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Catalytic Variables for Improving Personnel Classification and Assignment

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INTRODUCTION

Placing personnel into jobs to maximize expected performance[1] of the organization is a basic problem in large organizations. The solution to this problem requires prediction of the expected performance of each person on each job. These estimates are frequently obtained by developing a separate performance prediction system for each job category.

The predictors in these separate systems consist of information about each person (e.g., age, aptitude scores, interests, experience). After the predictions are made for each person on every possible job, it is desirable to assign each person to a job to maximize expected future performance. This can be accomplished by one of several available computing algorithms (Langley, Kennington, & Shetty, 1974).

If it is necessary to use different sets of prediction weights to make accurate predictions for the various jobs then there is interaction among the people and jobs, and it is important to pay careful attention to the assignment[2] process. However, if it is possible to predict performance accurately using the same set of weights for all jobs, then all possible assignments of personnel to jobs will yield the same overall average performance.

The importance of interaction between people and jobs has been described by Ward (1983). Recognition of the significance of interaction in the predicted payoff array highlights the fact that a constant can be added (or subtracted) from any row or column of the person-job predicted payoff array without changing the particular configuration of assignments of persons to jobs which maximizes the payoff.

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[1] We make no distinction among productivity, payoff, and performance.

[2] Assignment refers to a general class of personnel actions that includes classification into alternative career fields or job types, assignment to specific job position or location, and other actions such as rotation from one billet to another.

By recognizing that the prediction equations can consist of two types of terms--those that represent the interaction of persons with jobs and those that are additive--we can refer to one set of predictor variables as interactive variables and the other as additive (or noninteractive) variables. Since the noninteractive variable terms can be removed from the operational prediction equations without limiting the assignment process, there is no requirement to have these variables available in calculating predicted payoffs for the optimal assignment of people to jobs. These noninteractive variables are required only to develop the prediction equations in conjunction with the interactive variables. When noninteractive variables increase the amount of interaction (i.e., differential classification potential) of the interactive terms we refer to these noninteractive variables as catalytic variables.[3] Catalytic variables are needed only to develop the weights to be used by the interactive variables, but are not required for making optimal assignments of people to jobs. Therefore, variables can be considered as potential catalytic variables when there is reason to believe that, when they are added to the prediction system in a noninteractive way, they may increase predictive accuracy and increase the person-job interaction and that there is good reason to consider eliminating them from the operational prediction equations. Candidates for catalytic variables are:

1. Variables that have been used operationally but must be eliminated because time is not available to collect the variables. For example, if it is necessary to reduce testing time for the ASVAB, it might be possible to use some subtests as catalytic variables for the others without loss of classification effectiveness. These catalytic subtests would be used in a noninteractive way to determine the weights for the interactive (or operational) subtests. The catalytic subtests would not be required for operational administration to new applicants.

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[3]The interaction by which we differentiate catalytic variables from interactive variables is between predictor variables and jobs (i.e., of variables represented in a set of regression equations to predict performance in several jobs, those having similar weights for the different jobs are catalytic; those having different weights for the different jobs are interacting). This interaction is contrasted with that occurring in the case of moderator variables where the interaction is between sets of predictor variables (i.e., the weights assigned to one set of predictors are a function of the values for the other set of variables (moderator) (Sanders, 1956). Suppressor variables, on the other hand, are variables which are not themselves significantly correlated with the criterion (job) variables, but which are significantly correlated with other predictor variables which are correlated with the criterion. These variables then "suppress" or control for predictor variance not related to the criterion variable(s) (Horst, 1941). Suppressor variables and catalytic variables are similar in that they both effect a change in the weights assigned other predictor variables when they enter the equation.

2. Variables that have been used experimentally but will not be used operationally. For example, the Vocational Interest Career Examination (VOICE) has been administered to Air Force personnel in conjunction with the Armed Services Vocational Aptitude Battery (ASVAB). Although the VOICE variables are not used operationally, the classification value of the ASVAB might be enhanced by using the VOICE scores as catalytic variables.

3. Some predictor variables may be very expensive. These variables may be collected on a small number of subjects in conjunction with less expensive interactive (operational) variables. The expensive variables can be used as catalytic variables to enhance the operational variables. Therefore, cost of the expensive variables is eliminated.

CATALYTIC VARIABLE CONCEPT

Description of Available Information

Assume that information is available for performance (on the job or at a school) for many individuals on many different jobs and that each person has performed on one and only one job. Also, assume that the same predictor information is available for all persons and that all performance measures are in the same units.

Let

Y_{ij} = the observed performance of person i on job j ($i = 1, \ldots, I_j$ and $j = 1, \ldots, J$).

${}_jX_{ik}$ = the observed value for interactive predictor variable k for person i who has performance Y_{ij} on job j ($k = 1, \ldots, K$).

${}_jC_{i\ell}$ = the observed value for potential Catalytic[4] predictor variable ℓ for person i who has performance Y_{ij} on job j ($\ell = 1, \ldots, L$).

U = a vector of 1s with dimension $I_1 + I_2 + \ldots + I_J = N$, the total number of individuals for whom criterion information has been obtained.

This information is shown in the arrays in Table 1.

[4] Catalytic variables will be more formally defined in a later section.

Table 1
The Observed Information

Performance Data Y_{ij} | U_i | **Interactive Predictors** $_jX_{ik}$ | **Catalytic Predictors** $_jC_{i\ell}$

1	2	...	j	...	J	U_i	1	2	...	k	...	K	1	2	...	ℓ	...	L
Y_{11}	–	...	–	...	–	1	$_1X_{11}$	$_1X_{12}$	...	$_1X_{1k}$	...	$_1X_{1K}$	$_1C_{11}$	$_1C_{12}$	...	$_1C_{1\ell}$	...	$_1C_{1L}$
Y_{21}	–	...	–	...	–	1	$_1X_{21}$	$_1X_{22}$	...	$_1X_{2k}$	...	$_1X_{2K}$	$_1C_{21}$	$_1C_{12}$	...	$_1C_{1\ell}$	...	$_1C_{1L}$
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
$Y_{I_1 1}$	–	...	–	...	–	1	$_1X_{I_1 1}$	$_1X_{I_1 2}$	...	$_1X_{I_1 k}$	...	$_1X_{I_1 K}$	$_1C_{I_1 1}$	$_1C_{I_1 2}$	...	$_1C_{I_1 \ell}$	...	$_1C_{I_1 L}$
–	Y_{12}	...	–	...	–	1	$_2X_{11}$	$_2X_{12}$	...	$_2X_{1k}$	...	$_2X_{1K}$	$_2C_{11}$	$_2C_{12}$	...	$_2C_{1\ell}$	...	$_2C_{1L}$
–	Y_{22}	...	–	...	–	1	$_2X_{21}$	$_2X_{22}$	...	$_2X_{2k}$	...	$_2X_{2K}$	$_2C_{21}$	$_2C_{22}$	...	$_2C_{2\ell}$	...	$_2C_{2L}$
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
–	$Y_{I_2 2}$	...	–	...	–	1	$_2X_{I_2 1}$	$_2X_{I_2 2}$	...	$_2X_{I_2 k}$	...	$_2X_{I_2 K}$	$_2C_{I_2 1}$	$_2C_{I_2 2}$	...	$_2C_{I_2 \ell}$	...	$_2C_{I_2 L}$
–	–	...	Y_{1j}	...	–	1	$_jX_{11}$	$_jX_{12}$	...	$_jX_{1k}$	...	$_jX_{1K}$	$_jC_{11}$	$_jC_{12}$	...	$_jC_{1\ell}$	...	$_jC_{1L}$
–	–	...	Y_{2j}	...	–	1	$_jX_{21}$	$_jX_{22}$	...	$_jX_{2k}$	...	$_jX_{2K}$	$_jC_{21}$	$_jC_{22}$	...	$_jC_{2\ell}$	...	$_jC_{2L}$
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
–	–	...	$Y_{I_j j}$	...	–	1	$_jX_{I_j 1}$	$_jX_{I_j 2}$	...	$_jX_{I_j k}$	...	$_jX_{I_j K}$	$_jC_{I_j 1}$	$_jC_{I_j 2}$	...	$_jC_{I_j \ell}$	...	$_jC_{I_j L}$
–	–	...	–	...	Y_{1J}	1	$_JX_{11}$	$_JX_{12}$	...	$_JX_{1k}$	...	$_JX_{1K}$	$_JC_{11}$	$_JC_{12}$	...	$_JC_{1\ell}$	...	$_JC_{1L}$
–	–	...	–	...	Y_{2J}	1	$_JX_{21}$	$_JX_{22}$	...	$_JX_{2k}$	...	$_JX_{2K}$	$_JC_{21}$	$_JC_{22}$	...	$_JC_{2\ell}$	...	$_JC_{2L}$
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
.	.	...	.	...	.	.	.	.	...	.	...	.	.	.	...	.	...	.
–	–	...	–	...	$Y_{I_J J}$	1	$_JX_{I_J 1}$	$_JX_{I_J 2}$	...	$_JX_{I_J k}$	...	$_JX_{I_J K}$	$_JC_{I_J 1}$	$_JC_{I_J 2}$	...	$_JC_{I_J \ell}$	...	$_JC_{I_J L}$

Notice that in the Y_{ij} array, a dash, –, indicates unknown performance information, since each person performs in one and only one job.

