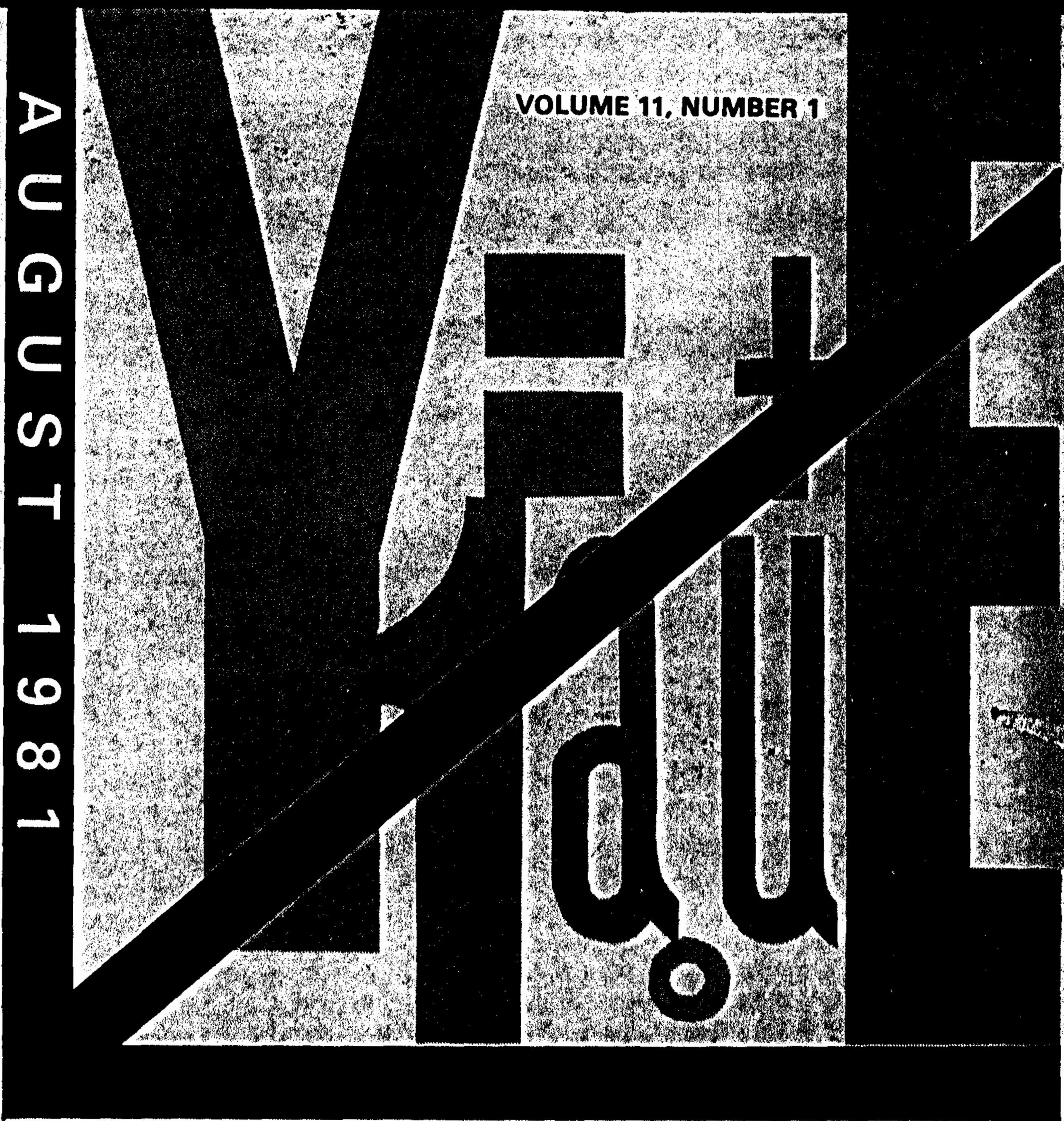

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EFFECTS OF AGE AT DELIVERY AND OTHER MATERNAL TRAITS ON THE COGNITIVE DEVELOPMENT OF CHILDREN: AN APPLICATION OF INTERACTION REGRESSION ANALYSIS TO A POLICY COMPLEX

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INTRODUCTION

Traditionally, the study of child development has defined the context within which children grow as the social environment. To some extent there is, at the moment, a re-discovery of genetic and constitutional elements influencing growth. One outcome has been recognition that development is influenced by factors in several domains and by the simultaneous interaction of many of them. As yet ill-considered is the role of personal characteristics; for example, the change to a relatively higher incidence of birth in very young women, the relevance of puberty in both young women and men (Zlatnick and Burmeister, 1977; Kiernan, 1977; Ryder and Westoff, 1971; Morse et all, 1975), demographic factors (Cohen, Gaughran, and Cohen, 1979), and psychodynamic processes (Quay, 1981) to consequent patterns of reproduction.

Related to this conservatism has been slowness to develop policies (Stringfellow, 1978) and programs (Wechsler, 1979) to assist teenagers who are pregnant. In terms of human development, we have scarcely studied the after effects on women of pregnancy occurring early in the reproductive years (Sauber, 1970). The impact of delivery at an early age on the offspring has been studied a little (Thompson, Cappleman and Zeitschel, 1979), and some work has been done on the impact of the problem on the social order (Furstenberg, 1976), and on children explicitly not planned for (Matejcek, Dytrych, and Schuller, 1979).

Data adapted from a 1977 report of the Census Bureau (see Table 1) show that there has been a substantial change since 1925 in the rate of live births. The change is evident in differential rates in women between forty and forty-nine years of age and in girls between ten and nineteen years of age. The lower portion of the age spectrum has been examined by Reinhold (1977), and by Ventura (1977) who found that the rate of illegitimacy in girls 15-17 years of age has almost doubled in the past decade. In the fifty years ending in 1975, according to Census Bureau data, the rate of live births per one thousand women age nineteen years and under, changed by age, and by race. For all females 15-19 years there was a slight drop in fertility. It was sharpest for white girls, declining from 54.3 births per thousand in 1925 to 47.7. In the case of black girls the rate in 1975 was slightly above that observed in 1925. At both the beginning and the end of the half century summarized in Table 1, and adapted from the original data, the rate for live births per one thousand women in young black females was double the rate for white. In the case of girls ages ten to fourteen years the overall rate for both races doubled

TABLE 1

ANNUAL RATE OF LIVE BIRTHS PER 1,000 WOMEN 10-19 AND 40-49 YEARS OF AGE, BY RACE FROM 1925 TO 1975[1]

	All Females		White Females		Black Females		All Females		White Females		Black Females	
Calendar Year	10 to 14 Years	15 to 19 Years	10 to 14 Years	15 to 19 Years	10 to 14 Years	15 to 19 Years	40 to 44 Years	45 to 49 Years	40 to 44 Years	45 to 49 Years	40 to 44 Years	45 to 49 Years
1925	0.6	62.0	0.3	54.3	3.1	112.5	30.1	4.2	29.8	3.6	33.1	9.6
1930	0.6	56.1	0.3	49.8	2.9	100.0	23.5	3.1	23.4	2.6	23.6	8.2
1935	0.6	50.2	0.3	43.2	2.9	101.4	18.4	2.4	18.1	2.0	21.3	5.9
1940	0.6	52.5	0.2	44.3	3.4	111.3	14.9	1.7	14.4	1.4	18.5	4.4
1945	0.7	49.7	0.3	41.1	3.7	108.5	16.3	1.5	15.9	1.3	19.0	3.2
1950	1.0	78.5	0.4	67.9	5.0	146.2	14.4	1.1	13.9	.9	17.7	2.2
1955	1.0	83.9	0.4	78.7	4.5	153.9	15.6	1.0	15.1	.9	20.3	1.7
1960	1.0	89.3	0.4	80.4	4.5	147.2	15.4	1.0	14.7	.9	21.1	1.8
1965	0.8	72.2	0.3	62.4	4.2	142.5	12.9	.8	12.2	.7	17.9	1.4
1970	1.2	69.3	0.5	58.2	5.2	140.0	8.3	.5	7.7	.5	12.5	1.0
1971	1.1	65.9	0.5	54.7	5.0	135.8	7.3	.5	6.6	.4	11.7	1.0
1972	1.2	63.2	0.5	52.1	5.1	131.5	6.4	.4	5.8	.3	10.0	.8
1973	1.3	60.7	0.6	50.1	5.3	124.6	5.6	.3	5.1	.3	8.8	.7
1974	1.2	58.9	0.6	48.9	4.9	118.3	5.0	.3	4.5	.3	7.7	.6
1975	1.2	57.2	0.6	47.7	4.8	113.0	4.4	.3	4.0	.3	6.8	.6

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[1] Adapted from U.S. Bureau of the Census Current Population Reports, Series P .25, No. 704, *Projections of the Population of the United States 1977 to 2050*, 1977.

in the half-century beginning in 1925. Between the races the pattern was different. The rate in black girls at all ages was ten times the rate in whites. Within the black group aged 10-14 years the rate rose from 3.1 per thousand in 1925, peaked in 1973, and was at 4.8 live births per one thousand women by 1975. Within the same age group of white females the rate increased steadily from 1925, and had reached .6 per one thousand women by 1975.

Among older women, the years from 1925 to 1975 showed a profound decline in rate of reproduction. Among women aged forty to forty-four years the rate for both white and black women dropped from 30.1 live births per 1,000 women in 1925 to 4.4 in 1975; the drop for blacks and whites was about the same. Among older women, ages forty-five to forty-nine, the overall change was also a drop from 4.2 in 1925 to .3 per 1,000 women in 1975. There was a proportionately greater drop among black women from 9.6 to .6 live births per 1,000 women. The drop for white women was from a lower baseline rate in 1925 of 3.6 live births to .3, 1975. Within the age groups CA 10-14, 15-17, and 17-19 Scott (1981) reports a decline in rate of births between 1970 and 1977 for all but the CA 10-14 year group.

Summarizing these data we note the following points:

1. Incidence rates per thousand are higher for black girls.
2. In both racial groups the rate at ages 15-19 years in 1975 was generally comparable to the rate in 1925.
3. The greatest rate increase in the 10-15 year group, using 1925 data as the baseline, is in white females.
4. In contrast, rates among women of both races age 40-49 years dropped, and to very low levels.

The population of teenagers becoming mothers is clearly a volatile group. Black females have had a higher birth rate for the fifty years summarized in Table 1 than white females. However, the group exhibiting the greatest change proportional to its own 1925 base rate is the youngest set of white females.

As we shall see from the review of the literature which follows investigators have tended to concentrate almost exclusively on the young, primiparous women ascribing to their youth a significance restricted to their obstetrical status. Interest has pursued the course of her pregnancy - rightly formulated as at risk - and ended with the obstetric outcomes.

To the behaviorist interested in the quality of answers to important human problems it is obvious that we do not understand answers unless we appreciate the methodology which yields them. More explicitly, we seek to appraise the samples, the statistical tests, and the simplicity or complexity of how the elements relate to each other. In the case of the problem addressed in this paper the tradition is one of comparing mean scores, of defining issues in univariate contexts, and of restricting the formulation of problems to what one might call 'exercises for one hand.'

In contrast to the simplicity of the corpus of literature addressing the growing problem of adolescent pregnancy is the complexity of the situation. A teenager is half-child plus half-adult. Some bear children but many more do not. Some are worriers who conceive anyway, and some are rigid people whose scrupulosity does not exclude conception. Some are bright and some are dull; some are rich and some are poor. To this list we add two comments. First, each of these traits exists to some degree, or at some level; second, these elements, plus others, exist and interact concurrently, and all rise and fall in salience across the span of adolescence. In this sense the social problem of understanding the problem of adolescent pregnancy, and for this discussion, its effects on children, is an excellent topic for applied statistical analysis. It is multivariate, existing across the span of adolescence for the mother; it covers the preschool years for the child until the socializing power of the classroom takes over. The problem is empirical, in the sense of yielding to statistical assessment using psychometric and demographic data.

In this paper we extend the scope of inquiry to cover the development of children, as a function of maternal age, to age five years. Also, we broaden the topic of maternal age at delivery to permit comparison of outcomes from delivery age thirteen to age forty-two years. Finally, we have adopted a multivariate strategy emphasizing both correlates of maternal age and the interaction of elements in a complex of nine maternal traits used as predictors.

