The Effect of the Violation of the Assumption of Independence When Combining Correlation Coefficients in a Meta-Analysis

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Meta-analysis is a technique for combining the summary statistics from viously conducted research studies. Pioneered by Gene V Glass (1976) a-analysis gives not only an indication of the direction of the results of : studies, but provides an index of the magnitude of the effect as well. ta-analyses are reported in terms of mean effect size, $\overline{ES}$. There are two es of effect sizes. An experimental effect size is the mean of the experintal group minus the mean of the control group divided by the standard viation,

$$ES = \frac{\bar{X}_E - \bar{X}_C}{s_X},$$

ile a correlational effect size is simply a correlation coefficient,

$$ES = r.$$

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Meta-analysis has been further refined by Hedges (1983), who has been developing techniques for using effect sizes as data points and then fitting regression models. The focus of this paper, however, will be the use of correlation coefficients in meta-analyses and the effect of the violation of the assumption of independence in these analyses.

Independence

A necessary assumption for the results of statistical analyses to be tenabl is independence. All inferential statistical techniques require independence of observations. By independence is meant that the probability of including one subject or data point will in no way affect the probability of including any oth subject or data point. Another way of defining independence is to say that the value of a variable for a subject is not predictable from the value of a variabl for any other subject.

So far independence has been defined in reference to primary studies perfor by researchers who draw a random sample of subjects, measure the subjects on variables of interest, and calculate statistics from the measured data using their hypothesized models. The meta-analysts, on the other hand, draw a sample of studies usually from journal articles, record the numerous statistics reported in each study, and calculate a statistic based on effect sizes or a meta-statistic from a data set of simple statistics. When jumping from the level of individual studies to combinatory techniques, studies parallel subjects and simple statistics parallel observations on variables. In the framework of combinatory methodology, then, independence means that the value of any statist which is included should in no way be predictable from the value of any other included statistic.

The typical study which is chosen for inclusion in a meta-analysis, however will yield more than one effect size or simple statistic. When the meta-analyst uses all the statistics available in a particular study to calculate the mean size, the assumption of independence is violated. Landman and Dawes (1982) ꞉ five ways in which the assumption of independence can be violated in meta- ꞉s. These five types of violations are as follows:

"1) Multiple measures from the same subjects, . . .
2) Measures taken at multiple points in time from the same subjects, . . .
3) Nonindependence of scores within a single outcome measure, . . .
4) Nonindependence of studies within a single article, . . . and
5) Nonindependent samples across articles" (pp. 506-507).

Kraemer (1983) specifically provides the caveat that "only one effect size tudy can be used to ensure independence" (p. 99) in meta-analyses. This that the ratio of effect sizes to studies in a meta-analysis should be n order to avoid violating this assumption. However, even a cursory review ıblished meta-analyses reveals that the assumption of independence is, in , seldom met.

꞉ose

The purpose of this study was to determine the effect of the violation the assumption of independence on the distribution of r and the distribution Fisher's Z. In this Monte Carlo simulation the following four parameters were ꞉d with the values specified:

N - the sample size within a study (20, 50, 100),

p - the number of predictors (1, 2, 3, 5),

rho(i) - the population intercorrelation among predictors (0, .3, .7),

rho(p) - the population correlation between predictors and criterion (0, .3, .7).

Predictor and criterion variables were generated to conform to all possible combinations of the parameters specified above and then correlated. The main parameter of interest was rho(i), since it was the index of nonindependence when it assumed a nonzero value in the multiple predictor cases. When only one predictor was used or when the intercorrelation among predictors, rho(i), equaled zero, then the assumption of independence was not violated.