Developing Prediction Equations From Interacting Variables

To determine the least squares regression weights in the usual manner, these data can be used to define the vectors (see Table 2) of N elements ($N = I_1 + I_2 + \ldots + I_J$):

Y = a vector containing the observed performance Y_{ij}.

U(j) = a vector with elements equal to 1 if the corresponding element of Y involves job j, 0 otherwise.

X(jk) = a vector with elements having a value for variable k if the corresponding element of Y is from job j, 0 otherwise.

E(1) = an error vector.

In this report, symbols in parentheses following a capital letter are used to distinguish vectors (e.g., U(j) (see Table 2) is a vector with elements equal to 1 if the corresponding element of Y is from job j or equal to 0 otherwise; X(jk) is a vector with elements equal to the value for variable k if an element of Y is from job j or equal to 0 otherwise; E(1) is an error vector for Model 1).

The regression equation coefficients can be determined by solving for the coefficients A_j, B_{jk} for $j=1, \ldots, J$, $k=1, \ldots, K$ in Model 1 shown below. $J(K+1)$ regression coefficients are in the model.

$$\begin{aligned} Y = \; & A_1U(1) + B_{11}X(11) + B_{12}X(12) + \ldots + B_{1k}X(1k) + \ldots + B_{1K}X(1K) \\ + \; & A_2U(2) + B_{21}X(21) + B_{22}X(22) + \ldots + B_{2k}X(2k) + \ldots + B_{2K}X(2K) \\ + \; & \ldots \\ + \; & A_jU(j) + B_{j1}X(j1) + B_{j2}X(j2) + \ldots + B_{jk}X(jk) + \ldots + B_{jK}X(jK) \\ + \; & \ldots \\ + \; & A_JU(J) + B_{J1}X(J1) + B_{J2}X(J2) + \ldots + B_{Jk}X(Jk) + \ldots + B_{JK}X(JK) + E(1). \end{aligned}$$

This <u>single</u> regression model determines a prediction equation for performance on <u>each</u> job from information on the predictor variables (X variables). However, the regression equation for <u>each</u> different job can be computed separately since the vectors associated with each job are orthogonal to the set of vectors associated with each and every other job.

The regression coefficients can be displayed in the array shown in Table 3.

Table 2
Vectors for Determining the Regression Coefficients

Y	U(1)	X(11)...X(1K)	U(2)	X(21)...X(2K)	...	U(J)	X(J1)...X(JK)
Y(Job 1)	1 … 1	X(Job 1) $_1X_{ik}$	0 … 0	0 ... 0 … 0 ... 0	...	0 … 0	0 ... 0 … 0 ... 0
Y(Job 2)	0 … 0	0 ... 0 … 0 ... 0	1 … 1	X(Job 2) $_2X_{ik}$	...	0 … 0	0 ... 0 … 0 ... 0
⋮	⋮	⋮	⋮	⋮	...	⋮	⋮
Y(Job J)	0 … 0	0 ... 0 … 0 ... 0	0 … 0	0 ... 0 … 0 ... 0	...	1 … 1	X(Job J) $_JX_{ik}$

Each vector has n elements

Table 3

The Array of Regression Coefficients

$$
\begin{array}{c}
A \\
\left[\begin{array}{c}
A_1 \\
A_2 \\
\cdot \\
\cdot \\
\cdot \\
A_J
\end{array}\right.
\end{array}
\begin{array}{c}
B \\
\left.\begin{array}{cccc}
B_{11} & B_{12} & \cdots & B_{1K} \\
B_{21} & B_{22} & \cdots & B_{2K} \\
\cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdots & \cdot \\
\cdot & \cdot & \cdots & \cdot \\
B_{J1} & B_{J2} & \cdots & B_{JK}
\end{array}\right]
\end{array}
$$

Using the Prediction Equations

After the prediction coefficients have been computed, they can be applied to the predictor information for future groups of personnel to predict future performance for each person on every job. The prediction equations should be applied to a set of people whose data were not used to calculate the regression coefficients. This analysis indicates the degree of confidence that should be placed in future predictors. Since Brogden (1955) has shown that for any assignment of people to jobs, the sum of the multiple regression criterion estimates equals the sum of the actual criterion scores, a further evaluation of the prediction equations can involve comparison of the average performance estimates with that performance from alternative assignments.

Once we have confidence in the prediction equations, the regression coefficients can be applied to a set of data obtained for a total of M subjects (see Table 4).

Let

U = a column vector of 1s of dimension M.

X = a matrix of predictor variables of dimensions M by K.

A = a column vector of regression coefficients of dimension J.

B = a matrix of regression coefficients of dimensions J by K.

A' = the transpose of A.

B' = the transpose of B.

The data set could be new or the same set upon which the prediction equation was developed, in which case $M = N = I_1 + I_2 + \ldots + I_J$.

Table 4

Predicted Performance Array

$$\underset{(M\times J)}{P} = \left[\begin{array}{c:c} \underset{(M\times 1)}{U} & \underset{(M\times K)}{X} \end{array} \right] \left[\begin{array}{c} \underset{(1\times J)}{A'} \\ \hdashline \underset{(K\times J)}{B'} \end{array} \right]$$

$$\begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1J} \\ P_{21} & P_{22} & \cdots & P_{2J} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ P_{M1} & P_{M2} & \cdots & P_{MJ} \end{bmatrix} = \left[\begin{array}{c:cccc} 1 & X_{11} & X_{12} & \cdots & X_{1K} \\ 1 & X_{21} & X_{22} & \cdots & X_{2K} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 1 & X_{M1} & X_{M2} & \cdots & X_{MK} \end{array} \right] \left[\begin{array}{cccc} A_1 & A_2 & \cdots & A_J \\ \hdashline B_{11} & B_{21} & \cdots & B_{J1} \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ \cdot & & \cdots & \cdot \\ B_{1K} & B_{2K} & \cdots & B_{JK} \end{array} \right]$$

Table 4 represents the computation of the predicted score matrix P of dimensions M by J. The predicted performance array P can be input into an optimization algorithm to assign persons to jobs to maximize total overall system performance.

Interaction Between Predictor Information and Jobs

It is important to observe the characteristics of the predicted performance array, P. If there is "no-interaction" between the people and jobs, then it makes no difference which persons are assigned to which jobs (Ward, 1983). "No-interaction" conditions between people and jobs in the array, P, means that

$P_{rt} - P_{ru} = P_{st} - P_{su} = V_{tu}$ (a common value) for $r=1, \ldots M-1$; $s=r+1, \ldots, M$; $t=1, \ldots, J-1$; $u=t+1, \ldots, J$

This can be written as

$$P_{rt} = P_{ru} + V_{tu}$$

and

$$P_{st} = P_{su} + V_{tu}$$

But the conditions for "no-interaction" are equivalent to

$$P_{rt} + P_{su} = P_{ru} + P_{st}$$

This indicates that the sum of the predicted performance values will be the same for all possible assignments of people to jobs.

The conditions for "no-interaction" imply that the regression weights for the corresponding predictors could be identical across all jobs (Ward, 1973, p. 143). It is very important to recognize that even though the weights for the corresponding predictors could be identical across all jobs and have the "no-interaction" conditions in P, it is not necessary that the corresponding weights be identical. For if there is linear dependence among the predictor vectors for a particular job, then there could be an infinite set of weights that would produce the same predicted values for that particular job. It is not possible, in general, to estimate the "amount of interaction" by examining the differences among the corresponding regression coefficients across all jobs.

On the other hand, if the "no-interaction" conditions are not true, it is said that there is "interaction" between the people and the jobs. If there is a "large amount" of interaction, then it is important to seek more optimal assignments. In the presence of such interaction, random assignments could result in extremely poor overall predicted performance. The amount of interaction can be investigated by imposing the restrictions of "no-interaction" on the prediction systems and examining the loss of predictive accuracy (error sum of squares) when using a single set of weights for all jobs. Imposing the restrictions for "no-interaction" will be discussed in the following section.