REVIEW OF THE LITERATURE

__Adolescent Pregnancy and Risk:__ Gill, Illsey, and Koplik (1970) have wisely pointed out that pregnancy across the adolescent years is not homogeneous. Obviously some nineteen-year old women deliver as teenagers only in the actuarial sense of being just under twenty. By that age a woman may no longer be a primiparae and so resemble pregnant women in general. On the other hand, evidence is abundant that all but late adolescent pregnancies are at risk for a variety of reasons. A series of studies which pivot at least implicitly on what Zlatnick and Burmeister (1977) term 'gynecologic age' (Hacker, 1952; Pollakoff, 1958; Aznar and Bennett, 1961; Bochner, 1962; Mussio, 1962; Claman and Bell, 1964; etc), summarized in Table 2, have shown that toxemia is a substantial risk for adolescents who are pregnant. Rates of fifteen and eighteen percent were reported by Claman and Bell (1962) and by Pollakoff (1958), respectively. Low birth weight in the issue is also an established possibility. Mussio's (1962) small series of young mothers had an eight percent incidence of low birthweight; this finding is confirmed by the study of a large population conducted by Wiener and Milton (1970). On the other hand, Pollakoff's larger series of almost

TABLE 2

RISKS IN UNDERAGE AND OVERAGE PREGNANCIES

Investigation	Toxaemia	Fetal Abnormalities	Genetic Defects	Premature Delivery	Delivery Risk	Perinatal Mortality	Birth Defects
Claman & Bell (1964)	*				*		
Mussio (1962)	*			*	*		
Bochner (1962)	*				*		
Aznar & Bennett (1961)	*			*		*	
Hacker et alii. (1952)	*				*		∞
Milusky & Atkins (1975)		*	*				
Stine, Rider & Sweenye (1964)				*	*	*	
Selvin & Garfinkel (1972)				*			
Casazza (1972)					*		
Shaffner et alii (1977)						*	
Jekel et alii (1975)						*	
MacMahon & McKeown (1953)							*

three hundred young adolescents had a low birthweight rate of eighteen percent. Obviously, even the lower figure is one child in eleven.

The pregnancy itself is likely to be shortest among mothers age fifteen to nineteen years, according to data from the British 1970 perinatal survey (Chamberlain, 1975). The effects of prematurity are well known and have been reviewed by one of us (Jordan, 1976). It is clear that small preemies remain at substantial risk for survival. Perinatal mortality in Pollakoff's (1958) young adolescents was 5.9 percent, and the increase in mortality in the case studied by Jekel et al., (1975) was nine-fold. Survivors remain at risk for behavioral-developmental problems in subsequent years.

Within the adolescent group it is necessary to point to a subgroup, very young females who conceive. In 1973 the President's Commission on Mental Retardation reported that twelve thousand girls under fifteen years had given birth in 1971. Further, the number represented an increase of 23.6 percent over the rate in 1968. Within this group risk is high. Deliveries are prolonged (Bochner, 1962; Mussio, 1962; Claman and Bell, 1964) and prenatal care is reduced substantially. As with many pregnant girls they tend to be anemic and to have weight problems (Dickens, et al, 1973; Hacket et al., 1952). The research indicates that the critical age within adolescence is being under fifteen at the time of delivery (Aznar and Bennett, 1961, Wiener and Milton, 1970; Morris, Udry, and Chase, 1975). A British study of such girls in southeast London reported that they contacted the public antenatal service much later than other primiparae, thereby placing themselves and their babies at undue risk (McEwan, Owens, and Newton, 1974). Similar findings for forty-two states and the District of Columbia have been reported by Ventura (1977).

Selvin and Garfinkel's (1972) study of 1.5 million birth certificates shows that risk for low birthweight is higher than average in first-borns. This element additionally predisposes young women to risk. In addition the research of Jekel et al., (1975) reported a 27 percent incidence of birth weight under 2500g. for subsequent pregnancies in primiparas under eighteen years. Since they were 95 percent black this may well be a social as well as obstetrical risk.

For purposes of balance it is necessary to point out that study of adolescent pregnancy reveals scattered aspects which are benign. In offering this observation we do not wish to diminish the predominant problem of risk to mother and child. However, the literature reveals that young mothers are less likely than other mothers to need caesarean sections (Briggs, Herron, and Thompson, 1962; Claman and Bell, 1964). In addition, Hacker (1952) reports a reduced incidence of stillbirths and fetal anomalies. In this last regard it is helpful to recall the absence of lowered delivery age in Eastman's (1962) survey of antecedents to cerebral palsy in children.

Risk in Issue of Adolescent Pregnancies. For the purpose of this paper, which is a consideration of the effects of delivery age, it is appropriate to report the child-centered outcomes of adolescent pregnancy. We begin at the most general level by noting the high association of adolescent pregnancy and illegitimacy. Berkov and Shipley (1971) have observed that California birth certificates no longer indicate illegitimacy and infer a change in social values about the topic.

The degree of change may be seen by contrasting the finding just reported with mid-Nineteenth century data. Hebeler's (1847) report on the population of Prussia, for example, reports illegitimacy rates; Berlin's rate of 18.62 per cent was the highest, and the state of public morals is deduced accordingly. Illegitimacy has not really been associated uniquely with young mothers (Kinch et al., 1969; Pakter, 1961) despite conventional views on the matter. However, occurrence of illegitimacy in the younger mother may compound the pregnancy with social complications.

Hardy's (1966) paper on development of children of young mothers reports a mean Binet IQ at age four of 82. Similarly, the risk of neurological abnormality is raised, according to the President's Commission on Mental Retardation (1972) for children at age one year. Lobl, Welcher, and Mellits (1971) concluded from their study of nearly 4600 cases that children were at risk for mental retardation when the delivery age was under fifteen years.

In the research reported by Thompson, Cappleman, and Zeitschel (1979) offspring of mothers under eighteen years were not grossly different; however, they were rated less alert, less cuddly, and poorer in motor performance. Field's (1981) research on children of young black mothers revealed lower lengths, weights, and scores on selected subtests of the Denver Development Screening Test, e.g. adaptability and gross motor performance. However, intervention via bi-weekly home visits were effective with an experimental subgroup. Broman's (1981) data were drawn from the Collaborative Perinatal Study to age seven years, and mothers were in two young age groups, 12-15 and 16-17 years at delivery. At age four years Broman reported the children had lower IQ scores, less advanced motor development, and a higher frequency of deviant behavior. At age seven years probands' WISC IQ scores were marginally lower in the white sample ($p<.06$), with no difference within the black sample. Children of teen mothers were more likely to be rated deviant in behavior. SES effects were large. Data at child age eight years were reported for one hundred and fifty seven black children, half of whom were born to mothers at or under age fifteen years. Academic achievement and rated behavior of probands was not abnormal, but the offspring of young mothers were absent from school more frequently. The writers' research (Jordan, 1970) on development of issue of very young pregnancies at age three years suggests generally lowered child attainments. From these studies we conclude that adolescent pregnancy alone, but also in interaction with the social context, places the issue at risk for developmental failure.

The Social Context as a Risk Element: It is evident from the literature that intervention programs for adolescent mothers can help them and their babies. Research reported by Sarrel and Klerman (1968), and by Jekel, et al., (1972) shows that, e.g., toxemia and perinatal loss can be reduced by five-sixths by careful programming. While this is so, the prevailing reality is that most adolescent primiparae receive little or no antenatal care. We know from the 1975 Federal survey of Health attitudes of persons twelve to seventeen years of age (N=6768) that only a minority of young people are in the habit of consulting physicians.

Ten percent of Scottish girls concealed pregnancy until delivery, according to Gill, Illsley, and Koplik, (1970), and many receive little prenatal care.

The problem of early pregnancy among black girls is widespread (Murdock, 1968). Furstenburg (1970) describes discovery of conception leaving black teenagers "incredulous" in two out of every three instances. Their subsequent attitudes seemed similar to those of white working and middle-class girls. However, their circumstances were far different, and it is appropriate to recall the generally poor outlook for children of non-white, non-middle class mothers. With the exception of Quay (1981) few investigators have speculated on the psychosocial mechanisms of teenage pregnancies. Quay establishes five subgroups of girls, with passive dependent, subcultural, psychopathic, manipulative, and situational styles of behavior.

The National Academy of Sciences (Kessner et al., 1972) reported an infant mortality rate for black women in New York City nearly two and one-half times the rate for white women. Similarly, Morris, Udry, and Chase (1972) have reported the incidence of low birth weight in black women to be twice that among white women based on a study of nearly eight thousand deliveries. Grant and Herald(1972) have asserted that social factors such as race and social class are more important than delivery age for all but the youngest teenage mothers.

From adverse social conditions flows the entire nexus of health care and social influences, a complex which has spread across the generations (Fairweather and Illsley, 1960). In saying this we recognize with Quay (1981) that teenage pregnancies are expressions of psychodynamics and may be pathological (Kinch et al., 1969). This is not to say that problems of adjustment are always found in adolescent pregnancy. Werner and Smith (1979) found that locus of control in pregnant teenagers, the sense of whether one's life is self-or other-determined, was perceived as external to their lives. Gabrielson et al., (1970) have reported attempts at suicide in New Haven teenagers. These attempts came after delivery and were unsuccessful. They can be viewed as maladaptive efforts to cope with crisis rather than expressions of a fundamental maladjustment which also contributed to the conception. Obviously, a suicide-intent mother - for the purposes of this review - is not a child-centered mother. The teenage pregnancy may well be an obstetric success but evolve into compromised child care because the child-mother has not completed her own cycle of growth.

<u>The Outlook:</u> In the 1920's the rate of delivery to young unmarried women was studied and found high. Reporting on Madison and Dane County in Wisconsin, Young, Gillin, and Dedrick (1934) reported that twenty percent of deliveries were to unmarried girls under seventeen years. Recently, as we show in Table 1, the Census Bureau (1977) has summarized fertility rates since the 1970's by age. The Bureau reports that the rate for all girls ten to fourteen years, but especially white girls, has doubled in the intervening half century. At the moment, according to DeJong and Sell (1977), the rate of increase in the fifteen to nineteen-year group is less than in older women.

It is quite evident that teenage pregnancies will continue to occur. The President's Commission on Mental Retardation (1973) reported a 6.3

percent increase in pregnancies among the age group 15-19 years in 1971. Similarly, Gill, Illsley, and Koplik (1970) report a two-fold increase in births to British women under twenty years of age between 1955 and 1970. Stine, Rider, and Sweeney (1964) have expressed the problem as an incidence of nine hundred teenage pregnancies per annum in a city of just under one million persons. The overall effect is clear; teenage pregnancies occur and are on the rise, and they are a social problem of growing concern to society. To some extent, reports Brann (1979), and Klerman (1980), abortion reduces the problem; but we speculate at the risk of creating a subsequent problem. In the case of young black girls, a population at risk for early conception, it is clear that marriage in the immediate future is unlikely. Furstenburg (1970) has reported that, even when sought, marriage is often postponed for economic reasons. McAnarnay (1978) reported that half of all such marriages dissolve. Hardy et alll (1978) reported that only 35 percent of teen mothers finished high school.

Older Women: A brief return to Table I indicates that older women, ages forty to forty-nine years, are not absent from the list of those delivering live children between 1925 and 1975. However, older women are likely to put their children at risk by virtue of advanced age and postmaturity in reproductive efficiency. This degree of risk in children of older women explains the curvilinear relationship between delivery age and risk described by Pasamanick and Lilienfeld (1956), and by Davis and Potter (1957). They report an increased risk of producing a retarded child when maternal age at delivery is over twenty-nine years. Collaborative Perinatal Study data presented by Marmol, Scriggins and

Vollman (1969) support this observation, and an elegant mathematical model developed by Burch (1969) explicates the case using Down's syndrome, especially, for this discussion at delivery ages 40-44 years. Begab (1974) identified delivery ages above thirty-five years as increasing the risk of Down's syndrome in children. Such women he points out, delivered thirteen percent of all babies but fifty percent of all cases of Down's syndrome.

