement scale of the observed variables. A correlation matrix, on the other hand, loses this information in the process of standardising the observed scores in terms of units of their estimated standard deviation. See Schwarzer (forthcoming) for a didactic example of the use of multitrait-multimethod models.

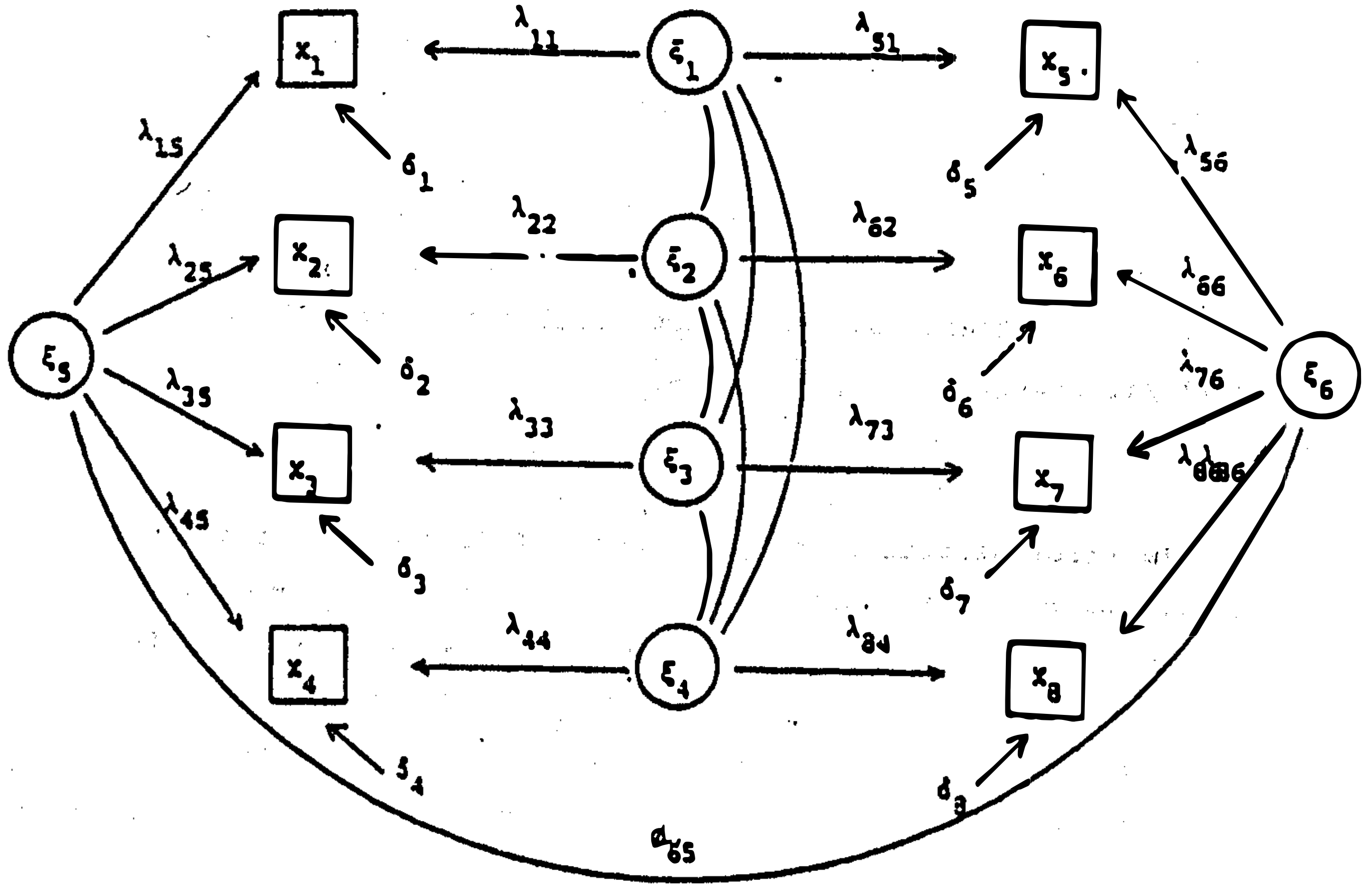

Figure 4.8 Multimethod Multitrait Model with Four Replicated Variable Pairs and Two Method Factors.