No-Interaction Situation

Assume that the "no-interaction" conditions are true for the predicted scores obtained from Model 1. This would be the case if:

$$
\begin{aligned}
B_{11} &= B_{21} = \ldots = B_{J1} = B_1 \\
B_{12} &= B_{22} = \ldots = B_{J2} = B_2 \\
&\vdots \\
B_{1k} &= B_{2k} = \ldots = B_{Jk} = B_k \\
&\vdots \\
B_{1K} &= B_{2K} = \ldots = B_{JK} = B_K.
\end{aligned}
$$

Since this is never exactly true for real data, we can obtain some indication of the extent of interaction by imposing these restrictions on Model 1 and obtain the restricted no-interaction regression model, Model 1r:

$$
\begin{aligned}
Y = \; & A_1U(1) + B_1X(11) + B_2X(12) + \ldots + B_kX(1k) + \ldots + B_KX(1K) \\
+ \; & A_2U(2) + B_1X(21) + B_2X(22) + \ldots + B_kX(2k) + \ldots + B_KX(2K) \\
+ \; & \ldots \\
+ \; & A_jU(j) + B_1X(j1) + B_2X(j2) + \ldots + B_kX(jk) + \ldots + B_KX(jK) \\
+ \; & \ldots \\
+ \; & A_JU(J) + B_1X(J1) + B_2X(J2) + \ldots + B_kX(Jk) + \ldots + B_KX(JK) + E(1r),
\end{aligned}
$$

which can be simplified to

$$\begin{aligned} Y = \; & A_1U(1) + A_2U(2) + \ldots + A_jU(j) + \ldots + A_JU(J) \\ & + B_1(X(11) + X(21) + \ldots + X(j1) + \ldots + X(J1)) \\ & + B_2(X(12) + X(22) + \ldots + X(j2) + \ldots + X(J2)) \\ & + \ldots \\ & + B_k(X(1k) + X(2k) + \ldots + X(jk) + \ldots + X(Jk)) \\ & + \ldots \\ & + B_K(X(1K) + X(2K) + \ldots + X(jK) + \ldots + X(JK)) + E(1r). \end{aligned}$$

Letting

$$\begin{aligned} X(1) &= X(11) + X(21) + \ldots + X(j1) + \ldots + X(J1) \\ X(2) &= X(12) + X(22) + \ldots + X(j2) + \ldots + X(J2) \\ & \ldots \\ X(k) &= (1k) + X(2k) + \ldots + X(jk) + \ldots + X(Jk) \\ & \ldots \\ X(K) &= X(1K) + X(2K) + \ldots + X(jK) + \ldots + X(JK), \end{aligned}$$

gives Model Ir

$$\begin{aligned} Y = \; & A_1U(1) + A_2U(2) + \ldots + A_jU(j) + \ldots + A_JU(J) \\ & + B_1X(1) + B_2X(2) + \ldots + B_kX(k) + \ldots + B_KX(K) + E(1r). \end{aligned}$$

If the sum of squares of the elements of restricted model error vector E(1r) is significantly larger than the sum of squares of the error E(1), then interaction exists. However, if no interaction (or a "small amount" of interaction) exists, it makes no (or little) difference which people are assigned to which jobs. To observe this, consider assigning any two persons, r and s, to any two jobs, say t and u. Under the assumptions that the prediction weights are identical, the predicted scores will be:

$$\begin{aligned} P_{rt} &= A_t + B_1 X_{r1} + B_2 X_{r2} + \ldots + B_K X_{rK} \\ P_{su} &= A_u + B_1 X_{s1} + B_2 X_{s2} + \ldots + B_K X_{sK} \\ P_{ru} &= A_u + B_1 X_{r1} + B_2 X_{r2} + \ldots + B_K X_{rK} \\ P_{st} &= A_t + B_1 X_{s1} + B_2 X_{s2} + \ldots + B_K X_{sK}. \end{aligned}$$

The total predicted performance of assigning person r to job t and person s to job u is the same as assigning person r to job u and person s to job t:

$$P_{rt} + P_{su} = P_{ru} + P_{st}$$

$$A_t + A_u + B_1 (X_{r1} + X_{s1}) + \ldots + B_K (X_{rK} + X_{sK}) =$$

$$A_t + A_u + B_1 (X_{r1} + X_{s1}) + \ldots + B_K (X_{rK} + X_{sK}).$$

It is necessary to have a large amount of interaction between people and jobs in order for alternative assignments to improve the total predicted performance. It is desirable to have a prediction system that provides accurate performance prediction and maintains a large amount of interaction between people and jobs. This observation leads to consideration of catalytic variables.

Introducing Catalytic Variables

In some situations it is possible to add new predictor information that will increase the accuracy of performance prediction, and also increase the amount of interaction between people and jobs. However, requiring additional predictor information can be expensive, difficult, or in some cases quite controversial. Therefore, it would be desirable to add additional predictor information on a small sample that would be required only for development of the prediction equations. But the new information would not be required for future operational assignment of people to jobs. Predictor variables that increase interaction but are not required for future operational use are referred to as catalytic variables.

Catalytic variables were identified in the Introduction without definition. They are shown in Table 1 and are designated (as described above) by:

$_jC_{i\ell}$ = the observed value for a potential catalytic predictor variable ℓ for person i who has performance Y_{ij} on job j (ℓ = 1, . . ., L).

We will augment Model 1 with the catalytic variables, but require that the coefficients associated with these variables be identical across all jobs. New vectors can be defined:

$C(j\ell)$ = a vector with elements having a value for catalytic variable ℓ if the person performed in job j; 0 otherwise.

Then, Model 2 can be written as:

$$
\begin{aligned}
Y = \; & A_1U(1) + B_{11}X(11) + B_{12}X(12) + \ldots + B_{1k}X(1k) + \ldots + B_{1K}X(1K) \\
+ \; & A_2U(2) + B_{21}X(21) + B_{22}X(22) + \ldots + B_{2k}X(2k) + \ldots + B_{2K}X(2K) \\
+ \; & \ldots \\
+ \; & A_jU(j) + B_{j1}X(j1) + B_{j2}X(j2) + \ldots + B_{jk}X(jk) + \ldots + B_{jK}X(jK) \\
+ \; & \ldots \\
+ \; & A_JU(J) + B_{J1}X(J1) + B_{J2}X(J2) + \ldots + B_{Jk}X(Jk) + \ldots + B_{JK}X(JK) \\
+ \; & W_1C(11) + W_2C(12) + \ldots + W_\ell C(1\ell) + \ldots + W_LC(1L) \\
+ \; & W_1C(21) + W_2C(22) + \ldots + W_\ell C(2\ell) + \ldots + W_LC(2L) \\
+ \; & \ldots \\
+ \; & W_1C(j1) + W_2C(j2) + \ldots + W_\ell C(j\ell) + \ldots + W_LC(jL) \\
+ \; & \ldots \\
+ \; & W_1C(J1) + W_2C(J2) + \ldots + W_\ell C(J\ell) + \ldots + W_LC(JL) + E(2),
\end{aligned}
$$

Where W_ℓ is the coefficient associated with catalytic predictor ℓ for all jobs $j = 1, \ldots, J$. Also, Model 2 can be rewritten as:

$$
\begin{aligned}
Y = \; & A_1U(1) + B_{11}X(11) + B_{12}X(12) + \ldots + B_{1k}X(1k) + \ldots + B_{1K}X(1K) \\
+ \; & A_2U(2) + B_{21}X(21) + B_{22}X(22) + \ldots + B_{2k}X(2k) + \ldots + B_{2K}X(2K) \\
+ \; & \ldots \\
+ \; & A_jU(j) + B_{j1}X(j1) + B_{j2}X(j2) + \ldots + B_{jk}X(jk) + \ldots + B_{jK}X(jK) \\
+ \; & \ldots \\
+ \; & A_JU(J) + B_{J1}X(J1) + B_{J2}X(J2) + \ldots + B_{Jk}X(Jk) + \ldots + B_{JK}X(JK) \\
+ \; & W_1(C(11) + C(21) + \ldots + C(j1) + \ldots + C(J1)) \\
+ \; & W_2(C(12) + C(22) + \ldots + C(j2) + \ldots + C(J2)) \\
+ \; & \ldots \\
+ \; & W_\ell (C1\ell) + C(2\ell) + \ldots + C(j\ell) + \ldots + C(J\ell)) \\
+ \; & \ldots \\
+ \; & W_L(C(1L) + C(2L) + \ldots + C(jL) + \ldots + C(JL)) + E(2).
\end{aligned}
$$

Define the new vectors

$$C(1) = C(11) + C(21) + \ldots + C(j2) + \ldots + C(J2)$$

$$\vdots$$

$$C(\ell) = C(1\ell) + C(2\ell) + \ldots + C(j\ell) + \ldots + C(J\ell)$$

$$\vdots$$

$$C(L) = C(1L) + C(2L) + \ldots + C(jL) + \ldots + C(JL).$$

Then Model 2 can be written as:

$$\begin{aligned} Y = \; & A_1U(1) + B_{11}X(11) + B_{12}X(12) + \ldots + B_{1k}X(1k) + \ldots + B_{1K}X(1K) \\ + \; & A_2U(2) + B_{21}X(21) + B_{22}X(22) + \ldots + B_{2k}X(2k) + \ldots + B_{2K}X(2K) \\ + \; & \ldots \\ + \; & A_jU(j) + B_{j1}X(j1) + B_{j2}X(j2) + \ldots + B_{jk}X(jk) + \ldots + B_{jK}X(jK) \\ + \; & \ldots \\ + \; & A_JU(J) + B_{J1}X(J1) + B_{J2}X(J2) + \ldots + B_{Jk}X(Jk) + \ldots + B_{JK}X(JK) \\ + \; & W_1C(1) + W_2C(2) + \ldots + W_\ell C(\ell) + \ldots + W_LC(L) + E(2). \end{aligned}$$

There are now J(K+1)+L regression coefficients to be computed. Notice that the new vectors C(1), C(2),. . .,C(L) are <u>not</u> orthogonal to any of the vectors used in Model 1. Therefore, the computational procedure for Model 2 is more complex than for Model 1.