Method

In this study dependent and independent correlations were generated between criterion and predictor variables. The values of the parameter p, the number of predictors, were one, two, three, and five, and path diagrams for each case appear in Figures 1 through 4 respectively. In these diagrams the G variables are the common generating variables used along with error to form the X variables or predictors, which are in turn combined along with error to produce the Y or criterion variables. The arrows between variables indicate the relationship among the endogenous variables. The associated lower case letters are the standardized regression coefficients for path analysis. The arrows which are not

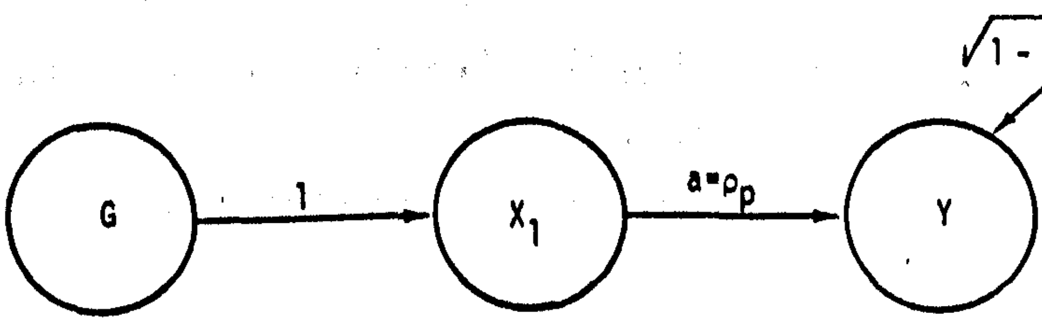

Figure 1. Path diagram for the one predictor case.

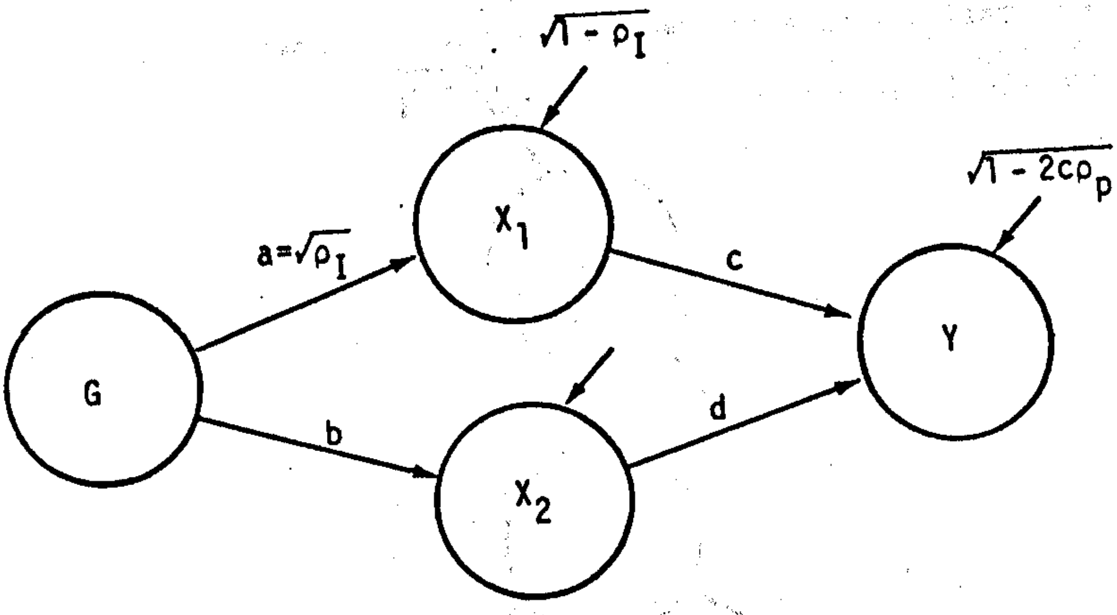

Figure 2. Path diagram for the two predictor case.