Risk In Issue of Over-Age Pregnancies. Advanced age in mothers is associated with perfectly normal children, of course, but there is a statistical association with a variety of conditions in addition to mental retardation. In the case of mothers over thirty-eight years of age the possibility of hare lip in the offspring is raised four-fold, in data analyzed by McMahon and McKeown (1953). In a 1972 analysis of over one million birth records in New York state by Selvin and Garfinkel being the sixth child or more in birth order, an event more probable in an older woman's pregnancies, is associated with a greater risk of premature delivery. Related to the period prior to delivery is the finding of Milusky and Atkins (1975). They reported the incidence of abnormality on fetuses of women over thirty-five years of age at one in ninety-six. In women over forty years the incidence of genetic disorders is highest, according to these investigators.

Policy. It is clear that current interest on age at delivery is concentrated on young mothers. Within that context the not unsubstantial corpus of literature on young pregnancies stresses the social and economic consequences for the mother. In our view, however current this formulation, it deals with a domain comparatively well studied. Appraising the literature the reader is struck by the relative absence of studies in which the child is the focus of attention; the studies of Oppel and Royston (1971), and Broman (1981) are exceptions. The result is that the current attention of policy-makers to the age of mothers is not matched with an opposite body of data which has been analyzed. For the policy maker the impact of delivery age needs to be understood within the complex of social and family variables which surround it. Equally, it is necessary to distinguish maternal age at delivery as an influence on children from the uncorrelated and correlated variables usually encountered. In this fairly conventional kind of matrix of social data policy needs to flow from variables which turn out to be important. The hazard is that it may be social class or ethnic group, not the nominal variable of interest, which is the salient function. Attention to the effect of maternal age on childrens' development in this paper is an attempt to clarify the elements of mother-and child-centered variables, with particular attention to the mental development of children born to young mothers.

PROBLEM

From our review of the literature we concluded that there were several aspects of the complex of maternal social and child variables which required attention. First, there was the scarcity of data over time on the development of children as a function of delivery age. Second, delivery age in relation to the mothers of emerging interest, *vide* Table 1, tends to be used within an attenuated span; that is, when young mothers are studied it is in isolation from older mothers, and comparative significance is unexamined. Third, maternal age at delivery tends to be pursued in isolation from other female traits, e.g. intelligence, values towards child rearing, and marital status. The absence of studies in which social and family data have been systematically gathered to appraise the effects of the independent variable, age at delivery, in primiparae is equally clear.

Many relevant studies have small samples and few have data at repeated intervals on the course of development. Essentially naive designs sometimes use samples with unidentified biases, as Vincent (1954) pointed out two decades ago. They also confound the effects of race, social class, and age at delivery. As a result, not the least aspect of the problem of adolescent and later pregnancies is the selection of a statistical model to apply to the data.

In the view of Werts and Linn (1970) problems of human development call for application of analytic schemes in which regression of variables is central. In their view, regression models can avoid the errors of specification, that is the failure to specify all of the variables which hypothetically influence a criterion.

The program of Bottenberg and Ward (1963) brought considerable utility to applied statistical analyses using regression. The technique is elucidated in the writings of McNeil, Kelly, and McNeil (1975), and Newman and Fraas (1978).

The major advantage of regression models is the opportunity to introduce variables of interest and, indeed, to create variables reflecting (e.g.) membership categories of theoretical interest, or as in Jordan (1971) vectors representing squared and cubed representations of non-linearity of regression. A third example is of particular importance in this study's report of applied statistical analysis, the question of interactions. In this situation, the effect of a variable depends on its own empirical role, and also depends, in the simplest case, on the role of another, interacting variable.

The fact that *primiparae*, or first-time mothers, are very young is an obvious fact of social significance. From the point of view of the Developmentalists the mothers' age at delivery, or conception - to sharpen the point, adds one more to the list of salient attributes of mothers, which includes intelligence, child-rearing values, and educational background, for example. The interaction of the new variable of interest with other variables obviously requires attention in models of development. Specific combinations are endless, even setting aside the question of non-linearity. Of particular relevance to this problem is the applicability of Koplyay's (1972) regression algorithm, with its attention to interactions. In this technique (whose procedures we describe shortly) the computer program erects interaction terms whose salience is assessed statistically. In unreported research we have found that the application of Koplyay's interaction regression to preschool development analyzed by multiple linear regression (Jordan, 1980a) is very helpful. Analyzing approximately two hundred sets of developmental data we have found that interaction analyses complement multiple regression analyses, while raising R^2 values. It is on the basis of this theoretical and empirical knowledge that we have concluded that interaction regression is particularly applicable to study of maternal age at delivery and its connection to other variables influencing child development.

PROCEDURE

The St. Louis Baby Study is an active, prospective study of newborns delivered in five St. Louis hospitals in the winter of 1966-67 (Jordan, 1980b). Within the data set are variables relevant to the question of effects of maternal attributes, especially age at delivery, on child outcomes. In Table 3 are listed the maternal variables of interest; they will be described in detail a little later. Equally, the data set contains measures of child development in two domains, intellectual and linguistic development. There are, as Table 2 also shows, thirteen measures, about equally divided between intellectual and linguistic growth. The child-age at which mother-and-child variables were gathered is shown in Table 3.

As a procedural note it is helpful to know that all probands were tested individually in their homes using examiners matched by race. Problems of tracing addresses within this non-captive cohort have been enormous. For that reason the cohort was split into two comparable groups (e.g. mean SES scores) at ages thirty-six and forty-two months. For any given chronological age the number of children for whom relevant data were available is the sum of the N for the two groups in that year. For example, the N for age three years is 756; at thirty-six months the number of children tested

TABLE 3

PREDICTOR AND CRITERION SERIES

		CRITERIA	
PREDICTORS	CHILD AGE	DOMAIN I INTELLECTUAL DEVELOPMENT	DOMAIN II LANGUAGE DEVELOPMENT
Marital Status Age Authoritarianism (AFI)	Birth		
Anxiety	6 Mos.		
	24 Mos.	(1) PAR *Intellectual* (2) PAR *Information* (3) PAR *Ideation* (4) PAR *Creativity*	(9) Mecham VLDS
IQ	30 Mos.		(10) Ammons FRPVT
FPS Subtests - *Conventional Social Role (CSR)* - *Denial Of Hostility (DH)* - *Basic Distrust (BD)* - *Moralistic Control (MC)*	36 Mos.	(5) PPVT (A)	(11) PAR *Communication*
	48 Mos.	(6) Preschool Inventory (Total) (7) Boehm T. of Concepts	(12) Preschool Inventory-*(AV)*
	60 Mos.	(8) WPPSI *Vocabulary*	(13) ITPA *Auditory Association*

was 380, and at forty-two months it was 376. For purposes of statistical analysis the birthday group was used; mothers took the Quick IQ test at child age thirty months.

The statistical model used to analyze the influence of a nexus of eight maternal traits on each of the thirteen criteria was regression analysis. The reasons were first, the overall relevance of regression analyses for development data (Werts and Linn, 1970), and, second, the pressing questions of how significant maternal traits interact with each other as influences on the two domains of child attainment.

Statistical Model: The statistical analysis selected was Koplyay's (1972) AID-4, an *interaction regression analysis*. In this technique the variance associated with a criterion measure is analyzed in terms of both the possible independent contribution of a given predictor, starting with the largest, and in terms of interactions between variables in a predictor set. Curvilinearity within complex regression models is also examined in this stepwise - like regression program. A nonsymmetrical branching process, based on variance analysis techniques, is used to subdivide the sample into a series of subgroups which maximize prediction of the dependent variable. The assumptions of linearity and additivity inherent in conventional regression techniques are not required.

The AID-4 interaction regression program operates by finding the predictor variable which when dichotomized will yield the lowest within group sum of squared deviations for the dependent variable. Essentially this is the dichotomization which accounts for more of the variance of the dependent variable, (i.e., has a larger correlation with the dependent variable) than any other dichotomization based on grouping the categories of a single predictor into two groups. Once this first dichotomization is complete, the AID-4 program searches for the next group with the now largest within group sum of squared deviations for the dependent variable. Searching and splitting continue so long as an eligible group has at least the specified minimum number of cases and a larger within group sum of squared deviations than a specified minimum proportion of the original sum of squared deviations. The Koplyay (1972) program requires prior specification of an acceptable increment in proportion of criterion variance, the number of cases minimally required to form a cell/term in the regression process, and also requires that variables be specified as monotonic or free-floating. In the analyses reported here an R^2 increment of .01, and a minimum cell size of 10 cases were stipluated.

The following measures were applied to mothers and probands at various ages from birth to child age sixty months (See Table 3):

Child Measures - *Intellectual* Domain:

1. Four subtests of the Preschool Attainment Record (Doll, 1966) were used at age 24 months; they were the *Informative, Ideation, Creativity,* subtests. The first three combined to yield an *Intellectual* domain score. This scale (PAR) is an extension of the Vineland Social Maternity Scale, and uses the mother as informant. It is important to note that we operationalized some questions and had children perform such tasks as bouncing a ball.
2. Form A of Dunn's Peabody Picture Vocabulary Scale (1965) employed at 36 months was a criterion.
3. The Preschool Inventory, developed by Caldwell (1970) was administered at 48 months.

4. The Vocabulary subtest of the Wechsler Preschool and Primary Scale of Intelligence (1968) was administered at 60 months and was used in this analysis.

<u>Child Measures - *Language* Domain:</u>

5. Mecham's Verbal Language Development Scale (1959) was administered using the mothers as informants at 24 months.

6. The Communication subtest of the PAR scale was administered to mothers at ages 36 months.

7. The *Association Vocabulary* subtest scores from the 48 month administrations of the Preschool Inventory were employed.

8. The *Auditory Associative* subtest of Kirk and McCarthy's (1961) Illinois Test of Psycholinguistic Abilities was administered at 60 months and used as a criterion.

<u>Maternal Measures:</u>

9. Authoritarianism in child rearing ideology was measured when mothers were at the end of confinement by using the *Authoritarian Family Ideology* Scale (AFI_{68}). This test has high reliability and is one of the set of measures in Ernhart and Loevinger's (1969) *Family Problems Scale* (FPS).

10. Maternal IQ was obtained by administering the Ammons' Quick Test (1962) at child age thirty months.

11-14. Also from the FPS, but used at 36 months, were the following subscales, employing the names arrived at by the authors: *Conventional Social Role (CSR), Denial of Hostility (DH), Basic Distrust (BD), and Moralistic Control (MC).*

15. Anxiety was measured at child age six months by administering

the Bendig (1956) short form of the Taylor Manifest Anxiety Scale individually to mothers.

Data Set: For the predictor set and each criterion a corpus of information at basic data-time points was created within the computer. An important aspect of the data set was its dynamic nature. That is, the predictor set and criterion series changed from child age to age. Looking at Table 3 we see that four of the eight predictors preceded in time the thirteen criteria. By age 36 months the predictor set incorporated the remaining predictors. This meant that criteria (5)-(8), (11)-(13), but not (1)-(4), or (9)-(10), could be studied by use of all eight predictors. In short, the predictor set was used prospectively and grew as the children grew. Alternatives to this approach are first, to use an abbreviated predictor set, one antecedent to all criteria; on the other hand, one could apply predictors without regard to the dynamics of their acquisition over four years. Such a step would be psychometrically feasible; but it would violate the integrity of the prospective longitudinal approach and it would constitute a degree of postdiction. Prospective study is difficult, and its virtues lie in part in the integrity of the resulting data set. Setting that virtue aside vitiates the painful acquisition of prospective data and violates the logic of using the prospective approach to begin with. The price to be paid is that not all predictors can be applied simultaneously to all criteria. In this study the predictive data set for any domain grows, as the children do, and follows the enlarging picture of childhood with parallel enlargement of the data subset of predictors.