The regression coefficients can be applied from Model 2 either to the data set from which the coefficients were derived or a new data set by augmenting the matrices X, A, and B with the two matrices

C = a matrix of potential catalytic predictor variables designated as ${}_jC_{i\ell}$ in Table 1.

W = a matrix of regression coefficients of dimension J by L with elements defined as shown below in Table 5 (i.e., the rows are identical).

W' = the transpose of W.

Then, a matrix of predicted values, Q, of dimension M by J, can be obtained as shown in Table 6.

Table 5

Regression Coefficients for Catalytic Variables

$$W = \begin{bmatrix} W_1 & W_2 & \cdots & W_L \\ W_1 & W_2 & \cdots & W_L \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ W_1 & W_2 & \cdots & W_L \end{bmatrix}$$

Observing Predicted Scores From Model 2

Consider again assigning any two persons r and s to jobs t and u. Then, the four predicted scores from Model 2 are:

$$\begin{aligned} Q_{rt} &= A_t + B_{t1}X_{r1} + B_{t2}X_{r2} + \dots + B_{tK}X_{rK} \\ &\quad + W_1C_{r1} + W_2C_{r2} + \dots + W_LC_{rL}. \\ Q_{su} &= A_u + B_{u1}X_{s1} + B_{u2}X_{s2} + \dots + B_{uK}X_{sK} \\ &\quad + W_1C_{s1} + W_2C_{s2} + \dots + W_LC_{sL}. \\ Q_{ru} &= A_u + B_{u1}X_{r1} + B_{u2}X_{r2} + \dots + B_{uK}X_{rK} \\ &\quad + W_1C_{r1} + W_2C_{r2} + \dots + W_LC_{rL}. \\ Q_{st} &= A_t + B_{t1}X_{s1} + B_{t2}X_{s2} + \dots + B_{tK}X_{sK} \\ &\quad + W_1C_{s1} + W_2C_{s2} + \dots + W_LC_{sL}. \end{aligned}$$

It can be observed that the difference between the two sums resulting from two different assignments, person r to job t and s to job u, and a second assignment of, person r to job u and s to job t, is given by:

$$(Q_{rt} + Q_{su}) - (Q_{ru} + Q_{st}) =$$

$$(B_{t1}X_{r1} + B_{t2}X_{r2} + \dots + B_{tK}X_{rK} + B_{u1}X_{s1} + B_{u2}X_{s2} \dots + B_{uK}X_{sK})$$
$$-(B_{u1}X_{r1} + B_{u2}X_{r2} + \dots + B_{uK}X_{rK} + B_{t1}X_{s1} + B_{t2}X_{s2} + \dots + B_{tK}X_{sK}).$$

Table 6

Predicted Performance Array With Catalytic Variables

$$
\underset{(M\times J)}{Q} = \left[\begin{array}{c:c:c} \underset{(M\times 1)}{U} & \underset{(M\times K)}{X} & \underset{(M\times L)}{C} \end{array}\right] \left[\begin{array}{c} \underset{(1\times J)}{A'} \\ \hdashline \underset{(K\times J)}{B'} \\ \hdashline \underset{(L\times J)}{W'} \end{array}\right]
$$

$$
\begin{bmatrix} Q_{11} & Q_{12} & \cdots & Q_{1J} \\ Q_{21} & Q_{22} & \cdots & Q_{2J} \\ \vdots & \vdots & \vdots & \vdots \\ Q_{M1} & Q_{M2} & \cdots & Q_{MJ} \end{bmatrix} = \left[\begin{array}{c:cccc:cccc} 1 & X_{11} & X_{12} & \cdots & X_{1K} & C_{11} & C_{12} & \cdots & C_{1L} \\ 1 & X_{21} & X_{22} & \cdots & X_{2K} & C_{21} & C_{22} & \cdots & C_{2L} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & X_{M1} & X_{M2} & \cdots & X_{MK} & C_{M1} & C_{M2} & \cdots & C_{ML} \end{array}\right] \left[\begin{array}{cccc} A_1 & A_2 & \cdots & A_J \\ \hdashline B_{11} & B_{21} & \cdots & B_{J1} \\ \vdots & \vdots & \vdots & \vdots \\ B_{1K} & B_{2K} & \cdots & B_{JK} \\ \hline W_1 & W_1 & \cdots & W_1 \\ W_2 & W_2 & \cdots & W_2 \\ \vdots & \vdots & \vdots & \vdots \\ W_L & W_L & \cdots & W_L \end{array}\right]
$$

The difference between these two payoff scores is determined only by the Bs and the Xs and there is no need to use the As, Ws, and Cs. The estimates of Bs in Model 2 were made using the information from the catalytic variables, Cs. Therefore, it is not necessary to know the values of Cs for making optimum assignments of future groups of people to jobs.

The addition of the new predictors (Cs) will make the interaction between people and jobs in the new array, Q, larger than the person-job interaction in the original array, P. Greater person-job interaction will allow for greater differential assignment potential. A hypothetical example is presented in the next section to illustrate the effect of a catalytic variable.

A Hypothetical Illustration of a Catalytic Variable

Assume that there are four jobs ($J = 4$), one interactive predictor variable ($K = 1$), and one catalytic predictor variable ($L = 1$). The data analysis might produce the following results for Model 1:

$$\begin{aligned} Y = \; & A_1 U(1) + B_{11} X(11) \\ & + A_2 U(2) + B_{21} X(21) \\ & + A_3 U(3) + B_{31} X(31) \\ & + A_4 U(4) + B_{41} X(41) + E(1). \end{aligned}$$

With numerical values for the As and Bs inserted, Model 1 becomes:

$$\begin{aligned} Y = \; & 6\,U(1) + .4\,X(11) \\ & + 5\,U(2) + .2\,X(21) \\ & + 3\,U(3) + .3\,X(31) \\ & + 1\,U(4) + .1\,X(41) + E(1). \end{aligned}$$

This regression model can be represented graphically as shown in Figure 1.

Adding the catalytic predictor variable C(1) to the prediction system might result in Model 2:

$$\begin{aligned} Y = \; & A_1 U(1) + B_{11} X(11) \\ & + A_2 U(2) + B_{21} X(21) \\ & + A_3 U(3) + B_{31} X(31) \\ & + A_4 U(4) + B_{41} X(41) + W_1 C(1) + E(2) \end{aligned}$$

With numerical values for the As, Bs, and W_1 inserted, Model 2 becomes:

$$Y = 0\,U(1) + 1.1\,X(11) + 2\,U(2) + .8\,X(21) + 4\,U(3) + .1\,X(31) + 5\,U(4) + .4\,X(41) + 3\,C(1) + E(2).$$

The regression model can be represented graphically, as shown in Figure 2, when the value of C(1) = 0. All other graphical representations would differ from Figure 2 by the amount 3 C(1).

Now, consider the assignment of one person with an interactive predictor value of 2 and a second person with an interactive predictor value of 8 to jobs 1 and 4. (Any other combination of persons and jobs could have been considered.)