	True score factors				method factors	Random errors
$x_1 =$	$\lambda_{11}\xi_1$				$+\ \lambda_{15}\xi_5$	$+\delta_1$
$x_2 =$		$\lambda_{22}\xi_2$			$+\ \lambda_{25}\xi_5$	$+\delta_2$
$x_3 =$			$\lambda_{33}\xi_3$		$+\ \lambda_{35}\xi_5$	$+\delta_3$
$x_4 =$				$\lambda_{44}\xi_4$	$+\ \lambda_{45}\xi_5$	$+\delta_4$
$x_5 =$	$\lambda_{51}\xi_1$				$+\ \lambda_{56}\xi_6$	$+\delta_5$
$x_6 =$		$\lambda_{62}\xi_2$			$+\ \lambda_{66}\xi_6$	$+\delta_6$
$x_7 =$			$\lambda_{73}\xi_3$		$+\ \lambda_{76}\xi_6$	$+\delta_7$
$x_8 =$				$\lambda_{84}\xi_4$	$+\ \lambda_{86}\xi_6$	$+\delta_8$

(8) Error Structure Analysis Model

Often in social science research the models employed to analyse data assume that there is zero correlation between the measurement error on one observed variable and other observed variables in the model. A trivial form of this assumption is found in the ordinary multiple regression model, which assumes that all observed variables are perfectly measured, thus there are no measurement errors which could be correlated. As noted in the discussion above, this assumption of independence between error terms is also made in the Classical Test Score model and in the Classical factor analysis model.

The error structure analysis model of Figure 4.9 is designed to investigate the validity of the assumption of independence between errors. Although the general LISREL framework does not formally require this assumption of independence, it is often specified in such models. This is usually because of the need to make restrictions on a model in order to identify it, and hence provide unique estimates of the parameters in the model.

Under certain conditions we would expect the assumption of independence between errors to be implausible. Particular conditions which may invalidate the assumption apply when there is a common method effect influencing the observed values for some or all of the variables, or where the observed variables are measured on repeated occasions, or when individuals' responses on a given variable are influenced by their scores on another variable, eg middle class pupils upgrading statements of their parent's education. In fact, correlated error across observed variables is to be expected whenever measurement is contaminated by systematic influences other than the true scores for those variables. In the case where the contaminating influences are not common across variables then we have a situation characterised by a test score