Hypotheses: The purpose of the investigation was to assess the influence of maternal age at delivery and the associated behavioral traits listed in Table 3 on the intellectual and linguistic development of children to age five years. Accordingly, we hypothesized that:

1. The developmental status of children as measured by standard tests would be (a) influenced primarily by maternal age, with (b) especial reference to delivery age of sixteen years and under.
2. The other maternal variables listed, in the order IQ, authoritarian family ideology, (from the *FPS* scale) marital status, and other *FPS* scales Basic Distrust, Conventional Social Role, Moralistic Control, and Conventional Social Role, anxiety, and marital status, would all be significant influences.
3. The interactions between these variables would be complex, and emphasizing, for the most part the role of maternal age.
4. Regression effects would be linear, with nonlinear terms in the regression equations in a smaller number of instances.
5. The models created for intellectual and linguistic criteria at any given age would show comparable configurations of antecedent maternal variables.

RESULTS

In this section we present the results of applying the predictor set of maternal age and related behavioral traits in a dynamic fashion to thirteen criterion measures. Eight of the measures are in the intellectual domain and five are in the linguistic domain.

Subjects: The analysis reported here is, in fact, thirteen discrete multivariate analyses using a common data base. The subjects reported in Table 4

TABLE 4

DESCRIPTIVE DATA ON THIRTEEN REGRESSION GROUPS

ble	Range of Means	Grand Mean	Range of S.D.'s
ctors:			
rital Status (%M)	87 - 90		
livery Age	25.63 - 26.39	25.83	6.25 - 6.71
thoritarianism	24.66 - 26.07	25.68	7.62 - 8.16
xiety	5.16 - 5.97	5.85	4.06 - 4.34
ick Test IQ		92	
- CSR	4.87 - 4.96	4.92	1.89 - 1.91
- DH	7.19 - 7.36	7.27	2.30 - 3.99
- BD	6.90 - 7.04	7.01	2.08 - 4.01
- MC	8.61 - 8.64	8.62	1.90 - 2.13

	N	Mean	S.D.	$\bar{M}$ SES	% Male	% Black
ia:						
- *Intellectual*	429	18.13	3.72	53.34	51	36
- *Information*	429	6.48	1.78	53.34	51	36
- *Ideation*	429	5.45	3.38	53.34	51	36
- *Creativity*	429	6.24	1.49	53.34	51	36
	428	19.65	4.75	53.31	51	36
s FRPVT	124	9.49	2.68	50.89	54	21
(A)	142	26.24	11.07	51.11	53	24
Communication	147	7.30	1.90	51.41	53	24
hool Inventory	141	34.36	12.00	51.37	53	25
Concepts T.	141	14.45	5.05	51.41	53	24
Assoc. Vocab.	132	5.87	2.80	50.48	53	25
Vocabulary	140	14.29	5.31	51.05	54	23
Aud. Assoc.	132	16.17	5.15	50.83	54	23

vary as a consequence of progressive extension of the elements in the predictor set and because the criterion changes for each analysis. The entire set of subjects have in common their membership in the same birth cohort. In contrast, a source of variation is that a child tested at one birthday might, theoretically, have been untested at another. The randomness of this event has been high, however. In Table 4 we see the mean and great mean values of sample characteristics. For the first five criteria listed in Table 4 there were over four-hundred subjects for each analysis. The number declined subsequently, due to inclusion of the 30-month Quick Test as a measure of mothers' verbal intelligence. In the remaining analyses the number of cases averages around one-hundred and forty. The sharp decline is a combination of the hazard of proposing testing to mothers as well as to their children. Splitting the sample to increase the intensity of tracing families, a step taken at three years, further restricted the size of the sample.

The means given for the child development criteria in Table 4 are very close to those we have reported elsewhere for the full 1966-67 birth cohort (Jordan, 1974). The social class range is broad and the average child in the group reported here is blue-collar middle-class. Attention is called to the differential rate of attrition by race in Table 4. This differential is a potential source of bias in this longitudinal data set, but conveys the hazards of prospective study using an inner city population.

At age two years the average score given in Table 6 for Mecham's Verbal Language Development Scale is 19.65. According to Mecham's norms (1958) this is just under 2.4 years language age. The PPVT (A) mean score of 26.24 yields an IQ of 95 at age three years. At age five years the WPPSI *Vocabulary* mean raw score of 14.29 falls, according to Wechsler's norms between test age 4 3/4 years and five years. This is quite similar to the mean IQ of 95 reported at age three years. Accordingly, the subjects are quite representative of youngsters in the preschool years, developmentally speaking.

Regression Models: Since the basic method of analysis is regression it seems appropriate to comment on the regression models generated before looking at variables discarded and variables found to have predictive significance. A basic aspect of this analysis has been concurrent extension of the predictor set as the criterion series evolved from ages two to five years.

In Table 5 there are relevant data; there, we see four important outcomes of the regression analyses. First, the bottom row shows the R^2 values of the models indicated in the columns. From left to right the R^2 values increase as a function of increasing age of the children; however, that observation can be expressed more analytically by formulating it as a function of the predictor series increasing concomitant to the maturation of the children. The R^2 values rise from statistically significant, but low, levels of .02 and .04 for criteria assessed at age two years to .44 and .41 for criteria at ages three and four years. Second, by domain, that is, intellectual and linguistic criteria sets, the R^2 values of the models are generally comparable. The third item which inspection of Table 5 reveals is the role of maternal child rearing values as contributors to the interaction regression models. When these predictive elements were added to the models at child age three years the R^2 values rose considerably. Fourth, some of the predictors listed in Table 5 occur more than once in regression models. For example, delivery age occurs twice in the simple two-variable model of the first criterion, 24 month PAR *Intellectual* scores. The second occurrence of a variable indicates curvilinearity of regression. In a few instances, e.g. in the regression model of the scores on the Ammons Full Range Picture Vocabulary Test at thirty months there is a polynomial, fourth power, degree of non-linearity. *Interactions.* Apart from the basic relevance of regression models for developmental data, a case well presented by Werts and Linn (1970), the approach has another advantage. It is the opportunity to use statistical models which replicate the reality of interactions among variables. The interaction version of regression analysis, developed by Koplyay (1972) and used in this inquiry, permits exploration of combinations of variables as independent terms in regression equations. In addition, the interactions can be supplemented by nonlinear expressions, and indeed, combined with them on occasion.

In Table 6 are the regression models used to test the influence of maternal traits, discretely, and combined as interaction terms, on thirteen criteria of child development to age five years. The models are arranged, like Table 5, so that the criteria range from ages two to five years; it will be recalled that the length of the predictor sets extends in parallel fashion, also in Table 5. In Table 6 the models increase in complexity as the predictor series lengthens, primarily through the apparently greater relevance of the later-added predictors; secondarily, this occurs through the greater possibility of interaction terms. Age of mothers at delivery *(MA)*, the variable of greatest interest in this inquiry, is present quite frequently. On the other hand, some terms are not, such as marital status, IQ and the *Basic Distrust* subscale of the Family Problems Scale.

Predictors: The variable of prime interest in the predictor set is maternal age at delivery. In Table 5 delivery age occurs in twelve of the thirteen models. Only the 24-month Verbal Language Development Scale (Mecham, 1958) omits this variable of prime interest. Further, when we examine the ordinal

PREDICTORS IN ORDER OF STATISTICAL SIGNIFICANCE IN THIRTEEN REGRESSION MODELS

Predictors		Criteria												
		24 Months					30 Months	36 Months		48 Months			60 Months	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Child Age	Maternal Variables	PAR *Intellectual*	PAR *Information*	PAR *Ideation*	PAR *Creativity*	Mecham VLDS	FRPVT	PPVT (A)	PAR *Communication*	Preschool Inventory	Boehm T. of Concepts	PI-*Assoc. Vocabulary*	WPPSI *Vocabulary*	ITPA *Audit. Association*
Birth	Marital Status (M)			2										
Birth	Delivery Age (DA)	1,3	3,4,5	1	2		1,3,4,5	3,5,6	3,5	2,4	2,3,9	2,3	2,8	4,6
	FPS-Authoritarianism(AFI)	2	1		1					6	5,7		3	2
6 Months	Anxiety (Anx.)		2			1	2	2	2	5		4,6,7	4,9	3,8
30 Months	IQ						6			3	4		7	
36 Months	FPS - *CSR*										8			
36 Months	- *DH*							4	1,4	8	6	1	5	7,9
36 Months	- *BD*							1		1	1		1	1
36 Months	- *MC*									7		5	6	5
R^2		.04	.08	.05	.06	.02	.28	.44	.37	.42	.32	.41	.23	.38

TABLE 6

REGRESSION MODELS

Y_{PAR} *Intellectual* = (DA) + (AFI) + (DA^2) + (DA * AFI) + (DA * AFI^2). R^2 = .04

Y_{PAR} *Information* = (AFI) + (Anx) + (DA) + (AFI * Anx) + (AFI * Anx * DA) + (AFI * Anx * DA^2).
+ (AFI * Anx * DA^3). R^2 = .08

Y_{PAR} *Ideation* = (DA) + (H) + (DA * H). R^2 = .05

Y_{PAR} *Creativity* = (AFI) + (DA) + (AFI * DA). R^2 = .06

Y_{VLDS} = (Anx) R^2 = .02

Y_{FRPVT} = (DA) + (Anx) + (IQ) + (DA * Anx) + (DA * Anx * IQ) + (DA^2 * Anx) + (DA^2) + (DA^3).
R^2 = .34

Y_{PPVT} (A) = (BD) + (DA) + (DH) + (BD * Anx) + (BD * Anx * DA) plus (BD * Anx * DH)
+ (DD * Anx * DH * DA). R^2 = .44

Y_{PAR} Communication = (BD) + (DH) + (DA) + (Anx) + (MC) + (AFI) + (BD * DH) + (BD * DH * DA)
+ (BD * DH * DA * Anx) + (BD * DH * DA * Anx * MC) + (BD^2 * DH * Anx * DA * MC) + (DH^2)
+ (DH * AFI) + (BD * DA * AFI). R^2 = .37

$Y_{Preschool}$ Inventory = (DH) + (BD) + (DA) + (AFI) + (IQ) + (Anx) + (MC) + (BD * DA) + (DH * IQ) + (DH * IQ * An
+ (DH * IQ * Anx * MC) + (DH * DA) + (DH * DA * Anx) + (DH * DA^2) + (DH * DA * AFI)
+ (DH^2 * DA^2 * AFI). R^2 = .42

$Y_{Test\ of\ Basic\ Concepts}$ = (DD) + (DA) + (AFI) + (CSR) + (IQ) + (DD * DA) + (BD * DA * AFI) + (BD * DA * AFI * CSR)
+ (BD * DA * IQ) + (BD * DA * IQ * DH) + (BD * DA^2). R^2 = .37

Y_{PI} - *Assoc. Vocab.* = (DD) + (DA) + (MC) + (Anx) + (DH * DA) + (BD * DA^2) + (BD + DA^2 + MC) + (BD * Anx)
+ (BD + Anx^2) + (DD * Anx^3). R^2 = .41

Y_{WPPSI} *Vocab.* = (DD) + (DA) + (AFI) + (Anx) + (IQ) + (MC) + (BD * DA) + (BD * AFI * DA) + (BD^2 * DA * AF
+ (BD * DA * Anx) + (BD * DA * Anx * MC) + (BD * IQ) + (BD * IQ * Anx). R^2 = .28

Y_{ITPA} - *Aud. Assoc.* = (BD) + (AFI) + (DA) + (Anx) + (MC) + (DH) + (BD * AFI) + (BD * AFI * DA) + (BD * AFI * A
+ (BD * AFI * Anx * DA) + (BD^2 * AFI * Anx) + (BD^2 * AFI * DH * Anx) + (BD * MC)
+ BD * MC * Anx). R^2 = .38

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role of delivery age as a source of variance within regression models we see that it occurs in first or second place in eight of the thirteen models. What Table 5 does not show, but which can be extracted from the computer printout generated by the Koplyay regression program is the level at which maternal age exercises its role. Within the total data set are mothers ages from thirteen years to forty-two years. This permits consideration of the levels of maternal age used in the regression analysis. Table 7 summarizes the eight instances of the role of delivery age when it occurs as the prime or second most important variable in the regression analyses. Here, we see the levels of delivery age which the regression analysis created incidental to developing complex regression models. The statistically significant, empirically derived, levels of maternal age at delivery, when a significant influence on child development, are shown. In Table 7 we see that maternal age played a significant role in six of eight criteria in the intellectual domain, and in two of five variables in the intellectual domain. The greater influence, this suggests, is the intellectual domain. In making this observation, however, it is necessary to observe that the grouping of criteria into two sets may overstate the relatively narrow distance between them. Vocabulary as used in the WPPSI is viewed here as intellectual, while vocabulary used in a different way in the ITPA is considered linguistic. We feel use of two domains as a superordinate concept to order the criterion series is helpful. We wish to point out our use is to make a *distinction* rather than to contrive a difference.