Using Model 1 gives the predicted values:

	Job 1	Job 4
Person with X = 2	$P_{21} = 6(1) + .4(2)$	$P_{24} = 1(1) + .1(2)$
Person with X = 8	$P_{81} = 6(1) + .4(8)$	$P_{84} = 1(1) + .1(8)$

Then, compare the predicted payoff sum obtained from assigning the person with X = 2 to job 1 and the person with X = 8 to job 4 with the predicted payoff sum obtained from assigning the person with X = 8 to job 1 and the person with X = 2 to job 4. Taking the difference gives:

$$\begin{aligned}(P_{21} + P_{84}) - (P_{81} + P_{24}) &= (6(1) + .4(2) + 1(1) + .1(8)) - (6(1) + .4(8) + 1(1) + .1(2)) \\ &= (.4(2) + .1(8)) - (.4(8) + .1(2)) \\ &= .4(2-8) - .1(2-8) \\ &= (.4-.1)\,(2-8) = -1.8.\end{aligned}$$

Observe that the difference betwen the two sums (-1.8) is determined only by the product of the difference between the B's (.4 and .1) and the difference between the Xs (2 and 8).

Making the same comparison using Model 2 gives the predicted values:

	Job 1	Job 4
Person with X = 2	$P_{21} = 0(1) + 1.1(2) + 3\,C(1,2)$	$P_{24} = 5(1) + .4(2) + 3\,C(1,2)$
Person with X = 8	$P_{81} = 0(1) + 1.1(8) + 3\,C(1,8)$	$P_{84} = 5(1) + .4(8) + 3\,C(1,8)$

Figure 1. Prediction equations using Model 1.
(Without Catalytic Variable)

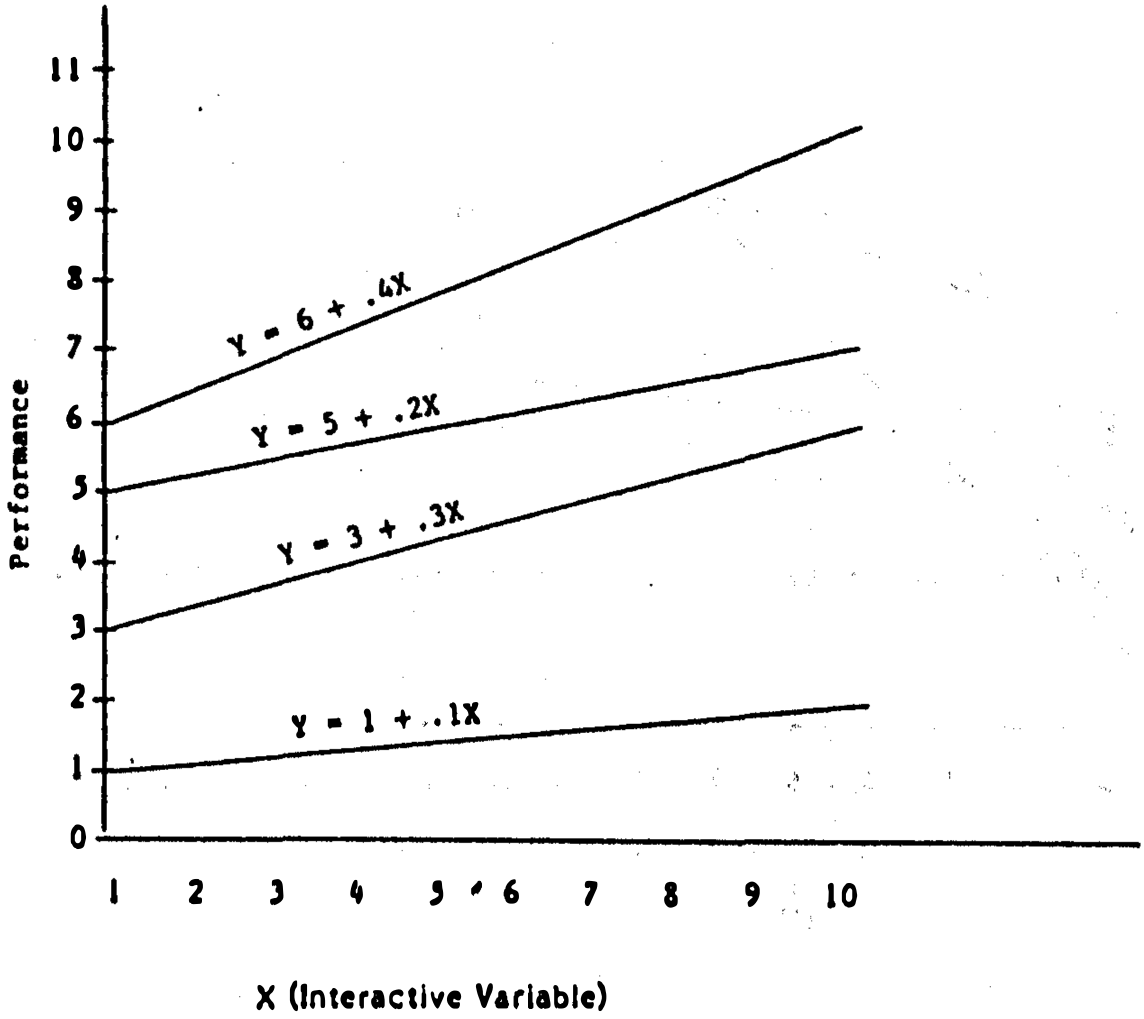
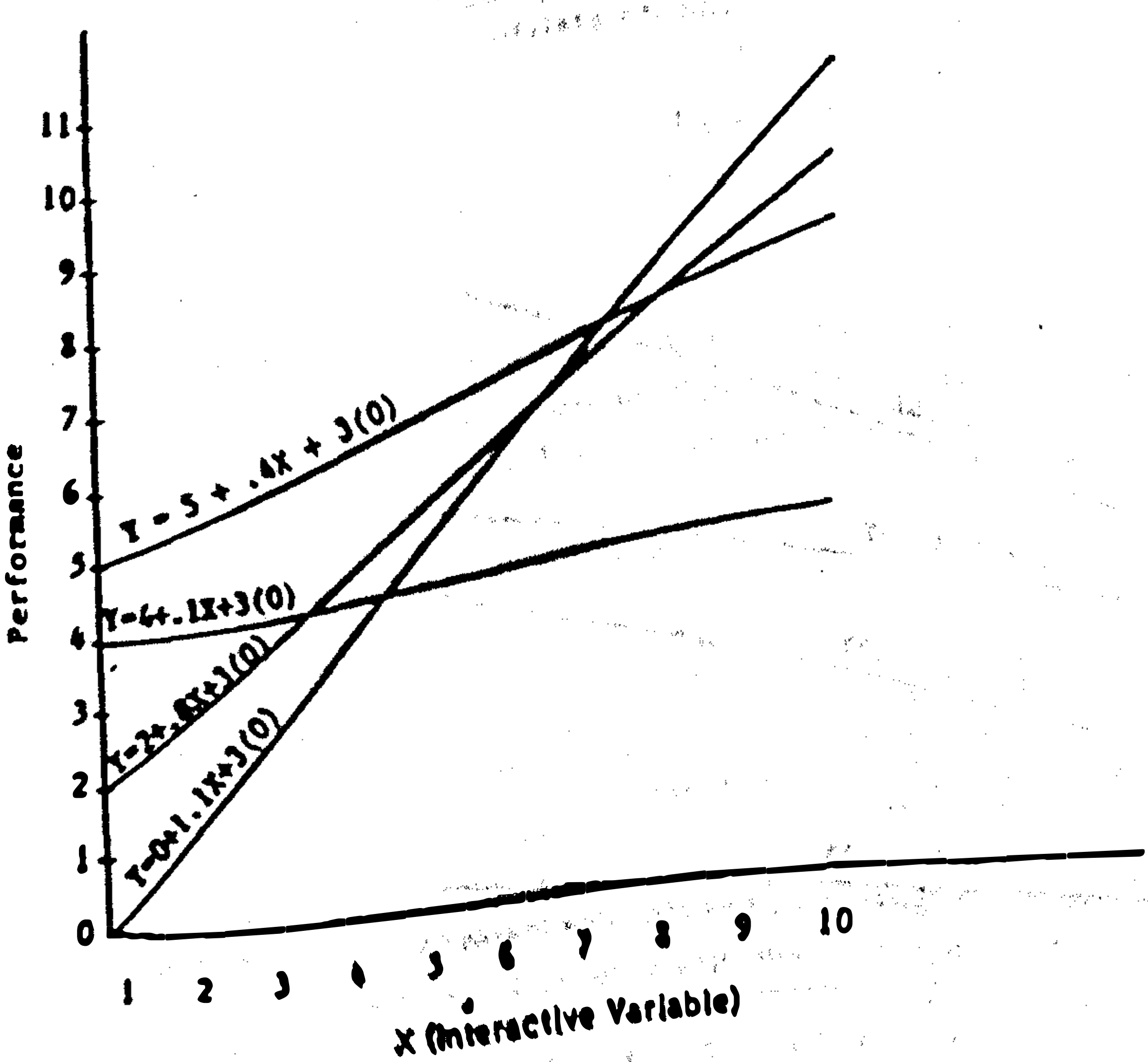