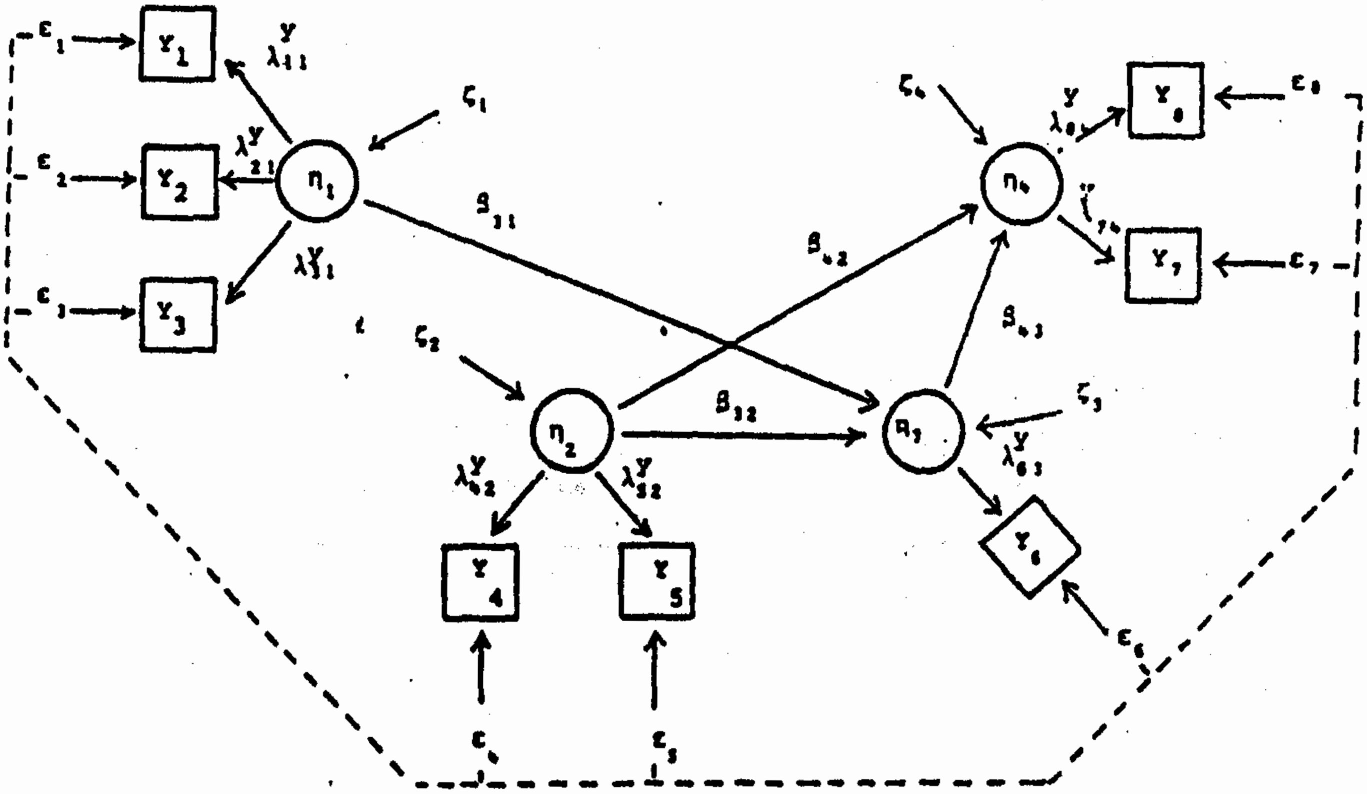

Figure 4.9 Error Structure Analysis Model

Measurement Models

general form $Y_i = \lambda^y_{ij} \eta_j + \varepsilon_i$

Structural Model

general form $\eta_i = \beta_{ji} \eta_j + \zeta_i$

Any set of error correlations, $r(\varepsilon_i, \varepsilon_j)$, can be specified and tested in this framework model with uncorrelated uniquenesses. Alternatively, if the errors or uniquenesses contain systematic influences (eg corresponding to test specific factors or method effects) across observed variables then they can be modelled as additional common factors underlying the observed variables (cf common factor analysis model). In many situations the theories being modelled do not contain the additional information required to specifically represent these additional factors, or the researcher is not particularly interested in them. In such cases the 'common' component of the error terms can be left unanalysed and unbiased estimates of the model obtained by allowing the errors to be freely correlated. However, in order to obtain unique estimates the model has to be identified and this is not possible if all error terms are allowed to be freely correlated within constructs. Identification with all errors correlated can be obtained, however, if we are prepared to place other types of restrictions on the measurement models, eg equality restrictions across some of the error variances. This specific assumption would imply that the different observed variables were all subject to contaminating influences of a similar pattern and magnitude. Even if such an assumption is considered plausible with respect to the random component of the uniquenesses, it will be much less plausible for any systematic components of their variation. Hence, it may not hold generally for such errors of measurement. Of course, how appropriate the specification of equal variances is for given data also depends on other conditions, such as the relative magnitudes of the systematic and random components of these error terms. A particular model in which these error terms are specified to be of equal variance, is the parallel measures test score model. However, we note that in the classical specification of this model the individual observed variables are assumed to be unidimensional, thus the error terms are assumed to contain only random measurement error, with no common or systematic component across error terms.

Alternatively by specifying the correlations between selected error terms to be a known (or assumed) fixed value (zero) we reduce the number of parameters to be estimated. Hence, the measurement models can be over-identified by the judicious specification of independence (zero correlation) between selected error terms in the model. The present type of model may be used to investigate the hypothesis of independence between specified error terms in a random subsample of the data. In a confirmatory study we would also turn to the knowledge available from the theory and other sources in order to decide which error terms are most likely to be correlated but, in an exploratory study it is necessary to empirically test hypotheses about the error structure among particular variables. These tests will, of course, be conditional upon the specified assumptions about the structure of errors among other variables in the model.

The general LISREL framework allows for correlated errors between observed variables within each of the endogenous and exogenous variable sets, but not across these two sets of variables. Thus, to allow for all posible specifications of such correlations the model in Figure 4.9 is parameterised with all observed variables and latent constructs as endogenous variables and constructs, respectively, in a recursive framework. All regression effects in the model are thus between Eta (η) constructs, and the error terms (ε_i) can then be specified with any correlational structure required. This parameterisation is therefore in terms of the right hand side only of the model in Figure 4.1, since parameterisation as a recursive model in the full framework would not allow for a specification of correlated errors between the two sets of error terms, (δ) and (ε), at either end of that model.