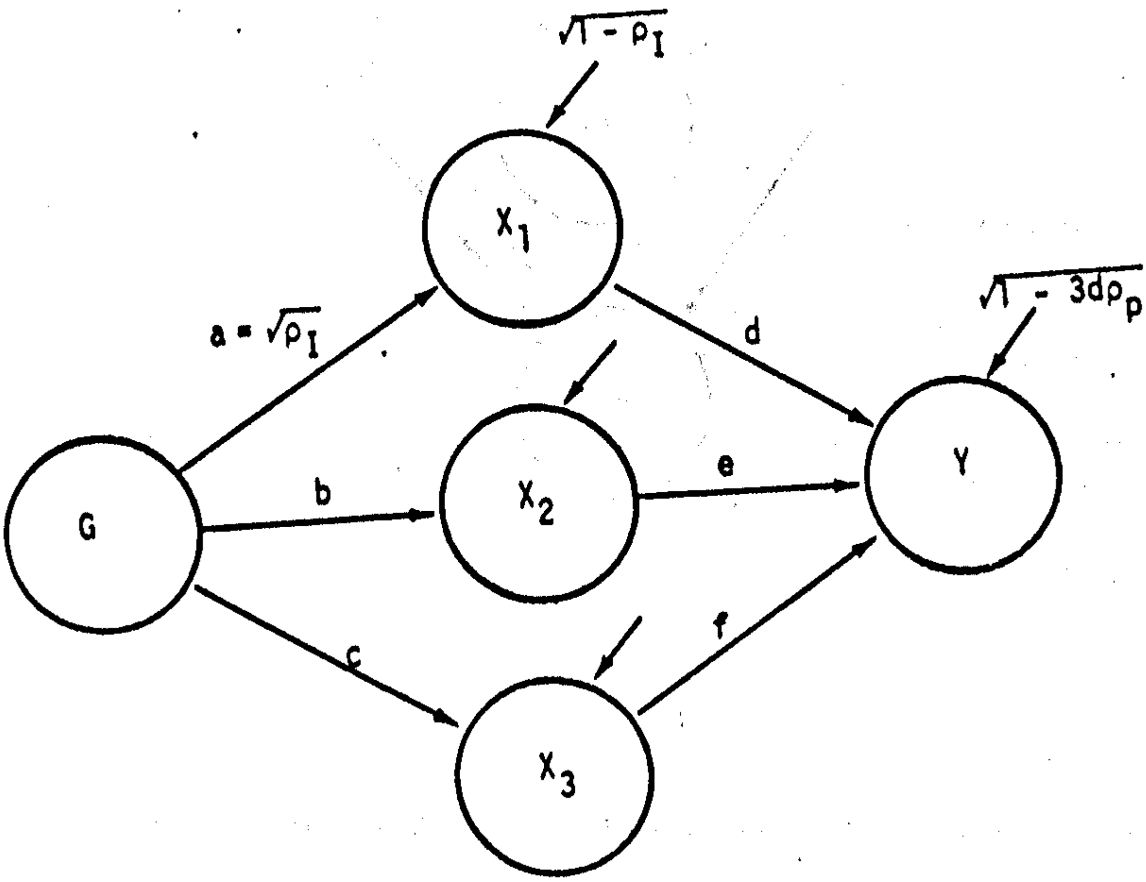

Figure 3. Path diagram for the three predictor case.