With regard to the levels of mother's age at delivery, Table 7 shows that virtually all of the groupings of ages were above twenty years. In the eight instances given in the Table no pattern can be discerned by primary or secondary role as a source of variance, nor by criterion domain and child age. In one instance, 24-month PAR *Ideation*, the split in delivery age occurs in the teens. A significant source of variance was found when 429 mothers' ages were split into a group of twenty-eight fifteen years of age and under, and into a second, larger group, the remaining 401 mothers age seventeen to forty-two years at the time of delivery. At age four years scores on the *Associative Vocabulary* subtest of the Preschool Inventory acquired a significant source of variance when grouped first into those under twenty-one years and into those aged twenty-two to forty-two years. Two other splits, those for the 30-month Full Range Picture Vocabulary Test, and for the 60-month WPPSI *Vocabulary* scores split at <23 : 24> years, at <25 :>26 years.

More interestingly, four of the seven analyses abstracted in Table 7 were instances of age groupings in the later twenties and middle thirties of delivery age. Two 48-month criteria, scores on the Boehm Concepts test, and on the Preschool Inventory, yielded significant sources of variance ascribed to delivery age when split into two groups at <28 : 29> years. Two 24-month criteria, scores on the PAR *Creativity* and *Intellectual* subtests, yielded significant amounts of variance when grouped at <34 : 35> years.

A significant overall finding is the range of these age groupings, and their tendency to occur in a manner emphasizing older than younger delivery ages as the locus of age effects.

An outcome of interest is the finding that some maternal traits hypothesized as relevant had little or no discernible role as sources of criterion variance. Marital status at the time of birth is an example. It was used in only one regression model, that was for 24-month PAR *Ideation* scores. The FPS subscale *Basic Distrust* was not used at all, and another subscale, *Conventional Social Role* was only used once, in the model for criterion, 48 month Boehm Test of Basic Concepts. Interestingly, in view of the psycholinguistic nature of the criterion series, maternal IQ (verbal) as measured by the Ammons Quick Test had little role in the regression models. In the four instances when it appeared, 30-month Ammons FRPVT, 48-month Preschool Inventory, and Boehm Concepts test, 60-month WPPSI Vocabulary, the contribution was not as a prime source of variance. In three of the four instances Quick Test IQ's were associated with lowered criterion scores at ages three and four years.

The predictors most frequently found significant were the variable of prime interest, maternal age at delivery of the proband (DA), and the FPS scale Denial of Hostility (DH). Anxiety, measured at six months was the next most frequently used predictor in the thirteen regression models. Of forty-four variables excluding interaction terms in the thirteen regression models seventeen are the predictors, delivery age and anxiety. Authoritarianism and denial of hostility account for another fourteen of the forty-four variables. For all thirteen criteria of psycholinguistic attainment to age five years the predictors delivery age, anxiety, authoritarianism, and denial of hostility account for thirty-one of the forty-four significant variables. Excluding combinations used as interaction terms these four variables out of nine in the predictor set examined account for three-quarters of the significant elements in the predictor sets.

DISCUSSION

Data. It is now appropriate to dilate on the significance of the findings just reported, beginning with the variable of prime interest, age of the mother at the time of delivery of her child. Current interest, plus a small body of research (Crumidy and Jacobziner, 1966; Oppel and Royston, 1971; Cutright, 1973) suggest that being a young mother has great significance for child development. Crumidy and Jacobziner (1966) have pointed out that young mothers tend to see their infants as dolls which are alive. The essence of dolls, we hazard, is that they are not creatures in their own right but essentially sources of entertainment for the person playing at being a mother. Cutright (1973) has pointed to the hazard of inhibited growth into mature womanhood for the girl plus the probability of having more children than average due to an early start. We note the findings of Oppel and Royston (1971). They report that children aged 6-8 years and 8-10 years were at risk when their mothers were under eighteen years delivery age. The risk was evident in reduced scores on measures of height, weight, IQ (Stanford-Binet, but not WISC), and reading; they were more likely to present behavior problems. More recent research by Belmont, Cohen, Dryfoos, Stein, and Zayac (1981) used data from the Collaborative Perinatal Study, and found no consistent evidence of adverse effects of maternal immaturity. Associated social disadvantages were the context within which lowered maternal age produced results.

Our findings, which treat a younger group of children, do not support the ideas that being a young mother, low delivery age, leads to discriminable performance on thirteen developmental criteria between two and five years of age. If anything, a case can be made for delivery age being a source of influence, but largely in the opposite direction.

At this point it is helpful to recall that our chosen statistical tool for analysis sets up mothers' delivery age so that the role of this predictor is established empirically by the process of maximizing the accounts of criterion variance. The data constitute the basis for the grouping of variables in interaction regression analysis. Delivery age arranged itself into groups separated, for the most part, at above average levels and below those levels as an empirical, inductive outcome. Our data, summarized in Table 7 shows that several of eight significant age effects arose when mothers were around thirty years of age. This suggests that a complex of factors centered around maturity as a mother may be a topic for consideration. Maturity, in this case, meaning above average age, rather than being merely beyond adolescence.

In the matter of related variables it is evident (see Table 5) that the values which mothers hold at the time of delivery, and thereby constituting base-line data for subsequent events, influence the behavior of their children. In the earliest years the Authoritarian Family Ideology scale and, in particular, the 1968 version of the Ernhart and Loevinger AFI '65 scale, accounted for criterion variance. This measure, which we have previously reported (Jordan, 1968), expresses a dimension of authoritarianism-liberalism, one in which, for example children's needs may or may not come before those of their parents. The authors of the Family Problems Scale say that:

> "A woman high on AFI apparently applies a punishment theory of learning to problems of character development and child rearing; the child cannot be trusted to choose his own foods, toilet train himself, or grow out of childish misdemeanors. Controls

are externally imposed rather than coming from within."
(Ernhart and Loevinger, 1969, p. 36).

Previous investigations using this same data bank have found significant curvilinear relationships between AFI values and the mother's age when age interacts with socioeconomic status (Jordan, 1968) and when age interacts with the mother's manifest anxiety level (Chovanec, 1968). The phenomenon identified by Jordan was that as age increased among low SES mothers authoritarianism decreased until ages 24-28 and then began to rise. But, at the high SES levels authoritarianism steadily decreased as age increased. The Chovanec findings were similar; that is, for highly anxious mothers, as age increased, authoritarian attitudes decreased until ages 30-33 when authoritarianism began to rise with further advancing age. However, for low anxious mothers authoritarian child rearing values steadily decreased as age increased. Both these reports support the maternal maturity hypothesis as a significant factor to be considered in studying child development. And, concurrently the age period when child rearing atittudes seem to change for the above mentioned segments of the population of mothers (i.e., the low SES mothers or the high anxious mothers) is in their later 20's and early 30's. From its role in the present investigation we conclude that the AFI values of mothers extend their influence beyond the perinatal phase of child development. The relationship is that authoritarianism is significantly and negatively correlated with high child criterion scores. In Table 8 we see the *r*'s for the AFI measure against the criteria which are direct measures of child performance e.g. 30 months and later. The *r*'s are relatively homogeneous and the levels of significance vary as a consequence of the differing degrees of freedom associated with the size of the sample used for each multivariate analysis.

TABLE 7

LEVELS OF MATERNAL AGE AND R^2 CONTRIBUTION WHEN HIGHLY SIGNIFICANT IN REGRESSION ANALYSES

Criterion	Domain		Age Levels and Ascribed Variance			
	(Intell.)	(Linguist.)	Primary Source	R^2 Contribution	Secondary Source	R^2 Contribution
24 Mos. PAR Intellectual	(I)		13-34 yrs. : 35-42 yrs.	.18		
24 Mos. PAR Ideation	(I)		13-15 yrs. : 17-42 yrs.	.01		
24 Mos. PAR Creativity	(I)				15-34 yrs. : 35-42 yrs.	
30 Mos. Ammons FRPVT		(L)	15-23 yrs. : 24-42 yrs.	.11		
48 Mos. Preschl. Inventory	(I)				16,20-28 yrs. : 29-37,40.42 yrs.	
48 Mos. Boehm Concepts	(I)				16,19-28 yrs. : 29-40.43 yrs.	
48 Mos. PI-Assoc. Vocab.		(L)			16,20,21 yrs. : 22-37,40.42 yrs	
60 Mos. WPPSI Vocab.	(I)				16,19-25 yrs. : 26-37,40-42 yrs.	

TABLE 8

CORRELATIONS OF AFI_{68}, AND *DENIAL OF HOSTILITY* SCALES AND PERFORMANCE CRITERIA

		Criteria							
		30 Months	36 Months		48 Months			60 Months	
Predictor		FRPVT	PPVT (A)	PAR *Communication*	Preschool Inventory	Boehm T. of Concepts	PI - *Assoc. Vocabulary*	WPPSI *Vocabulary*	ITPA *Auditory Association*
AFI_{68}	r	-.36***	-.40***	-.37***	-.40***	-.35***	-.37***	-.21*	-.37***
Denial of Hostility	r	-	-.43***	-.44***	-.49***	-.39***	-.44***	-.29**	-.46***
	df	2/122	2/140	2/145	2/139	2/139	2/376	2/138	2/389

*=<.01
**=<.001
***=<.0001

Among the other subscales of the Family Problems Scale is *Denial of Hostility*. A prominent role as a source of variance began to appear with the 36-month criteria; DH was evident as a prime source of variance in five of the last seven criteria. In discussing this FPS subscale Ernhart and Loevinger observe that it,

> "deals with harmony and discord in the home. It appears to measure a Pollyanna-like denial of problems involving hostility and related negative affects such as jealousy" (1969, p. 38).

In the analyses reported here low scores on this predictor were associated with high scores on several developmental criteria. The relevant criteria are those concurrent and subsequent to administration of the FPS scales at child age three years. Consulting Table 8 once more shows the highly significant relationship obtaining between the DH subscale and criterion scores. All are highly significant and at relatively consistent levels.

In connection with maternal values and increased delivery age there is the question of the relevance of the number of children a woman has delivered and its impact on child rearing attitudes in general and child attainment in particular cited research (Chovanec, 1968; Jordan, 1968) suggests that values change with age; but advancing age also tends to mean that women have more children. The number of children a woman has borne is a variable of interest in child development. It arises as the topic of size of a family or sibship, and it arises in the literature on birth order in children. In our data set it exists for probands as birth order information. Such material is both child-and mother-significant. Our intent has been to deal solely with mother-significant traits, but we wish to note that we are aware of the double - or ambiguous meaning of birth order data. Given an intent to deal less narrowly with maternal traits we advocate inclusion of birth-order data in order to consider the effects of multiparous states in women. Conceivably, the process of maturing as a woman, and the parallel or interacting state of acquiring more children, can contribute to our knowledge of how maternal traits influence childrens' development. Authoritarianism (AFI) certainly alters over time.

The Bendig version of the Taylor Anxiety Scale, administered six months post partum was of some influence throughout the criterion series. However, in only one instance, that of the mothers' reports of language attainment through Mecham's (1959) interview instrument, did the anxiety scale play a role. In that regression analysis, in which neither IQ nor the four FPS scales were included as predictors, anxiety scores were the sole source of variance. Low anxiety scores were associated with slightly higher language scores. A more explicit association, but still within a weak model ($R^2=.02$) was that between highest anxiety scores and depressed linguistic scores.

The variables emerging as of little utility as ascribable sources of variance are marital status, the FPS scale *Moralistic Control (MC)*, and IQ. In the case of the MC scale, described by Ernhart and Loevinger as "...concerned with impulse control..a laisser faire involvement," a low and weak relationship within an interaction term was evident at four and five years. In the case of IQ it is interesting to note that the Quick Test used to generate a maternal IQ is related to the criterion Full Range Picture Vocabulary criterion used at 30 months of age. This predictor was barely evident (see Table 5) in the regression analyses.