(9) Analysis of Covariance (ANCOVA) Model

The LISREL model can be used to estimate ANOVA and ANCOVA models. The prime feature of these two models which contrasts them with all of the models so far discussed in this paper is their focus on the parameters measuring means and their comparison across (treatment) groups. The other models discussed so far all focus on the covariance structure of variables which exists within a single group of subjects, although it is a straightforward move to compare structural models across groups also. Thus the essential difference is that the former models focus on covariance structures while ANOVA and ANCOVA both primarly focus on mean structures.

The ANOVA model was developed for the analysis of data from controlled experiments in which subjects could be randomly allocated to treatments. Randomised assignment allows the researcher to assume that variables which are not explicitly controlled within the experiment do not systematically influence the dependent variable across treatments (groups). However, much of social and behavioral science data describes situations in which the ideal conditions of experimental research cannot be satisfied. Research based on such data is sometimes called 'quasi-experimental', which alludes to the fact that it is possible to employ procedures in the analysis of the data to control statistically for the influence of extraneous factors which may have influenced the scores on the dependent variable. In order to exercise statistical control in this way it is of course necessary to have collected data on these extraneous influences. This is one of the reasons why quasi-experimental research is sometimes said to be weaker than experimental research: in quasi-experiments the researcher needs to know before hand which non-experimental factors to collect data on, whereas in a controlled experiment there is no requirement that the relevant extraneous influences be known at all, since the randomised allocation of subjects to treatments nullifies any effect they could have had.

The analysis of covariance (ANCOVA) model was designed to do the same job for quasi-experimental data as that which the analysis of variance (ANOVA) model did for data from controlled experiments; to test for a difference in the effects of two treatments. This involves a test of the difference in the mean scores for the treatment and control groups after treatment of the former group, given control for covariates which may also have influenced the outcome.

Assuming that we have data on the relevant extraneous factors (covariates), so that statistical control is possible for the influence of these, the usual multivariate ANCOVA analysis model makes two important assumptions about the data on the two groups. First, it assumes that the variances and covariances are equal across the groups. Second, it assumes that the relationship between the covariates and the outcome (dependent variable) are the same across groups. Now, because the condition of randomised assignment to treatment groups is not satisfied it is likely that the first of these assumptions will often be violated. For instance, children from one school are likely to differ from those in another school with respect to their entry abilities, social class, motivation etc. Only if statistical matching on the relevant variables is carried out can we be reasonably assured that the first assumption will be approximately satisfied. The second assumption is often the one which is least tenable and it is the one which is most crucial for accurate estimates. It implies that the control group and treatment group have the same regression lines with respect to the effect of the covariates on the dependent variable. This assumption may be more plausible in the situation where the treatment effect is null but where the treatment is 'effective' it seems plausible that the relationships will often differ between the control and treatment groups. Indeed the treatment may have the effect of changing just this relationship, eg compensatory education programmes may aim to raise the performance of disadvantaged children relative to non-disadvantaged children through special programmes (eg Head Start) and this treatment may also result in a changed relationship between social class and educational attainment at the end of the programme. Social class will, however, be one of the covariates which a researcher may wish to control for in any evaluation of the effect of the programme (treatment) (Sorbom, 1982).

The above two assumptions are made so as to achieve identification in he usual ANCOVA model, however, other identifying restrictions may be more appropriate to the research problem at hand. Thus the flexability of a facility allowing for the specification of an alternative set of identifying restrictions would be an advantage in such analyses.

Although not originlly designed with such analyses in mind the LISREL model can be parameterised to estimate the ANCOVA model (Sorbom, 1982; Joreskog and Sorbom, 1981; Horn and McArdle, 1980).

Figure 4.10 presents an augmented path diagramatic representation of a ANCOVA model. The analysis of differences between groups is carried out by using the LISREL facility to conduct a simultaneous multiple group analysis. This facility allows for the fitting and estimation of the structures in several groups simultaneously through the use of a joint estimation function. The differences in the mean structures between groups is tested by comparing the difference in fit between a model in which the mean parameters are specified to be equal across groups and one in which they are allowed to be estimated independently of the values of each other (ie Free, in LISREL notation). This model employs the chi square ratio test which is known to be sensitive to departures from the model assumption of multivariate normality.