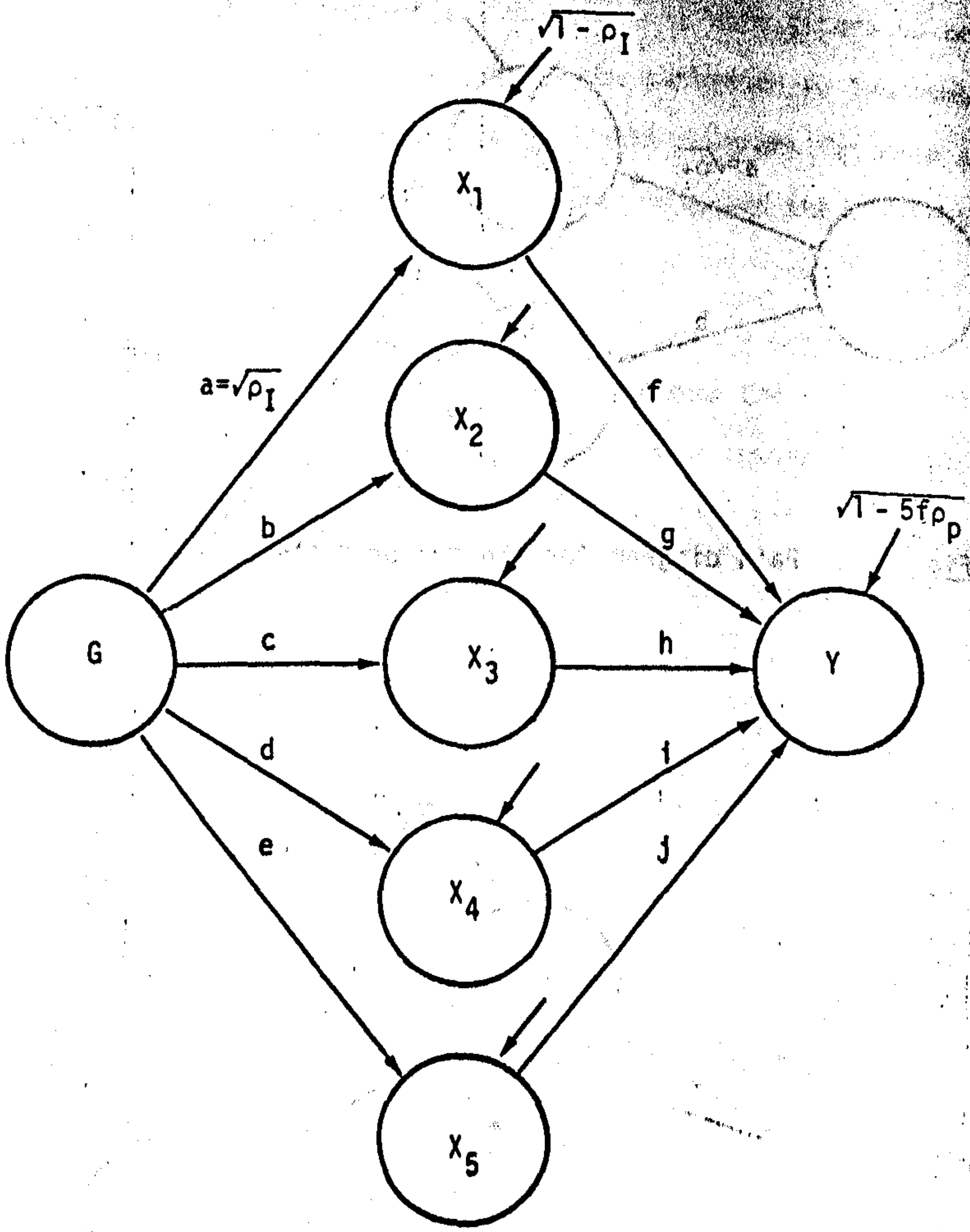

Figure 4. Path diagram for the five predictor case.

nected indicate exogenous variation, and those coefficients are given as well.

The following algorith derived by Knapp and Swoyer (1967) was used to erate correlated vectors of numbers:

$$Y = aX + \sqrt{1 - a^2}Z$$

ere X = a vector of randomly chosen numbers from the standard normal distribution,

Z = another vector of randomly chosen numbers from the standard normal distribution, and

a = the desired correlation between X and Y.

In the unique one predictor case, the intercorrelation among predictors ould not be varied since only one predictor was present. Therefore, independence xists in this case. Here the X1 vector was set equal to G, a vector of randomly hosen standard normal deviates, so the path coefficient between G and X1 is one. he path coefficient between X1 and Y, a, was set equal to the population correlation etween predictors and criterion, rho(p). Since a = rho(p), the error coefficient for Y was $\sqrt{1 - a^2}$ or $\sqrt{1 - \text{rho}(p)^2}$. The Y vector was then created as follows:

$$Y = aX1 + \sqrt{1 - a^2}Z$$

where Z = a vector of randomly chosen numbers from the standard normal distribution. The vectors for X1 and Y were then correlated.