Figure 2. Prediction equations using Model 2. (With Catalytic Variable)



Note - the graph is for C(1) = 0

Then, comparison of the two sums gives the difference

$$(P_{21} + P_{84}) - (P_{81} + P_{24}) = (0(1) + 1.1(2) + 3\,C(1,2) + .5(1) + .4(8) + 3\,C(1,8))$$
$$- (0(1) + 1.1(8) + 3\,C(1,8) + .5(1) + .4(2) + 3\,C(1,2))$$
$$= (1.1(2) + .4(8)) - (1.1(1,8) + .4(2))$$
$$= 1.1(2-8) - .4\,(2-8)$$
$$= (1.1-.4)\,(2-8) = -4.2.$$

Again the difference between the two sums (-4.2) is determined only by the product of the difference between the Bs (1.1 and .4) and the difference between the Xs (2 and 8). The values of the catalytic weight $W'_1 = 3$ and the catalytic values C(I) are not needed for the comparison. The difference (-4.2) using Model 2 is larger absolute value than the difference (-1.8) using Model 1. Comparison of other differences would indicate a tendency for Model 2 differences to be larger than the corresponding differences of Model 1. This would be true because the amount of interaction exhibited in Model 2 is greater than the amount of interaction in Model 1. The comparison of interactions in Model 2 (with) and Model 1 (without) potential catalytic predictors is discussed later.

In this hypothetical illustration, the introduction of the catalytic variable has increased the amount of person-job interaction (and possibly significantly increased predictive accuracy). But having performed its catalytic function, the catalytic variable and its regression weight are no longer required to make optimal assignments that maximize the sum of the predicted performance values.

No-Interaction Situation Using Catalytic Predictors

We can assume no-interaction (i.e., the regression coefficient for each predictor variable is the same for all jobs) and write the same restrictions as before:

$$B_{11} = B_{21} = \dots = B_{J1} = B_1$$
$$B_{12} = B_{22} = \dots = B_{J2} = B_2$$
$$\vdots$$
$$B_{1k} = B_{2k} = \dots = B_{Jk} = B_k$$
$$\vdots$$
$$B_{1K} = B_{2K} = \dots = B_{JK} = B_K$$

However, imposing these restrictions on Model 2, we obtain Model 2r:

$$Y = A_1U(1) + B_1X(11) + B_2X(12) + \ldots + B_kX(1k) + \ldots + B_KX(1K)$$
$$+ A_2U(2) + B_1X(21) + B_2X(22) + \ldots + B_kX(2k) + \ldots + B_KX(2K)$$
$$+ \ldots$$
$$+ A_jU(j) + B_1X(j1) + B_2X(j2) + \ldots + B_kX(jk) + \ldots + B_KX(jK)$$
$$+ \ldots$$
$$+ A_JU(J) + B_1X(J1) + B_2X(J2) + \ldots + B_kX(Jk) + \ldots + B_KX(JK)$$
$$+ W_1C(1) + W_2C(2) + \ldots + W_\ell C(\ell) + \ldots + W_LC(L) + E(2r)$$

Simplifying as before we obtain Model 2r:

$$Y = A_1U(1) + A_2U(2) + \ldots + A_jU(j) + \ldots + A_JU(J)$$
$$+ B_1X(1) + B_2X(2) + \ldots + B_kX(k) + \ldots + B_KX(K)$$
$$+ W_1C(1) + W_2C(2) + \ldots + W_\ell C(\ell) + \ldots + W_LC(L) + E(2r).$$

If the sum of squares of the elements of restricted model error vector E(2r) is significantly larger than the sum of squares of the error vector E(2), interaction exists. If more interaction exists in Model 2 (when compared to Model 2r) than exists in Model 1 (when compared to Model 1r), the "potential" catalytic predictors may be truly called catalytic.

Comparing Models With and Without Catalytic Predictors

The catalytic effect of predictors that have been added noninteractively to a prediction system can be investigated by comparing the error sum of squares from the four models (1, 1r, 2, and 2r). Alternately, the squared multiple correlations, R_1^2, R_{1r}^2, R_2^2, R_{2r}^2, from the four models can be compared. In each of these models we have:

SSE_1 (sum of squares of error for Model 1) $= N\hat{\sigma}_y^2(1-R_1^2)$,

SSE_{1r} (sum of squares of error for Model 1r) $= N\hat{\sigma}_y^2(1-R_{1r}^2)$,

SSE_2 (sum of squares of error for Model 2) $= N\hat{\sigma}_y^2(1-R_2^2)$, and

SSE_{2r} (sum of squares of error for Model 2r) $= N\hat{\sigma}_y^2(1-R_{2r}^2)$.

Then, computing the differences

$$D_1 = SSE_{1r} - SSE_1 = N\theta_y^2(R_1^2 - R_{1r}^2)$$

$$D_2 = SSE_{2r} - SSE_2 = N\theta_y^2(R_2^2 - R_{2r}^2)$$

provides a basis for examining the catalytic effect of additional predictors. D_1 is the sum of squares associated with interaction without potential catalytic predictors and D_2 is the sum of squares associated with interaction in the presence of potential catalytic predictors.

It is necessary to devise ways to decide if the additional predictor variables have a catalytic effect for differential classification of people to jobs. Observe that D2 is larger than D1 only when $(R_2^2 - R_1^2)$ is greater than $(R_{2r}^2 - R_{1r}^2)$. This means that when the potential catalytic variables are added to the interactive form of the operational variables, they must increase the accuracy of prediction by a larger amount than when they are added to the noninteractive form of the operational variables. Therefore, even if $(R_2^2 - R_1^2)$ is significantly large (i.e., absolute prediction is improved with the addition of the catalytic variables), there could be a decrease in person-job interaction when the potential catalytic variables are added (See Horst (1954, 1955) for discussion of differential vs absolute prediction). In this case, there would be less reason to consider using the additional variables in the catalytic form. On the other hand, if D2 is larger than D1 (and $(R_2^2 - R_1^2)$ is greater than $(R_{2r}^2 - R_{1r}^2)$) we can say that there is an increase in the amount of interaction with the inclusion of the catalytic variables. In such a case we would want to use additional variables in catalytic form.

It is possible to introduce consideration of a super prediction model (Model S) and its squared multiple correlation, R_S^2. This model allows for the investigation of the increase in predictive accuracy and interaction when the potential catalytic variables (Cs) are allowed to have different weights across all jobs (i.e., to join the Xs). Other comparisons among the squared multiple correlations (R_1^2, R_{1r}^2, R_2^2, R_{2r}^2, R_S^2) might be helpful in making decisions about the proper role of the potential catalytic variables. For example:

- If D_2 is much larger then D_1 (indicating increased interaction), and
- if R_2^2 is much larger than R_1^2 (indicating an increase in predictive accuracy), and
- if R_S^2, is insignificantly larger than R_2^2, then we might conclude that the variables would perform very well using only their additive, catalytic form (Model 2).

Further study and experience is needed to develop descriptive, statistical, and practical methods of decision-making about catalytic effects.

APPLICATION OF THE CATALYTIC VARIABLE CONCEPT

The procedure for introducing catalytic predictor variables will be illustrated with data from the military. The first example involves four jobs, one interactive variable (aptitude test) and four potential catalytic variables.

Description of the Information from Example I

Y_{ij} = Performance measure of individual i on job j:

There are 500 individuals from each job providing a total of 2000 individuals.

${}_jX_{ik}$ = the observed interactive predictor (aptitude test score) for individual i who has performance Y_{ij} on job j. (With one interactive variable, k=1.)

${}_jC_{i\ell}$ = the observed value for potential catalytic predictor variable ℓ for person i who has performance Y_{ij} on job j (ℓ = 1, 2, 3, 4).

(In the example, each catalytic variable is a mutually exclusive, categorical, binary-coded predictor variable.)

U = a vector of 1s with dimension 2000.

The example data would appear as displayed in Table 7.

Developing Prediction Equations from the Interacting Variable

For the example, the least squares regression weights can be determined in the usual manner by defining the predictor vectors:

Y = a vector containing the observed performance Y_{ij} with N = 2000 elements.

U(j) = 1 if an element of Y is from job j; or 0 otherwise, j = 1, 2, 3, 4.

X(jk) = an ability test value if an element of Y is from job j; or 0 otherwise.

E(1) = an error vector.