An alternative formulation of the ANCOVA model is presented in Figure 4.11. In this model the mean structures of the group are not specified, rather the treatment effect is assessed by the influence of the variable 'T', which represents group membership, on the dependent construct. In this model the data is analysed as if it was all from the same group but, that members of the group differ on the characteristic which denotes exposure to the treatment. In the case where the data are drawn from N groups this

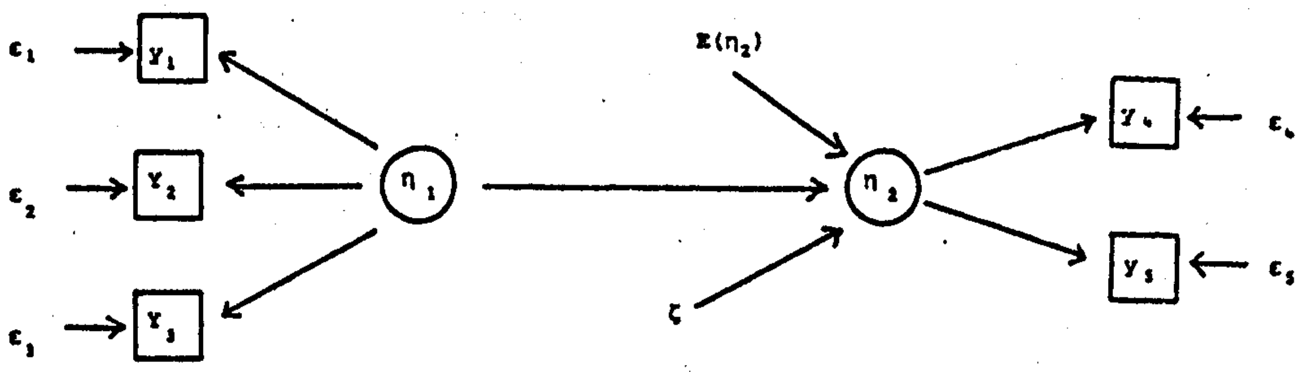

Figure 4.10 ANCOVA Model with Explicit Structure on Mean of Dependent Variable

η_1 latent covariate

Y_1-Y_3 observed covariates

η_2 latent dependent construct

Y_4, Y_5 observed indicators for dependent construct

$E(\eta_2)$ mean of latent dependent construct

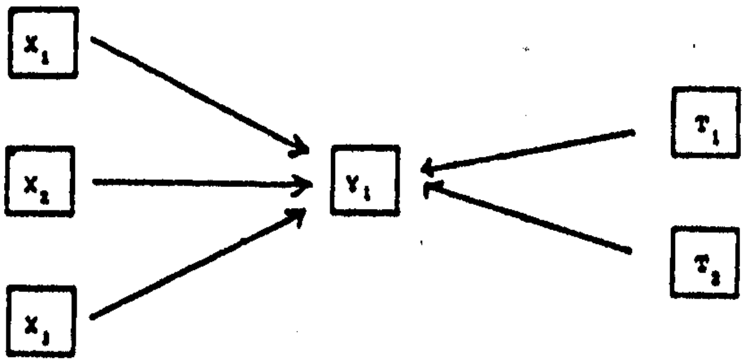

Figure 4.11 Model to Test for Effect of Group Membership on Dependent Variable (measurement error omitted)

X_1 - X_3 observed covariates

Y_1 dependent variable

T_1, T_2 dummy variable indicating group membership formulation requires that group membership be denoted by N-1 dummy variables. Different strategies of coding the dummy variables can be employed to investigate various contrasts between groups (Kenny, 1979; Bray and Maxwell, 1982). The advantage of this model over the former is that it removes the need to rely on the chi square ratio statistic in assessing the statistical significance of the treatment effect, which is assessed instead in this model by the standard error for the parameter indicating the effect of group membership.

If the principal focus of an investigation is to estimate the magnitude of the means for different groups, in addition to the testing of differences between them, then the explicit modelling of the structures of means is necessary, as in the former model. Indeed, it is probably always necessary to model the data by the explicit structures on means if it is the means of groups which are the focus of the research. Only through such models is it possible to assess the substantive importance of any difference between groups, in addition to testing for the statistical significance of any difference between means. The model in Figure 4.11 allows for the statistical testing of the difference between being in one group as opposed to another. It does not furnish any direct information on the means.