We turn next to the regression models of development generated by means of the predictor series and data set gathered by prospective means. There are statistical models in which the locus of attention tends to be the F-statistic and its attendant probability. In regression analyses the amount of criterion variance attributable to the predictor remains in prime focus. The low R^2 values listed in Table 6 are the product of models based on reasonably lengthy predictor sets. Even when the regression interactions are complex, as in models 6-8, 12, and 13, the R^2 values are low. Were the object of the exercise to raise R^2 values to maximum levels it would have been quite simple to incorporate a perinatal social class score based on occupation, education, and income source, already in the data archive into the predictor set. The effect of that however, in an interaction model would have been to blur the intended concentration on the complex of maternal age at delivery and associated maternal traits. Under the circumstances the attenuated R^2 values are correlates of the unclouded picture of maternal traits and their influence on development of children over several years.

Our investigation has examined the influence of a wide span maternal age at delivery plus selected other maternal traits measured post-natally on thirteen psycholinguistic criteria from two to five years. Among our hypotheses has been the nature of interactions and a prime role among them for delivery age. In the first four analyses of the scores from the Preschool Attainment Record there are many interactions (see Table 6). They illustrate several complex phenomena only really analyzable in regression analyses. One is the matter of complex interaction terms. In the PAR *Information* model in Table 6 there is the three-term interaction (AFI * Anx * DA). The same equation also contains a non-linear term within an interaction; it is (AFI * Anx * DA^3). In the first set of analyses, the four PAR criteria, the variable of prime interest, delivery age (DA) is prominent in interaction terms. It is also prominent in the regression models presented for the subsequent developmental criteria. Addition of the Family Problems Scale predictors attenuates the role of delivery age, but does not end it. In general, the models used to account for the variance of the intellectual domain criteria are comparable to those created for the linguistic criteria in the selection of predictors and in their combinations as interaction terms.

Finally, we return to the methodological question which is at the heart of this study, the question of the applicability of the Koplyay *interaction regression* model to study of human problems in development.

It is evident that there is a class of problems in which the contribution of complex terms in regression models cannot be set aside. The terms themselves may be composed of two, three, or more variables. The form of the variables includes the possibility of cubed and quadratic polynomials, especially when anthropometric data are the subject of analysis (Joossens and Brems-Heyns, 1975). However, only the actual analysis, rather than the nature of the problem being addressed, can show how beneficial the interactions are as explanations of the variance surrounding the criterion.

The range of topics to which interaction analysis may be applied is wide. Problems suitable for analysis are those in which the range of possible interactions is wide, requiring a parsimonious but not overly-economic processing of data. By this we mean that the combination of variables may be large, and no combinations should be ruled out in advance, except for the most complex which would defy interpretation anyway. The variables themselves may be discrete or continuous. In the latter case the Koplyay program accummulates a wide range of levels of a variable, e.g. our spread of maternal ages at delivery ranging from thirteen to forty two years. Put more broadly, the application of interaction regression is not a matter of substantive area; it is more a question of the hypothetical importance of interactions among variables, given the opportunity to test the statistical significance of fine levels of scores within variables. Finally, the technique is applicable to situations where the investigator is willing to let the data sort themselves out, in the best sense. The investigator specifies certain arbitrary but not irrational constraints and then allows the data to aggregate and cluster on a mathematical basis.

Policy. In our view the importance of age at delivery, especially in young primiparae, lies in its putative implications for social policy. To some extent, society in any generation has little control over the age at which females can conceive. Menarche occurs much sooner in the late twentieth century than it did in previous generations. In females in the 1966-67 cohort of the St. Louis Baby Study menarche appeared, on the average, at 146.40 months - 12.20 years. However, Nature merely sets the stage, and it is the circumstances of decisions made by the young within specific social contexts which lead to early motherhood. Quay (1981) has shown that adolescent pregnancy may express several personal themes. Some lead to termination of pregnancy and others are followed by parenthood. In the context of this report motherhood occurs within social settings with meanings for both mother and child. The challenge is to form policies for obstetric care and to raise the educational level which will mold care-giving. Appropriate social policy will assist mother and child through attention to nutrition in early pregnancy followed by sustained attention to social stimulation of the young child. In the case of the latter action should be attention to stimulating the child through, for example, a program of home visitation in which the commonly held view of immutability of development is challenged. This policy item accepts that lower-class orientation to human growth is quite conservative, and that young, poor mothers can be taught that their action can stimulate cognitive growth. Simultaneously, we need to stimulate mothers to continue their own personal and academic growth. Avoidance of subsequent and immediate pregnancies, and continuation of education and acquisition of skills, are elements of social policy for young mothers. In the case of the policy elements aimed at mother and child the social matrix of the young pregnancy, rather than the early gynecological age of the mother, is the element with the most social implications. For young non-mothers we join Morris (1981) in asserting that the best social policy is that which defers the first birth.

SUMMARY

We summarize the findings of this inquiry into application of interaction regression analysis to the effects of delivery age and related traits on development of children in the format of the hypotheses which structured the investigation.

1. The range of ages at delivery is wide; and the salience of this variable and others of hypothetical importance makes interaction regression a useful technique to apply to the data.
2. Among a complex of maternal traits age of delivery is a salient influence on child attainment on psycholinguistic criteria between ages two and five. However, we do not report that teenage is a significant range among delivery ages. In some respects the opposite may be more likely, with delivery age in the twenties, and especially the late twenties, being an empirically generated formulation of greater significance.
3. Among the other traits in the predictor set the values examined by means of the Family Problems scales were significant. In particular, the Denial of Hostility and Basic Distrust scales. Anxiety was also a significant predictor. The scales Moralistic Control, and Conventional Social Role were not significant. Verbal IQ's of mothers and their marital status at delivery were not significant.
4. Interactions of variables within regression models were complex. High order interactions and non-linear terms were observed. No uniformly influential element in the interaction terms was observed, although the significant predictors were evident as discrete elements and as elements of interaction terms.

5. Regression effects were non-linear, for the most part. In selected instances even complex regression models yielded low accounts of criterion variance.
6. The thirteen criteria employed at ages two to five years were classed as *intellectual* and *linguistic*. In both instances the patterns of variables identified as significant were comparable. The domains were not, however, incompatible as areas of child development.
7. Suggestions for public policy arising from the review of the literature and from original data of the St. Louis Baby Study (Jordan, 1982) have been presented. Attention to nutrition in early pregnancy and attention to educational and social stimulation for young mothers and their children are advocated.

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A NOTE ON PROPORTIONAL CELL FREQUENCIES IN A TWO-WAY CLASSIFICATION

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Abstract - A proportional, but non-equal two-way data set is analyzed, comparing the full rank model solution to the fitting constants, hierarchical model and unadjusted main effects solutions. The latter three models yield identical results; the full rank model, yielding different results, is shown to be testing different main effect hypotheses.

Several writers have explored different approaches to the analysis of disproportionate cell frequencies data in a two-way (or higher) layout. One such solution, the "full rank model" solution, as described by Timm and Carlson (1975) has been purported to be the "best" solution to the traditional two-way design; Overall, Spiegel and Cohen (1975) appear to concur in this position. One rather interesting circumstance is that, for proportional, but non-equal cell entries, the full rank model solution fails to yield an additive solution. While this problem has been pointed out before (see Overall and Spiegel, 1969; also, Williams 1977), a simple example together with the sums of squares should be helpful.

Consider the following data (taken from Williams, 1974, p. 77):

ACT Scores

Sex	College		
	Arts and Sciences	Education	Engineering
Male	20	21	21
	18	17	22
	18	19	16
	16	14	18
	21	12	23
	22	26	
	24	28	
	28	21	
	29	14	
	16	15	
	18		
	13		
	15		
	18		
	17		

Sex	College		
	Arts and Sciences	Education	Engineering
Female	19	23	27
	17	29	24
	17	21	22
	16	17	
	18	15	
	27	13	
	14		
	15		
	16		

Several different procedures could be effected to code the data or obtain suitable solutions. Because contrast coding is an effective means to obtain a solution for the full rank model approach of Timm and Carlson, contrast coding is used for the other solutions as well. In addition to the Y (criterion) variable, five other variables can be defined:

X_1 = 1 if male, -1 if female;

X_2 = 1 if in the College of Arts and Sciences, 0 if in the College of Education, -1 if in the College of Engineering;

X_3 = 0 if in the College of Arts and Sciences, 1 if in the College of Education, -1 if in the College of Engineering;

$X_4 = X_1 \cdot X_2$; and

$X_5 = X_1 \cdot X_3$.

Six models can be defined:

$Y = b_0 + b_1X_1 + e_1$, (1)

$Y = b_0 + b_2X_2 + b_3X_3 + e_2$, (2)

$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + e_3$, (3)

$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_4$, (4)

$Y = b_0 + b_1X_1 + b_4X_4 + b_5X_5 + e_5$, (5) and

$Y = b_0 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_6$. (6)

In equations 1 through 6, the b's are regression coefficients specific to an equation (b_0 will likely be different for the different equations; so also the other b's are specific to an equation); the e's are error terms associated with each equation.

Table 1 shows the sums of squares for the various approaches to analyzing the data on sex and college.

It can be noticed in Table 1 that the main effect for sex and sex independent of college are equal, but are unequal to sex independent of college and interaction; a similar result occurs for the college effect. If either an unadjusted main effects solution, a fitting constants solution or a hierarchical model are completed, an additive solution is found. See Table 2. (The terminology for type of solution is the same as in Williams, 1972).

However, if a full rank model solution (as suggested by Timm and Carlson) is executed a non-additive model results. See Table 3.

The difference in the solutions shown in Tables 2 and 3 are that different hypotheses are being tested. It can be shown (see Williams, 1977) that the solution in Table 2 corresponds to the one proposed by Jennings (1967); the hypothesis for sex differences is given by (in terms of sample means)

$$\frac{n_1\bar{Y}_1 + n_2\bar{Y}_2 + n_3\bar{Y}_3}{n_1 + n_2 + n_3} = \frac{n_4\bar{Y}_4 + n_5\bar{Y}_5 + n_6\bar{Y}_6}{n_4 + n_5 + n_6} \qquad (7)$$

where the n's and $\bar{Y}$'s correspond to the cells in the two way layout. $\bar{Y}_1$ is the mean of males in arts and science, $\bar{Y}_2$ is the mean of males in education, and $\bar{Y}_3$ is the mean of males in engineering; means for females ($\bar{Y}_4$, $\bar{Y}_5$, $\bar{Y}_6$) are similarly defined.