A different procedure was used for data generation in the multiple predictor cases. In Figure 2, path coefficients a = b and c = d. In Figure 3, a = b = c and d = e = f. In Figure 4, a = b = c = d = e and f = g = h = i = j. In these three diagrams the correlations between any two predictors is equal to the product of the path coefficients connecting those two predictors with the generating variable or the quantity, a^2, since all the coefficients between generating variables and predictors are equal. For the correlation between two predictors to equal rho(i), the path coefficient, a, was set equal to $\sqrt{\text{rho}(i)}$. Then all the X vectors were generated as follows:

$$X(i) = \sqrt{a}G + \sqrt{1 - a}Z(i)$$

Where X(i) = a vector of values for a predictor and i assumes incremental values for vectors from one to p, the number of predictors,

a = rho(i) = the population intercorrelation among predictors,

Z(i) = a vector of randomly chosen standard normal deviates and i assumes incremental values for vectors from one to p, the number of predictor

The following points concern the generation of the Y vectors. First it should be noted that each Y is a linear combination of the p predictors plus error. The weight of that combination is c in Figure 2, d in Figure 3, and f in Figure 4. Second, it should be noted that correlation coefficients can be reconstructed from the standardized regression coefficients in a path diagram. In Figure 2, the correlations between the two predictors and the criterion can be reconstructed as follows:

$$r_{yx_1} = c + abd,$$

$$r_{yx_2} = d + bac,$$

but since c = d, and a = b = $\sqrt{rho(i)}$, the correlation between Y and any predictor X(i), can be written as follows:

$$r_{yx_i} = c + \rho(i)c = c(1 + \rho(i)).$$

Also since r_{yx_i} is an estimate of rho(p), that value can be substituted into the equation so that it can be solved for c as follows:

$$\rho(p) = c(1 + \rho(i))$$

$$c = \frac{\rho(p)}{1 + \rho(i)}.$$

In Figure 3 in parallel fashion, the correlations between the three predic and the criterion can be reconstructed as follows:

$$r_{yx_1} = d + abe + acf,$$

$$r_{yx_2} = e + bcf + bad,$$

$$r_{yx_3} = f + cbe + cad,$$

since $a = b = c = \sqrt{rho(i)}$, and $d = e = f$, the correlation between Y and any predictor, X(i), can be written as follows:

$$r_{yx_i} = d + \rho(i)d + \rho(i)d = d(1 + 2\rho(i)).$$

so since r_{yx_i} is an estimate of rho(p), that value can be substituted into the equation so that it can be solved for d as follows:

$$\rho(p) = d(1 + 2\rho(i)),$$

$$d = \frac{\rho(p)}{1 + 2\rho(i)}.$$

In Figure 4 the last obvious parallel exists. The correlations between the five predictors and the criterion can be reconstructed as follows:

$$r_{yx_1} = f + abg + ach + adi + aej,$$

$$r_{yx_2} = g + baf + bch + bdi + bej,$$

$$r_{yx_3} = h + caf + cbg + cdi + cej,$$

$$r_{yx_4} = i + daf + dbg + dch + dej,$$

$$r_{yx_5} = j + eaf + ebg + ech + edi,$$

but since $a = b = c = d = e = \sqrt{rho(i)}$, and $f = g = h = i = j$, the correlation between Y and any predictor, X(i), can be written as follows:

$$r_{yx_i} = f + \rho(i)f + \rho(i)f + \rho(i)f + \rho(i)f = f(1 + 4\rho(i)).$$

Again r_{yx_i} estimates rho(p) so with the appropriate substitutions the solution for f is as follows:

$$\rho(p) = f(1 + 4\rho(i)),$$

$$f = \frac{\rho(p)}{1 + 4\rho(i)}.$$

So far in generating the Y variables in the two, three, and five predictor cases, the weights of the combinations, c, d, and f, respectively, have solutio But in each case a weight for the error term is needed. In the Knapp and Swoye algorith, the value a^2 can be viewed as r^2, the amount of variance accounted for so $1 - a^2$ is the amount of variance not accounted for and $\sqrt{1 - a^2}$ is the weight the error vector, Z.