In addition to the flexability in specifying the most appropriate identifying restrictions for the particular model at hand, these formulations of the ANCOVA model allow for the investigation of differences in mean structures within the specific context of the covariance structural relationships in the groups and the assessment of model fit. A model which does not fit satisfactorally may indicate that important covariates have been omitted, hence that the difference between groups may be highly sensitive to the particular specification of the model employed in the analysis of the data. Analyses with the standard ANCOVA model leave the researcher substantially uninformed as to these dangers to inferences and conclusions drawn from the analysis. In addition the standard ANCOVA model is not always appropriate when subjects are assigned to treatments on the basis of particular non-random strategies, where there may be error of measurement in the variables or, when the number of subjects varies significantly across groups. Each of these problems can be separately and jointly addressed in the formulation of models in a a more general structural modelling context, although the restrictions required to identify a particular model will always mean choosing between a fixed set of alternative constraints on the model.

5.00 Discussion and Other Issues

The approach of this paper has been to give the reader a non-technical conceptual overview of the scope of the LISREL model and to discuss a selection of models commonly employed in social science within this framework. These models have been discussed in a heuristic way so that researchers interested in extending their modelling techniques to include the more general approach of LISREL can see where it fits into the general application of linear models.

Anyone who refers to the more detailed literature will soon realise that there are several issues not so far mentioned. Problems may be found in the failure of the iterative estimation process to converge to stable estimates. The estimates for some parameters may occasionally be implausible even when the model has an acceptable overall fit. A particular example of this is Heywood solutions, ie negative estimated variances for the errors of measured variables (ε_1 δ); or for construct residuals (ζ).

Methodological problems in deciding on criteria for comparing the measures of fit for different models based on the same data are still to be resolved. The issues rest on the criteria for comparing models which are not nested, that is, more or less restricted versions of one another, and upon the susceptability of some of the measures of fit to sample size. Version V of the program does, however, include new measures of model fit. We will have to wait to see how well these measures distinguish between the fit of models which are not nested. The sampling distribution for these new indicies of model fit are unknown, thus they do not provide an opportunity for the statistical testing of the differences between models.

In practice, ad hoc solutions to these problems are adopted to suit the particular investigatory situation. In some cases more general 'rules of thumb' are being developed to cover a wider range of applications. Many of these issues are as unresolved and important for other forms of the linear model and for some other methods of estimating the sub-models discussed in this article, also.

This brings us to the one area in which the LISREL method of estimation makes more restrictive assumptions about the variables and data than some other methods. Maximum liklihood estimation produces estimates with desirable properties provided the data conform to a multivariate normal distribution. This is generally considered to be a highly restrictive assumption to make in respect of data in social science situations. Methods of assessing the properties of data in this respect are available but are not widely known or applied (Gnanadesikan, 1977). Models based on estimation procedures employing Ordinary Least Squares or Generalised Least Squares techniques make less restrictive distributional assumptions about the data. Version V of the LISREL program has an option for selecting an estimation procedure based on the method of Unweighted Least Squares (ULS). However the technical and computational problems of computing sampling information about the precision of the estimates (standard errors) have not yet been solved. Thus the ULS estimates are accompanied by less information on the fit of the model and on the precision of parameter estimates than the ML estimates. In some cases it will be useful to compute model estimates by both methods. Where they are simiar we can be more confident that they are not artefacts of the method, but for data of a highly skewed nature it is best to opt for the ULS estimates as the ML method is known to be sensitive to the effects of skewness. If, however, the researcher is prepared to assume that the observations are drawn from an underlying true distribution which is not significantly skewed, ie one which is relatively normally distributed, then it is posible to utilise an option in the program to treat the measurement scale of the observed variables as ordinal in nature and to rescale the values so that the variable is normally distributed. In doing this the weights given by the categories in the measuring scale for the observed measures are disregarded and new weights calculated from the frequency of the observations in each category. The correlation between two such ordinal variables is then calculated from a contingency table and is known as a polycholoric correlation. These correlations are used in the same way as the ordinary (Pearson product-moment) correlation coefficient as intermediate input for further analysis by ULS or ML methods in the program.