Since proportionality holds, the numerator and denominator of the left side of equation 7 can be multiplied by $\frac{n_4}{n_1}$ (or by $\frac{n_5}{n_2}$ or $\frac{n_6}{n_3}$ or any combination thereof, since the proportion is the same):

$$\frac{\frac{n_4}{n_1}n_1\bar{Y}_1 + \frac{n_4}{n_1}n_2\bar{Y}_2 + \frac{n_4}{n_1}n_3\bar{Y}_3}{\frac{n_4}{n_1}(n_1 + n_2 + n_3)} = \frac{n_4\bar{Y}_4 + n_5\bar{Y}_5 + n_6\bar{Y}_6}{n_4 + n_5 + n_6}. \qquad (8)$$

Since $\frac{n_4}{n_1} = \frac{n_5}{n_2} = \frac{n_6}{n_3}$, equation 8 can be simplified:

Table 1

Two-Way Solution for Proportionate Cell Frequency

Source of Variation	df	SS	MS	F
Sex	1	$SS_1 = .14$	.14	.01
Sex (Independent of College)	1	$SS_3 - SS_2 = 49.24 - 49.10 = .14$	.14	.01
Sex (Independent of College and Interaction)	1	$SS_4 - SS_5 = 107.42 - 95.36 = 12.06$	12.06	.57
College	2	$SS_2 = 49.10$	24.55	1.16
College (Independent of Sex)	2	$SS_3 - SS_1 = 49.24 - .14 = 49.10$	24.55	1.16
College (Independent of Sex and Interaction)	2	$SS_4 - SS_6 = 107.42 - 34.91 = 72.51$	36.26	1.72
Interaction	2	$SS_4 - SS_3 = 107.42 - 49.24 = 58.18$	29.09	1.38
Within	<u>42</u>	$SS_{DEV_4} = 885.83$	21.09	
Total	47	$SS_T = 993.25$		

Table 2

Summary Table for the Unadjusted Main Effects Solution, Fitting Constants Solution and Hierarchical Model With Proportional Data

Source of Variation	df	SS	MS	F
Sex	1	.14	.14	.01
College	2	49.10	24.55	1.16
Interaction	2	58.18	29.09	1.38
Within	42	885.83	21.09	
Total	47	993.25		

Table 3

Summary Table For Full Rank Model Solution With Proportional Data

Source of Variation	df	SS	MS	F
Sex (Independent of College & Interaction)	1	12.06	12.06	.57
College (Independent of Sex & Interaction)	2	72.51	36.26	1.72
Interaction	2	58.18	29.09	1.38
Within	42	885.83	21.09	
Total	47	1028.58 ≠ 993.25		

$$n_4\overline{Y}_1 + n_5\overline{Y}_2 + n_6\overline{Y}_3 = n_4\overline{Y}_4 + n_5\overline{Y}_5 + n_6\overline{Y}_6. \quad (9)$$

While equation 9 could be expressed in several alternative forms, it is clear that the number of members in a cell are incorporated into the hypothesis. The full rank model solution addresses a different hypothesis. For the sex effect, the hypothesis tested is (in terms of sample means)

$$\frac{\overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3}{3} = \frac{\overline{Y}_4 + \overline{Y}_5 + \overline{Y}_6}{3}. \quad (10)$$

Note that equation 10 tests a hypothesis regarding means that suggest all groups have the same number of members, even if they do not. The actual mean for males is 19.33 and for females is 19.44. The cell means are $\overline{Y}_1 = 19.53$, $\overline{Y}_2 = 18.70$, $\overline{Y}_3 = 20$, $\overline{Y}_4 = 17.67$, $\overline{Y}_5 = 19.67$ and $\overline{Y}_6 = 24.33$. Thus $\frac{\overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3}{3} = 19.41$ and $\frac{\overline{Y}_4 + \overline{Y}_5 + \overline{Y}_6}{3} = 20.55$. It is this writer's opinion that the additive solution is more likely to be of interest than the solution found through the full rank model solution suggested by Timm and Carlson.

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MULTIPLE LINEAR REGRESSION VIEWPOINTS
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UNDERSTANDING PARTIAL REGRESSION COEFFICIENTS IN THE PRESENCE OF CORRELATED REGRESSORS

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ABSTRACT

The interpretation of partial regression coefficients in the presence correlated regressors causes difficulty for students in the social sciences. nce correlation among regressors is the typical case in the social sciences, is presents a considerable instructional problem. This article presents an planation of the partial regression coefficient in the presence of correlated gressors that is a simple and direct extension of the case where regressors e mutually orthogonal. The interpretation presented emphasizes the rela- nship between the partial regression coefficient and the simple regression efficient. An example using SAS computer package is provided.

troduction

The extension of the principles and techniques of simple linear gression to multiple linear regression frequently results in confusion and isunderstanding for students in the social sciences. The major problem ea concerns the understanding of the regression coefficients when regres- s are moderately correlated. In most texts on regression analysis (Cohen d Cohen, 1975; Draper and Smith, 1966; Kerlinger and Pedhazur, 1973) e extension from simple to multiple regression is discussed via the special se where the regressors are uncorrelated. Pedagogically, this is appro- riate since it requires the introduction of a minimum of new concepts. owever, in the actual analysis of data in the social sciences, correlated gressors are far more the rule than the exception. Unfortunately, it is the conceptual leap from independent regressors to correlated regressors at there exists the greatest lack of clarity in explanation. An example of is confusion is the belief displayed not only be beginning students, but practicing researchers, that the order of entry of the variables into a

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stepwise regression procedure affects the resultant regression weights when the full model is estimated! This confusion is exacerbated by the treatment of stepwise regression output in statistical computer packages such as SPSS (although, to the credit of the SPSS authors, they provide the best explanation we have found to date on the problem of correlated regressors) (Nie, et al., 1975).

The purpose of this paper is to present a lucid explanation of partial regression coefficients in the presence of correlation among the regressors. Our goal is to bridge the gap between a purely verbal explanation such as ". . . the increase in Y for a unit increase in X holding all other variables constant. . ." and a purely mathematical explanation such as:

$$B_{Y1\cdot 2} = \frac{r_{Y1} - r_{Y2}r_{12}}{1 - r_{12}^2}\left(\frac{S_Y}{S_1}\right)$$

Although both of these approaches are technically correct, neither provides a particularly good intuitive understanding of what is involved in multiple regression with correlated regressors.

Simple and Partial Regression Coefficients

It is our experience that the simple regression coefficient is readily comprehended by students as they approach multiple regression, and that an explanation of the partial regression coefficient in terms of a simple regression coefficient is heuristically appealing to students. Such a transition is clear and direct in the case of mutually orthogonal regressors. This multiple regression setting reduces to a series of simple regression equations (as in Draper and Smith, 1966, pp. 107-115). That is, the partial regression coefficient is identical to what it would be in a simple regression.

Our purpose here is to show that a similar reduction can be used even when regressors are correlated. The presentation below demonstrates how this would be done. It might reasonably follow the mutually orthogonal setting in a regression course.

Consider a regression with dependent variable Y, and three moderately correlated regressors X_1, X_2, and X_3:

(1) $$\hat{Y} = B_1X_1 + B_2X_2 + B_3X_3 + B_0$$

Since the regressors are correlated, it is obvious that the coefficient B_1 will not have the same value as a simple regression coefficient from the regression of Y on X_1 (alone). However, B_1 will be identical to the coefficient obtained from a simple regression of Y on the residuals of X_1 (say, X_1') after the collinearity with X_2 and X_3 has been removed. This can be accomplished by regressing X_1 on X_2 and X_3:

$$\hat{X}_1 = a_2X_2 + a_3X_3 + a_0$$

then $$\hat{X}_{1i}' = X_{1i} - \hat{X}_{1i} .$$

The same procedure is followed for X_2 and X_3. A new equation:

$$\hat{Y} = B_1'X_1' + B_2'X_2' + B_3'X_3' + B_0' \tag{2}$$

can be shown to yield exactly the same regression coefficients as equation (1). That is, $\underset{\sim}{B}' = \underset{\sim}{B}$. The pedagogical advantage gained by creating equation (2) is that the $\underset{\sim}{X}_i$'s are mutually orthogonal and the B_i's can be understood as in the mutually orthogonal case. Thus, the partial regression coefficient is the simple regression coefficient of Y on the residuals of X_1 after the effects of X_2 and X_3 have been removed from X_1.

The utility of this approach to understanding regression coefficients is that it allows the student to link his comprehension of the partial regression coefficient to the firmer ground of the simple regression coefficient. This is particularly useful when such concepts as suppressor variables, multicollinearity, and shrinkage in r-squared are discussed.

An Example

An example of this approach with three regressors using the SAS statistical package is presented below:

```
(JCL)

DATA SAMPA;

INPUT  Y  X1  X2  X3;

CARDS;

   (insert data)

PROC GLM; MODEL  Y = X1  X2  X3;

PROC GLM; MODEL  X1 = X2  X3;
```

```
OUTPUT OUT = SAMPB  RESIDUAL = RESID;
DATA SAMPC; MERGE SAMPA SAMPB;
PROC GLM; MODEL Y = RESID;
DATA SAMPA;
PROC GLM; MODEL X2 = X1  X3;
OUTPUT OUT = SAMPD  RESIDUAL = RESID;

DATA SAMPE;  MERGE SAMPA  SAMPD;
PROC  GLM;  MODEL  Y = RESID;
DATA SAMPA;
PROC  GLM;  MODEL  X3 = X1 X2;
OUTPUT OUT = SAMPF  RESIDUAL = RESID;
DATA SAMPG; MERGE SAMPA  SAMPF;
PROC GLM;  MODEL  Y = RESID;
//
```

The first PROC GLM statement results in the standard multiple regression output for the full model. The second PROC GLM regresses X_1 on the remaining independent variables and calculates the residuals, while the third PROC GLM performs the regression of Y on the residual of X_1. Students can now verify that the regression coefficient for residuals is identical to that for X_1 in the original model. The remaining PROC GLM statements calculate the coefficients for X_2 and X_3 in the same manner. Although the layout for calculating all regression coefficients is presented here for completeness, calculation of only one or two of these may be sufficient for instruction.

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SPECIFICATION BIAS IN CAUSAL MODELS WITH FALLIBLE INDICATORS

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ABSTRACT

In the bivariate case, measurement error in the independent variable produces an attenuated estimate of the true regression coefficient. In the multivariate case, the bias which results from specifying, incorrectly, a model with no measurement error will produce biased estimates which are predictable in neither their direction nor magnitude. This paper demonstrates some of these biases in a causal model of educational attainment.

Educational researchers have known for a long time that measurement errors in independent variables cause regression estimates to be biased. In the bivariate case, measurement error in the independent variable produces attenuated regression estimates. In the multivariate situation, however, neither the size nor direction of the bias is predictable, unless one knows in advance the magnitude and nature of the errors. This paper examines the implications of measurement error in a socioeconomic model of educational attainment. Wolfle and Lichtman (1981) estimated models of educational attainment for whites, blacks, and Mexican-Americans using

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estimates of structural parameters corrected for measurement error. Their report, however, concentrated on comparisons of interethnic differences in coefficients, and nowhere did they demonstrate that their LISREL-produced estimates differ in important ways from multiple regression estimates of the same model. Using the Wolfle and Lichtman (1981) model of educational attainment, this paper demonstrates that regression estimates will differ substantially from estimates that are corrected for the existence of measurement error.

The importance of bias created by ignoring measurement error is a point of some controversy. Jencks, et al. (1972, p. 336) concluded that the effects of random measurement error in a model of intergenerational mobility were relatively unimportant. In contrast, Bielby, Hauser and Featherman (1977) found that random measurement errors among nonblack men yielded regression estimates biased from 9 to 16 percent. For black men, however, Bielby, et al., found evidence of nonrandom errors, which yielded estimates whose biases were substantially larger than those for nonblacks. They concluded that, "because of the differing structures of response error among black and nonblack men, ignoring those structures leads to an exaggeration of black-nonblack returns to schooling and to understatement of racial differences in total and conditional inequality of occupational attainment" (Bielby, Hauser and Featherman, 1977, p. 1277). In addition, Wolfle (1979) has compared regression estimates in a model of educational attainment to LISREL-produced estimates corrected for measurement error using data from the National Longitudinal Study of the High School Class of 1972. Among whites, he found random measurement error produced regression estimates biased as much as 200 percent.

Concerned that differential levels of measurement bias would affect ir substantive conclusions about differences in the educational process whites, blacks, and Mexican-Americans, Wolfle and Lichtman (1981) ed a general method of the analysis of covariance structures (Jöreskog d Sörbom, 1978) to generate structural parameter estimates free of asurement error bias in a model of educational attainment.

This paper examines the size and importance of measurement error ases in the Wolfle and Lichtman (1981) model as a demonstration of the sts involved in ignoring measurement error. In order to do this, the rameters in the Wolfle and Lichtman model have been reestimated with dinary least squares regression. These new estimates have then been mpared with the LISREL (corrected) estimates reported by Wolfle and ichtman (1981).

HE MODEL

The basic model of educational attainment used in this analysis is hown diagrammatically in Figure 1. The variables of interest are shown ithin ellipses, and include father's occupational status, father's education, mother's education, number of siblings, sex, ability, academic reparation, college plans, and educational attainment. The arrows manating from the ellipses to mnemonic labels describe the measurement portion of the LISREL model, and are described in detail in Wolfle and Lichtman (1981). The variables used in the ordinary least squares regression are described below.