Table 7

Observed Information for Example 1

	Performance Data					Interactive Predictor	Catalytic Predictors			
	Y_{ij}				U_i	$_jX_{ik}$	$_jC_{i\ell}$			
	1	2	3	4			1	2	3	4
	55	-	-	-	1	32	1	0	0	0
	63	-	-	-	1	65	0	0	0	1
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
($I_1 = 500$)	82	-	-	-	1	52	1	0	0	0
	-	46	-	-	1	49	0	0	1	0
	-	69	-	-	1	66	0	1	0	0
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
($I_2 = 500$)	-	72	-	-	1	38	0	0	0	1
	-	-	62	-	1	53	0	1	0	0
	-	-	93	-	1	59	0	1	0	0
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
($I_3 = 500$)	-	-	87	-	1	62	0	0	0	0
	-	-	-	43	1	54	0	0	0	1
	-	-	-	76	1	47	1	0	0	0
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.
($I_4 = 500$)	-	-	-	82	1	59	0	0	0	1
($N = 2000$)										

Notice that the - in the Y_{ij} array indicates unknown performance information, since each person performs in one and only one job. However, the 0 values for the mutually exclusive categorial variables represent nonmembership in the particular category.

Then the regression coefficients can be determined by solving for the regression coefficients A_1, A_2, A_3, A_4, B_{11}, B_{21}, B_{31}, B_{41} in regression Model 1. Observe that K = 1 in this example, since there is only one interactive predictor variable. Model 1 (for example) is:

$$
\begin{aligned}
Y = \; & A_1\, U(1) + B_{11}\, X(11) \\
& + A_2\, U(2) + B_{21}\, X(21) \\
& + A_3\, U(3) + B_{31}\, X(31) \\
& + A_4\, U(4) + B_{41}\, X(41) + E(1).
\end{aligned}
$$

As indicated previously, this single regression model determines a prediction equation for performance on each of the four jobs. However, the regression equation for each job can be computed separately since the vectors associated with each job are orthogonal to the set of vectors associated with the other three jobs. The vectors are illustrated in Table 8.

Table 8

Vectors for Determining the Regression Coefficients for Example 1

Y	U(1)	X(11)	U(2)	X(21)	U(3)	X(31)	U(4)	X(41)
55	1	32	0	0	0	0	0	[illegible]
63	1	65	0	0	0	0	0	[illegible]
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
82	1	52	0	0	0	0	0	[illegible]
---	---	---	---	---	---	---	---	--
46	0	0	1	49	0	0	0	
69	0	0	1	66	0	0	0	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
72	0	0	1	38	0	0	0	[illegible]
---	---	---	---	---	---	---	---	--
62	0	0	0	0	1	53	0	
93	0	0	0	0	1	59	0	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
87	0	0	0	0	1	62	0	
---	---	---	---	---	---	---	---	-
43	0	0	0	0	0	0	1	
76	0	0	0	0	0	0	1	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
.	.	.	.	.	.	.	.	
82	0	0	0	0	0	0	1	

The regression coefficients can be displayed as shown in Table 9.

Using the Prediction Equations for Example 1

The prediction equations can be used to determine the predicted performance of each of M persons on each of the four jobs. The predicted performance matrix P of dimension M by 4 is computed by the matrix multiplication as shown in Table 10.

The predicted performance array P can be put into an optimization algorithm to assign persons to jobs to maximize total system performance. In the example shown, there are only four jobs represented and M people. Usually, there are job quotas for each job such that the sum of the job quotas is equal or very nearly equal to the total number of people to be assigned (M in this case).

Interaction Between Predictor Information (ability test measure) and Jobs

As mentioned above, if there is no interaction between persons and jobs, we would have:

$$B_{11} = B_{21} = B_{31} = B_{41} = B_1.$$

Since this is never exactly true for any real data, some indication of the extent of interaction can be obtained by imposing the restrictions indicated above and solving the restricted no-interaction regression model, Model 1r:

$$\begin{aligned} Y = \; & A_1U(1) + B_1X(11) \\ & + A_2U(2) + B_1X(21) \\ & + A_3U(3) + B_1X(31) \\ & + A_4U(4) + B_1X(41) + E(1r), \end{aligned}$$

which can be simplified to

$$\begin{aligned} Y = \; & A_1U(1) + A_2U(2) + A_3U(3) + A_4U(4) \\ & + B_1(X(11) + X(21) + X(31) + X(41)) + E(1r). \end{aligned}$$

Table 9

The Array of Regression Coefficients for Example 1

$$
\begin{bmatrix}
A & B \\
A_1 & B_{11} \\
A_2 & B_{21} \\
A_3 & B_{31} \\
A_4 & B_{41}
\end{bmatrix}
$$

Table 10

Predicted Performance Array for Example 1

$$
\underset{(M\times 4)}{P} = \left[\begin{array}{c:c} \underset{(M\times 1)}{U} & \underset{(M\times 1)}{X} \end{array}\right] \left[\begin{array}{c} \underset{(1\times 4)}{A'} \\ \hdashline \underset{(1\times 4)}{B'} \end{array}\right]
$$

$$
\begin{bmatrix}
P_1 & P_2 & P_3 & P_4 \\
P_{21} & P_{22} & P_{23} & P_{24} \\
\vdots & \vdots & \vdots & \vdots \\
P_{M1} & P_{M2} & P_{M3} & P_{M4}
\end{bmatrix}
=
\left[\begin{array}{c:c}
1 & X_{11} \\
1 & X_{21} \\
\vdots & \vdots \\
1 & X_{M1}
\end{array}\right]
\left[\begin{array}{cccc}
A_1 & A_2 & A_3 & A_4 \\
\hdashline
B_{11} & B_{21} & B_{31} & B_{41}
\end{array}\right]
$$

Letting $X(1) = X(11) + X(21) + X(31) + X(41)$ give Model 1r:

$$Y = A_1U(1) + A_2U(2) + A_3U(3) + A_4U(4) + B_1X(1) + E(1r).$$

If the sum of squares of the elements of restricted model error vector E(1r) is "significantly" (statistically and/or practically) larger than the sum of squares of the error vector E(1) (R^2_{1r} smaller than R^2_1), then interaction exists. If interaction is not indicated, individuals can be assigned (e.g., arbitrarily or randomly) to any job without affecting total predicted performance.

Introducing Catalytic Variables

Catalytic variables were defined earlier as predictor variables that increase interaction between people characteristics (i.e., interacting variables) and jobs, but are not required for future operational use (i.e., do not interact themselves with jobs) to optimally classify people into jobs.

In our example, Model 1 will be augmented with four catalytic predictor variables. However, as indicated earlier, the regression coefficients associated with each of these four catalytic predictor variables must be the same for all four jobs. There should be no interaction between catalytic predictor variables and jobs.

Then, four catalytic predictor vectors can be defined in our example.

C(1) = a vector for catalytic variable 1, which, in the example, is a binary-coded predictor having a value of 1 if the observation comes from the first mutually exclusive category and 0 otherwise.

C(2) = a vector for catalytic variable, 2 which, in the example, is a binary-coded predictor having a value of 1 if the observation comes the second mutually exclusive category from and 0 otherwise.

C(3) = a vector for catalytic variable 3, which is defined similar to C(1) and C(2).

C(4) = a vector for catalytic variable 4, which is defined similar to C(1), C(2), and C(3).

Then, the final form of regression Model 2 above can be written as:

$$
\begin{aligned}
Y = \; & A_1U(1) + B_{11}\,X(11) \\
& + A_2U(2) + B_{21}\,X(21) \\
& + A_3U(3) + B_{31}\,X(31) \\
& + A_4U(4) + B_{41}\,X(41) \\
& + W_1C(1) + W_2C(2) + W_3C(3) + W_4C(4) + E(2).
\end{aligned}
$$

The predictor vectors C(1), C(2), C(3), and C(4) are generally not orthogonal to the other vectors. Therefore, the computational procedure for Model 2 is more complex than for Model 1. It is important to note that the least squares estimates of the values for A_1, A_2, A_3, A_4, B_{11}, B_{21}, B_{31}, and B_{41} are not generally the same in Models 1 and 2. After solving for the coefficients in Model 2, the predicted performance matrix Q of dimension M by 4 from the matrix multiplication, as shown in Table 11, can be obtained.

No-Interaction Between Predictor Information and Jobs Using Catalytic Predictors

The hypothesis of no-interaction can be investigated as before by assuming in Model 2 that:

$$B_{11} = B_{21} = B_{31} = B_{41} = B_1$$

and imposing these restrictions obtain the restricted model, Model 2r:

$$
\begin{aligned}
Y = \; & A_1\,U(1) + B_1\,X(11) \\
& + A_2\,U(2) + B_1\,X(21) \\
& + A_3\,U(3) + B_1\,X(31) \\
& + A_4\,U(4) + B_1\,X(41) \\
& + W_1\,C(1) + W_2\,C(2) + W_3\,C(3) + W_4\,C(4) + E(2r).
\end{aligned}
$$

As before, if the sum of squares of the elements of restricted model error vector E(2r) is "significantly" (statistically and/or practically) larger than the sum of squares of the error vector E(2) (R^2_{2r} smaller than R^2_2), then it can be concluded that interaction exists. If more interaction exists in Model 2 (when compared to Model 2r) than exists in Model 1 (when compared to Model 1r), the "potential" catalytic predictors can be truly called catalytic.