In this section I have attempted to indicate some of the areas in which the methodology of LISREL modelling raises unresolved questions about such methods in general, however, I also wish to stress that the new awareness of such issues is itself a major step forward. A set of papers which aim to present didactic discussions of various features of structural modelling and the robustness of the estimators used in estimating models is in preparation (Cuttance and Ecob, forthcoming).

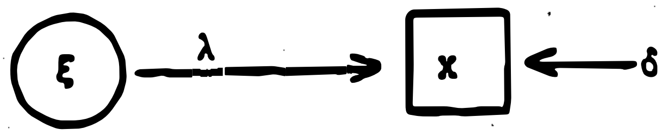

Figure 2.1 LISREL Measurement Error Model

$$x = \lambda\xi + \delta$$

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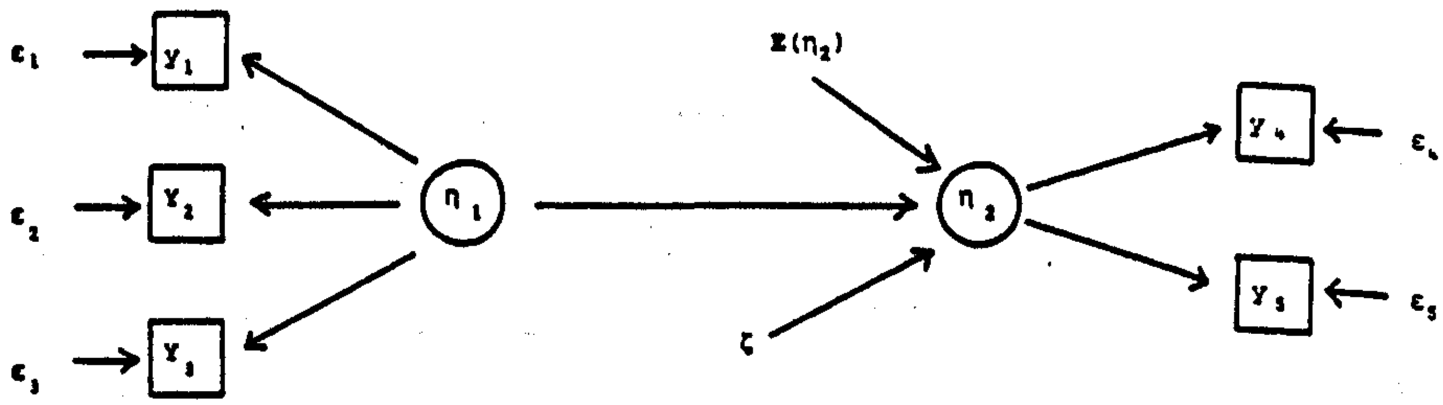

Figure 4.10 ANCOVA Model with Explicit Structure on Mean of Dependent Variable

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Y_1-Y_3 observed covariates

η_2 latent dependent construct

Y_4, Y_5 observed indicators for dependent construct

$E(\eta_2)$ mean of latent dependent construct

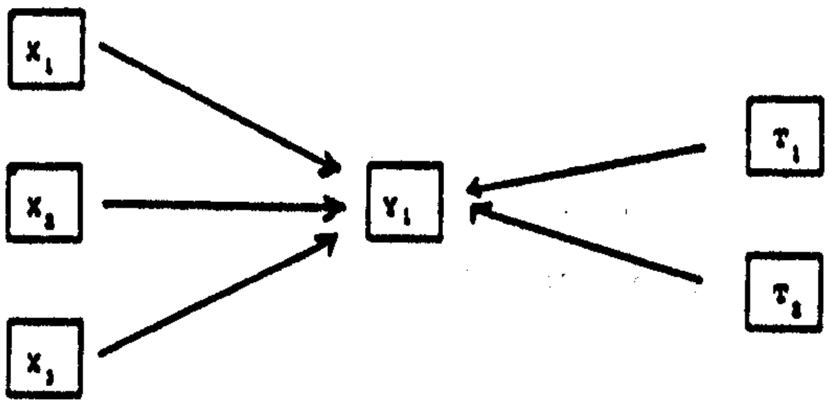

Figure 4.11 Model to Test for Effect of Group Membership on Dependent Variable (measurement error omitted)

X_1 - X_3 observed covariates

Y_1 dependent variable

T_1, T_2 dummy variable indicating group membership

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