In the three multiple predictor cases studied here, formulas for the R^2 val are given below:

$$R^2_{y \cdot 12} = c\rho_{yx_1} + c\rho_{yx_2} = 2c\rho(p),$$

$$R^2_{y \cdot 123} = d\rho_{yx_1} + d\rho_{yx_2} + d\rho_{yx_3} = 3d\rho(p),$$

$$R^2_{y \cdot 12345} = f\rho_{yx_1} + f\rho_{yx_2} + f\rho_{yx_3} + f\rho_{yx_4} + f\rho_{yx_5} = 5f\rho(p).$$

The Y variables were generated as follows:

$$Y = c(X1 + X2) + \sqrt{1 - 2c\rho(p)}Z,$$

$$Y = d(X1 + X2 + X3) + \sqrt{1 - 3d\rho(p)}Z,$$

$$Y = f(X1 + X2 + X3 + X4 + X5) + \sqrt{1 - 5f\rho(p)}Z.$$

Correlations between the criterion variables and each of the predictors were then calculated in the multiple predictor cases

The number of replications was chosen by solving for n_r in the formula for the standard error of the mean of the correlation coefficient given below:

$$\sigma_{\bar{r}} = \frac{\sqrt{\frac{(1 - \rho^2)^2}{n_s}}}{\sqrt{n_r}}.$$

lue for $\sigma_{\bar{r}}$ was arbitrarily set at .01, which was deemed sufficiently for precision in this study. In this formula, ρ is the population lation, rho(p), and was set equal to zero. The symbol, n_s, is the sample and was set equal to 20. Substituting these values into the equation ed n_r, the number of replications, to assume the largest value that would ssible among the values for parameters, rho(p) and n_s, that were chosen for study. The solution for n_r, the number of replications, was 500.

For each combination of N, p, rho(i), and rho(p) and for all r and Z ributions, the means, medians, and standard deviations were calculated.

lts

The means, medians, and standard deviations of the correlation coefficients all values of rho(i), rho(p), and the number of predictors, p, when N=20 ar in Table 1. The same information when N = 50 and N = 100 appears in les 2 and 3 respectively.

The means, medians, and standard deviations of the Fisher's Z transformation the correlation coefficients for all values of rho(i), rho(p), and the ber of predictors, p, when n = 20 appear in Table 4. The same information n N = 50 and N = 100 appears in Tables 5 and 6 respectively.

Inspection of these tables shows that when the population correlation fficient, rho(p), equals zero both the mean of r and the median of r hover und that value and neither is consistently higher or lower than the other. wever, when rho(p) assumes a nonzero value the median of r is usually larger an mean r. This is because r is a biased statistic and its distribution is gatively skewed when rho(p) is positive. This ordering of the mean and the dian when rho(p) is not zero does not occur in the Fisher's Z distribution.

As N increases both the mean of r and the mean of Z are better estimators f the parameter rho(p). This follows from the Central Limit Theorem. Both he median of r and the median of Z tend to be better estimators of the population

Table 1

Means, Medians, and Standard Deviations for Correlation Coefficients When N = 20

		rho(i)								
		0			.3			.7		
p	rho(p)	$\bar{r}$	Md_r	SD_r	$\bar{r}$	Md_r	SD_r	$\bar{r}$	Md_r	SD_r
1[a]	0	.015	.007	.230						
	.3	.294	.322	.206						
	.7	.690	.706	.126						
2	0	.002	.011	.225	-.004	-.007	.223	.002	-.004	.234
	.3	.300	.316	.214	.296	.299	.208	.297	.311	.209
	.7	.683	.698	.129	.692	.714	.125	.695	.710	.117
3	0	.001	.003	.230	-.009	-.013	.233	.002	-.007	.228
	.3	.295	.313	.213	.289	.305	.214	.295	.316	.211
	.7	b			.686	.703	.126	.687	.703	.126
5	0	-.002	-.004	.233	.008	.007	.227	.004	.000	.221
	.3	.293	.309	.216	.307	.320	.208	.292	.303	.202
	.7	b			b			.694	.714	.120

[a]With one predictor nonzero rho(i) values are undefined.

[b]This combination would generate data which are undefined.