Table 11

Predicted Performance Array With Catalytic Variables for Example 1

$$\underset{(M\times 4)}{Q} = \left[\underset{(M\times 1)}{U} \;\vdots\; \underset{(M\times 1)}{X} \;\vdots\; \underset{(M\times 4)}{C}\right] \begin{bmatrix} \underset{(1\times 4)}{A'} \\ \hdashline \underset{(1\times 4)}{B'} \\ \hdashline \underset{(4\times 4)}{W'} \end{bmatrix}$$

$$\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} \\ \vdots & \vdots & \vdots & \vdots \\ Q_{M1} & Q_{M2} & Q_{M3} & Q_{M4} \end{bmatrix} = \left[\begin{array}{c:c:cccc} 1 & X_{11} & C_{11} & C_{12} & C_{13} & C_{14} \\ 1 & X_{21} & C_{21} & C_{22} & C_{23} & C_{24} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & X_{M1} & C_{M1} & C_{M2} & C_{M3} & C_{M4} \end{array}\right] \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ \hdashline B_{11} & B_{21} & B_{31} & B_{41} \\ \hdashline W_1 & W_1 & W_1 & W_1 \\ W_2 & W_2 & W_2 & W_2 \\ W_3 & W_3 & W_3 & W_3 \\ W_4 & W_4 & W_4 & W_4 \end{bmatrix}$$

Comparing Interactions With and Without Catalytic Predictors for Examples

As indicated above, the catalytic effect of predictors that have been added noninteractively to a prediction system can be investigated by comparing the error sum of squares from the four models (1, 1r, 2, 2r).

For the example 1 data with N = 2000 we obtained:

$$D_2 - D_1 = N\hat{\sigma}_y^2((R_2^2 - R_{2r}^2) - (R_1^2 - R_{1r}^2))$$

$$D_2 - D_1 = N\hat{\sigma}_y^2((.1930 - .1785) - (.1832 - .1705))$$

$$= N\hat{\sigma}_y^2\,(.0018) = 2000\,\hat{\sigma}_y^2\,(.0018).$$

The fact that D_2-D_1 is greater than zero indicates that some catalytic effect is due to the four predictors C(1), C(2), C(3), and C(4). Also, the increase in absolute predictive accuracy (R_2^2 - R_1^2) is statistically significant. Other information would be required to decide if the catalytic variables are practically useful.

A second random sample of 2000 subjects was chosen and the analysis was repeated. The difference from Sample 2 was:

$$D_2 - D_1 = N\hat{\sigma}_y^2((R_2^2 - R_{2r}^2) - (R_1^2 - R_{1r}^2))$$

$$D_2 - D_1 = N\hat{\sigma}_y^2\,((.1649-.1539)-(.1571-.1471))$$

$$= N\hat{\sigma}_y^2\,(.0010) = 2000\,\hat{\sigma}_y^2\,(.0010).$$

The second sample also indicates statistically significant increase in absolute prediction, and an increase in the amount of interaction (D_2 greater than D_1). This suggests the possibility of using the catalytic variables.

<u>Example of Noncatalytic Effects</u>

Example 2 has been chosen to illustrate "potential" catalytic variables that result in decrease in interaction and, therefore, become noncatalytic variables. This example consists of three jobs, one interactive variable (aptitude test) and 2 potential catalytic variables. There are a total of 7043 people in the example, 2317 subjects from job 1, 1836 subjects from job 2, and 2890 subjects from job 3.

For this example we can compute the difference between the interaction sum of squares without and with potential catalytic variables:

$$D_2 - D_1 = N\hat{\sigma}_y^2\,((R_2^2 - R_{2r}^2) - (R_1^2 - R_{1r}^2))$$

$$D_2 - D_1 = N\hat{\sigma}_y^3((.3647-.3623) - (.3385-.3349)$$

$$D_2 - D_1 = N\hat{\sigma}_y^2(-.0012)$$

$$= 7043\hat{\sigma}_y^2\,(-.0012).$$

The negative value of D_2-D_1 indicates that there is a decrease in person-job interaction (differential prediction) when the potential catalytic variables are added. However, there is a statistically significant increase in absolute predictive accuracy. It would be doubtful that the addition of the catalytic variables would be of practical value in this case.

Catalytic Effects in Operational Situations

The actual catalytic effect in an operational situation depends on the particular set of people and jobs under consideration. The predicted scores P (without potential catalytic predictors) and the predicted scores Q (with potential catalytic predictors) should be computed for a particular set of people and jobs. The interaction sum of squares for the P matrix (designated D_p) can be compared with the interaction sum of squares for the Q matrix (designated D_q) in the same manner as above. As before, it is suggested that, if D_q is larger than D_p, then, for this particular set of people and jobs, the additional predictor variables have a catalytic effect.

As the interaction between people and jobs increases, it becomes more important to assign the "right person to the right job."

CONCLUSIONS

If there is no interaction between people characteristics and jobs in the prediction of job performance, then it makes no difference in overall system performance which people are assigned to which jobs. To increase interaction (and, therefore, differential assignment potential), it is usually necessary to add new variables to the operational variables in the prediction system. The addition of new variables can be costly, time consuming, and frequently controversial. The approach described herein suggests adding predictor variables in a noninteractive way to the operational (interacting) predictors to increase the possibility of more interaction between people and jobs. If these additional noninteractive variables can increase interaction, they are called catalytic variables. Catalytic variables (which enter the prediction system in an additive way) are not required for use in the assignment of people to jobs to maximize overall system performance.

The statistical and practical significance of the catalytic effects approach should be studied to develop guidelines for making cost-benefit decisions about the use of catalytic variables.

To gain more knowledge about the catalytic process, data already collected for people, jobs, and potential catalytic variables should be studied.

Data sets involving performance measures requiring a wide variety of attributes, and a large number of different jobs should be used to maximize the prospects of finding catalytic predictors.

REFERENCES

Brogden, H. F. (1955). Least squares estimates and optimal classification. Psychometrika, 20(3), 249-252.

Horst, P. (1941). The prediction of personal adjustment. Social Science Research Bulletin, 48.

Horst, P. (1954). A Technique for the development of a differential prediction battery. Psychological Monographs, No. 380.

Horst, P. (1955). A Technique for the development of a multiple absolute prediction battery. Psychological Monographs, No. 390.

Langley, R. W., Kennington, J., & Shetty, C. M. (1974). Efficient computational devices for the capacitated transportation problem. Office of Naval Research, Naval Research Logistics Quarterly, 21(4), 637-647.

Sanders, D. R. (1956). Moderator variables in prediction. Educational and Psychological Measurement, 16, 209-222.

Ward, J. H., Jr. (1983). Strategies for capitalizing on individual differences in military personnel systems. In R. C. Sorenson (Ed.). Human individual differences in military systems (NPRDC Spec. Rep. 83-30). San Diego: Navy Personnel Research and Development Center.

Ward, J. H., Jr., & Jennings, E. (1973). Introduction to linear models. Prentice-Hall Inc., Englewood Cliffs, NJ.

SUMMARY

Organizations have a fundamental problem of placing personnel into jobs to maximize expected performance. Whether or not placing people in specific jobs really makes a difference in overall expected system performance depends on the interaction of people characteristics with jobs. It is desirable to increase the interaction of the people characteristics, as measured by predictor tests, with the jobs.

The purpose of this effort is to suggest a procedure for using one set of performance predictor variables in a simple noninteractive way to enhance the differential classification potential (person-job interaction) of a set of operational predictor variables. The noninteractive variables are required only in determination of the regression coefficients for the operational predictors, but are not required for operational use in future differential classification actions.

Separate equations are developed to predict performance on each job. The equations are determined so that the weights for the operational predictors are allowed (if necessary) to vary across the various jobs. However, one set of predictors (the potential catalytic variables) is required to have the same regression weights across all jobs (noninteractive). If this noninteractive set of predictors can increase the amount of person-job interaction in the new predicted performance values, then the potential for improved assignment has been increased. These noninteractive variables are called catalytic.

Since catalytic variables are used in prediction systems in a noninteractive way, they are not required for future use in the classification system. Therefore, this procedure will allow personnel classification system developers to use a set of catalytic predictors to enhance the differential classification potential of a set of operational (interactive) predictors, but not require these catalytic predictors for future classification. If catalytic variables can be found, savings in time and money might be possible with little loss in classification effectiveness of the operational predictors.

This approach should be applied to prediction situations in which data are already available and it is desirable to enhance the classification effectiveness of a set of operational predictors without requiring the operational use of the catalytic variables.