Table 2

Means, Medians, and Standard Deviations for Correlation Coefficients When N = 50

		rho(i)								
		0			.3			.7		
p	rho(p)	$\bar{r}$	Md_r	SD_r	$\bar{r}$	Md_r	SD_r	$\bar{r}$	Md_r	SD_r
1[a]	0	.001	-.001	.141						
	.3	.303	.305	.128						
	.7	.697	.705	.073						
2	0	.005	.000	.142	-.001	-.003	.140	.004	.005	.149
	.3	.294	.307	.132	.300	.305	.131	.304	.305	.130
	.7	.697	.705	.075	.694	.703	.076	.696	.703	.069
3	0	.002	.001	.139	.007	.003	.145	.001	-.002	.142
	.3	.294	.301	.130	.295	.300	.130	.295	.300	.136
	.7	b			.696	.703	.075	.694	.700	.076
5	0	-.002	-.001	.143	-.006	-.009	.144	-.005	-.007	.141
	.3	.299	.303	.129	.300	.305	.129	.295	.300	.128
	.7	b			b			.699	.705	.071

[a]With one predictor nonzero rho(i) values are undefined.

[b]This combination would generate data which are undefined.

Table 3

Means, Medians, and Standard Deviations for Correlation Coefficients When N = 100

		rho(i)								
		0			.3			.7		
p	rho(p)	$\bar{r}$	Md_r	SD_r	$\bar{r}$	Md_r	SD_r	$\bar{r}$	Md_r	SD_r
1[a]	0	.008	.005	.108						
	.3	.299	.303	.091						
	.7	.698	.701	.053						
2	0	.004	.003	.099	-.008	-.009	.101	.009	.012	.097
	.3	.297	.303	.091	.304	.308	.091	.303	.303	.088
	.7	.700	.704	.051	.699	.703	.053	.699	.703	.048
3	0	-.005	-.009	.098	.002	.002	.102	-.001	.000	.097
	.3	.301	.305	.092	.302	.305	.092	.300	.302	.088
	.7	b			.698	.701	.050	.695	.699	.050
5	0	-.002	-.002	.099	.003	.001	.100	-.003	-.002	.100
	.3	.295	.298	.093	.296	.302	.093	.302	.306	.094
	.7	b			b			.699	.702	.051

[a]With one predictor nonzero rho(i) values are undefined.

[b]This combination would generate data which are undefined.

Table 4

Means, Medians, and Standard Deviations for Fisher's Z Transformation of the Correlation Coefficients When N = 20

		rho(i)								
		0			.3			.7		
p	rho(p)	$\bar{Z}$	Md_Z	SD_Z	$\bar{Z}$	Md_Z	SD_Z	$\bar{Z}$	Md_Z	SD_Z
1[a]	0	.016	.007	.243						
	.3	.317	.334	.233						
	.7	.885	.879	.237						
2	0	.002	.011	.238	-.004	-.007	.235	.002	-.004	.247
	.3	.327	.327	.246	.321	.309	.240	.323	.321	.242
	.7	.873	.864	.242	.890	.895	.241	.893	.887	.230
3	0	.001	.003	.244	-.009	-.013	.246	.002	-.007	.241
	.3	.321	.324	.244	.313	.315	.244	.321	.327	.242
	.7	b			.879	.874	.242	.880	.873	.241
5	0	-.002	-.004	.246	.009	.007	.240	.004	-.001	.233
	.3	.319	.319	.248	.334	.331	.240	.316	.313	.231
	.7	b			b			.891	.895	.229

[a]With one predictor nonzero rho(i) values are undefined.

[b]This combination would generate data which are undefined.

Table 5

Means, Medians, and Standard Deviations for Fisher's Z Transformation of the Correlation Coefficients When N = 50

		rho(i)								
		0			.3			.7		
p	rho(p)	$\bar{Z}$	Md_Z	SD_Z	$\bar{Z}$	Md_Z	SD_Z	$\bar{Z}$	Md_Z	SD_Z
1[a]	0	.001	-.001	.144						
	.3	.319	.315	.144						
	.7	.876	.877	.144						
2	0	.005	.000	.145	-.001	-.003	.142	.004	.005	.152
	.3	.309	.317	.146	.316	.315	.147	.320	.315	.146
	.7	.877	.877	.145	.870	.873	.147	.873	.873	.136
3	0	.002	.001	.141	.007	.003	.148	.001	-.002	.145
	.3	.309	.310	.146	.310	.310	.145	.311	.309	.152
	.7	b			.874	.874	.145	.870	.867	.149
5	0	-.002	-.001	.146	-.006	-.009	.147	-.005	-.007	.144
	.3	.315	.313	.145	.316	.315	.145	.310	.310	.143
	.7	b			b			.878	.877	.141

[a]With one predictor nonzero rho(i) values are undefined.

[b]This combination would generate data which are undefined.

Table 6

Means, Medians, and Standard Deviations for Fisher's Z Transformation of the Correlation Coefficients When N = 100

		rho(i)								
		0			.3			.7		
p	rho(p)	$\bar{Z}$	Md_Z	SD_Z	$\bar{Z}$	Md_Z	SD_Z	$\bar{Z}$	Md_Z	SD_Z
1[a]	0	.008	.005	.110						
	.3	.311	.313	.101						
	.7	.870	.869	.102						
2	0	.004	.003	.101	-.008	-.009	.102	.009	.012	.098
	.3	.309	.312	.100	.317	.318	.101	.316	.313	.098
	.7	.874	.875	.100	.873	.872	.104	.872	.874	.094
3	0	-.005	-.009	.099	.002	.002	.103	-.001	.000	.098
	.3	.313	.315	.102	.315	.315	.103	.313	.312	.097
	.7	b			.870	.869	.097	.863	.865	.097
5	0	-.002	-.002	.100	.003	.001	.101	-.003	-.002	.101
	.3	.308	.308	.103	.309	.311	.102	.315	.316	.105
	.7	b			b			.871	.872	.100

[a]With one predictor nonzero rho(i) values are undefined.

[b]This combination would generate data which are undefined.

parameter, rho(p), as N increases as well. Both the mean and the median are consistent estimators. It should be remembered here that when r equals zero, Fisher Z also equals zero. However, when r is .3, Z is .31; and when r is .7, Z is .867.

Inspection of the tables shows that there is no discernible trend in mean r, mean Z, median r, and median Z over levels of rho(i) or levels of p. This seems to indicate that nonindependence of the data does not affect the estimation of the population parameter, rho(p). This is, of course, only for the case when the same parameter is being estimated by all the data.

When evaluating the standard deviations they should be referenced to the known expected values in the cases when independence is not violated. For the r distribution, the standard error of r can be found by substituting the values for the parameters used in this study into the following formula:

$$\sigma_r = \sqrt{\frac{(1 - \rho(p)^2)^2}{n}} .$$

Therefore, the standard error of r when rho(p) is 0 and N is 20 is approximately .224. The standard error of r when rho(p) is .3 and N is 20 is approximately .204. The standard error of r when rho(p) is .7 and N is 20 is approximately .114. When rho(p) is 0 and N is 50 the standard error of r is approximately .141. When rho(p) is .3 and N is 50 the standard error of r is approximately .129. When rho(p) is .7 and N is 50 the standard deviation is approximately .072. The standard error of r when rho(p) is 0 and N is 100 is .1. The standard error of r when rho(p) is .3 and N is 100 is approximately .091. Finally, the standard error of r when rho (p) is .7 and N is 100 is approximately .051.

Inspection of Tables 1, 2, and 3 shows that all the standard deviations are close to their expected values. The largest deviation of the standard deviation from its expected value was .015 and that was in an independent case. This deviation is of no practical concern. There is some improvement as N increases se standard deviations are consistent estimators, but there are no apparent es over levels of rho(i) or p.

For the Fisher's Z distribution, the values of the standard deviations can und by substituting the values for the parameter used in this study into ollowing formula:

$$\sigma_{Z_r} = \frac{1}{\sqrt{N - 3}} .$$

efore, the standard error of Z when N is 20 is approximately .243. The dard error of Z when N is 50 is approximately .146. Finally, the standard r of Z when N is 100 is approximately .102.

Again inspection of Tables 4, 5, and 6 shows that all the standard deviations very close to their expected values. There is some improvement in the estimates I increases, but there are no apparent changes over either levels of rho(i) or p.

clusion

The general conclusion, then, is that nonindependence does not affect the imation of either the measures of central tendency or the standard deviations correlation coefficients and for Fisher's Z transformation of the correlation fficients when the same population paremeter is being estimated.

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