The model is a fully recursive set of structural equations in which ability is dependent upon five exogenous variables plus a residual

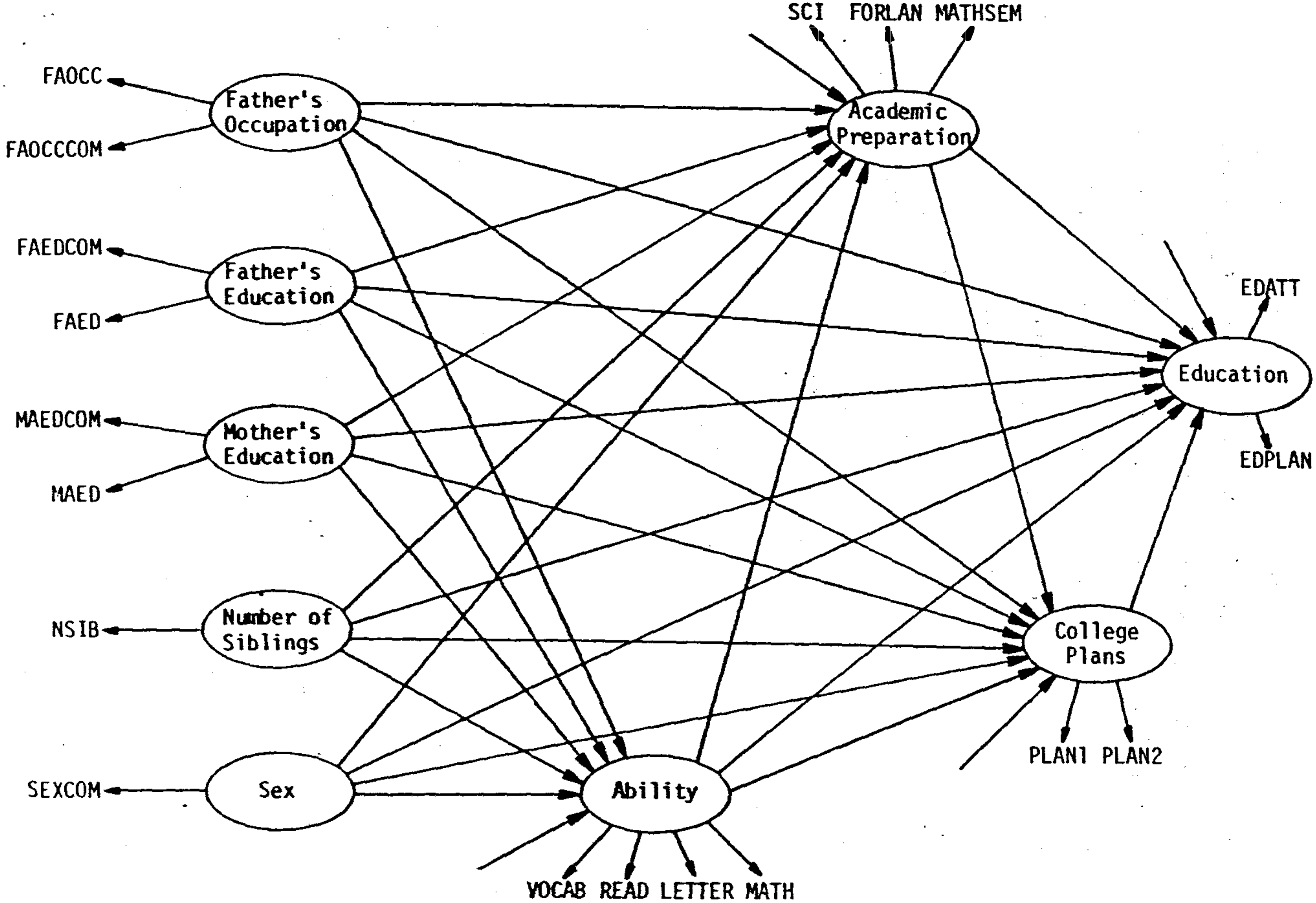

Figure 1. Structural Equation and Measurement Models of Educational Attainment

Among 1972 High School Graduates

sturbance term. Academic preparation is dependent upon ability, five ogenous variables, and a residual term. College plans is dependent upon ility, academic preparation, five exogenous variables, and a residual. inally, educational attainment is dependent upon all of the preceding ariables in the model, plus a residual term.

HE DATA

Data for this study were drawn from the National Longitudinal Study f the High School Class of 1972 (see Levinsohn, et al., 1978). The NLS, hich has been and continues to be supported by the National Center for Education Statistics, was designed to provide data on a large cohort of high school seniors, and to follow these students as they made the move from high school into their early years of adulthood. The data file included base-year survey and test-score data collected in 1972, along with follow-up surveys in 1973, 1974, and 1976 (the 1978 follow-up data are now available, but had not yet been made public at the time of our analysis). The analysis reported here is restricted to white NLS respondents. As with most other analyses of the process of socioeconomic achievement, pairwise present correlations were used to estimate the parameters of the model; the average number of whites in the analysis was 11,743.

In estimating the parameters of the model using ordinary least squares regression, in some cases only one manifest measure was used in place of the LISREL latent variable, and in other cases a simple summated scale was computed. The variable used to measure father's occupation was FAOCCCOM (V2468), a composite variable measured in terms of Duncan's (1961)

SEI scale as revised to match the 1970 census occupation classification (Hauser and Featherman, 1977). The variable was a composite of each individual's responses to base-year and first-year follow-up questionnaire items which asked respondents to indicate their father's main occupation.

The variables used to represent father's and mother's education were FAEDCOM (V1627) and MAEDCOM (V1628), respectively. These also were composite scores based on base-year and first follow-up questionnaire items. The category responses for these two variables were recoded to the number of years of schooling completed.

Values for the number of siblings were obtained by summing the values of four questions, which asked respondents to indicate the number of older brothers, younger brothers, older sisters, and younger sisters. Completing the specifications of the exogenous variables, sex was measured by the base-year and first follow-up composite variable, V1626. This dummy variable was coded zero for males and unity for females, so that positive coefficients emanating from this variable indicate higher values on the dependent variable for women.

The dependent variable, ability, was computed by taking the arithmetic average of four subtest scores administered to the respondents in the base year. The four measures were tests of vocabulary, reading, letter-groups, and mathematics. Academic preparation was also computed as an average of three indicators. The variables included in this computation were the number of semesters of science taken between July 1, 1969, and high school graduation (V0046), the number of semesters of foreign language completed in the same period (V0053), and the semesters of mathematics (V0074).

College plans were indexed by an NLS routing question (V0385), to h respondents indicated how they intended to spend the largest part heir time in the year after leaving high school. Anyone who planned ttend a two-year or four-year college or university either full time art time was given a value of unity on the college plans variable. other respondents were given a value of zero.

Finally, educational attainment (V1854) was measured from responses a question asked in the 1976 follow-up in which respondents were asked indicate the highest level of education or training they had received. category responses were recoded to years of completed schooling.

ECTS OF RANDOM MEASUREMENT ERROR

Measurement errors can be of many kinds. One kind exists when ors in one variable are correlated with the values of another variable. example, Mare and Mason (1980) have shown that women report their her's occupation with greater error than men, apparently because the her's occupation is more salient for young men than women. Another nd of reporting error exists when the errors in one variable are rrelated with errors in another variable. Bielby, Hauser and Featherman 77), for example, found that blacks overstate the consistency among eir own status characteristics and those of their fathers.

It is also possible for measurement errors to be uncorrelated with ything else. For example, Bielby, Hauser and Featherman (1977) found at a status attainment model for nonblacks with correlated errors ovided no better fit to the observed covariances than did a model with ll error covariances specified to be zero. They concluded that measurement errors of status characteristics for nonblack men were strictly random. Wolfle and Lichtman (1981) reached the same conclusion for whites regarding the randomness of measurement errors in their model of educational attainment; they found that the most likely candidates for correlated errors yielded a model only marginally better in its fit than a model with only random errors.

As a result, the comparison of the Wolfle and Lichtman (1981) parameter estimates to ordinary least squares regression estimates will indicate some of the biasing effects of random errors of measurement in multivariate analyses. If there exist other kinds of errors, one should expect to find structural parameter estimates substantially affected by the nature of such errors.

Even with random errors, biases can sometimes be severe. For example, suppose both father's occupation and education were reported with random error. If these two fallible indicators were then to be included in a regression equation predicting variation in ability, the estimates of their structural effects would be biased toward zero, thus underestimating the dependence of ability upon these two social background variables. Moreover, it is unlikely that both father's occupation and education are measured with equal reliability, and to the extent that these errors of measurement are not equal, the regression estimates of ability on father's education and occupation will be either inflated or deflated. With unequal reliabilities, therefore, the bias in regression estimates is not necessarily toward zero (as in the bivariate case); the direction and magnitude of the bias depends on the relative reliabilities of the independent variables, and unless one knows what these are in advance, the biasing effects are unpredictable.

Consider now the regression of academic preparation on ability and exogenous variables. If the exogenous variables are reported with om errors, the joint dependence of both ability and academic prepara- on the exogenous variables will be underestimated. As a result, dependence of academic preparation on ability will be overstated, use the effects of the exogenous variables are understated.

It is reasonable to suspect that all the variables in this model reported with random error. Moreover, it is also reasonable to ose that their reliabilities differ. As a result, all the regression mates are likely to be biased, but it is unreasonable to suppose that magnitude of bias is necessarily either small or consistently in one ction.

ULTS

Table 1 shows two sets of estimated structural parameters in a model ducational attainment. For each dependent variable, the top row of ficients are corrected (LISREL) estimates as reported by Wolfle and htman (1981); the second row of coefficients for each dependent variable, wn in parentheses, are ordinary least squares (OLS) regression estimates. king first at the proportion of explained variance for each dependent iable, note that the method of least squares understates the true lained variance by 19 to 30 percent. This occurrence results from the mination of a considerable amount of random error in the corrected REL estimates of the variances and covariances among the latent variables.

Examining the effects of the five background characteristics on lity, as predicted the OLS estimates underestimate the effects

Table 1. Estimates of Parameters in a Model of Educational Attainment: White 1972 High School Graduates (N = 11,743)

Dependent Variable	Predetermined Variables								R^2
	Father's Occup.	Father's Educ.	Mother's Educ.	Number Siblings	Sex	Ability	Acad. Prep.	College Plans	
Ability	.047	.381	.572	-.262	.185				.16
	(.029)	(.374)	(.462)	(-.245)	(.534)				(.13)
Academic Preparation	.006	.011	.012	-.034	-.510	.119			.45
	(.004)	(.022)	(.024)	(-.036)	(-.405)	(.105)			(.32)
College Plans	.003	.008	.013	-.014	.031	.017	.090		.44
	(.001)	(.017)	(.013)	(-.016)	(.001)	(.017)	(.074)		(.31)
Educational Attainment	.001	.032	.035	-.025	-.029	.030	.115	1.786	.68
	(.001)	(.040)	(.048)	(-.042)	(-.004)	(.036)	(.134)	(1.379)	(.50)

Note: Corrected LISREL estimates are shown without parentheses. Ordinary least squares regression estimates are shown in parentheses.

of these variables. For example, the OLS estimate for the effect of father's occupation on ability is .029, whereas the corrected estimate is .047. This is a negative bias of about 38 percent. The OLS estimates of the effects of father's education, mother's education, and number of siblings on ability are also negatively biased by about 2 percent, 19 percent, and 6 percent respectively. The only OLS coefficient for ability which is not negatively biased is the effect from sex, but this may be due to the different ways in which the ability variable was constructed. LISREL gave the greatest weight to a mathematics test score in the construction of the latent ability variable, which when balanced against three manifest measures of verbal ability yielded estimates that suggest there is no sex effect on ability. The construction of the ability measure for the OLS equations gave these four subtests equal weight, giving verbal expression, on which women excell, more weight than math. Thus, the regression estimate shows a positive effect.

Looking at the effects on academic courses taken in high school, it is seen once again that the OLS estimate of father's occupational status is negatively biased by about one-third. OLS estimates of parental education are, however, positively biased, both by about 100 percent. This positive bias causes the dependence of academic preparation on parental education to be overstated in the OLS analysis; as a result, the effect of ability on academic preparation is understated by about 12 percent.

A similar pattern seems to develop when looking at the effects on college plans. The structural coefficients for father's occupation and academic preparation are negatively biased by 67 percent and 18 percent, respectively, but father's education is positively biased by about 100 percent.

Turning to the dependent variable of primary interest, educational attainment, the OLS estimates of the effects of parental education are once again positively biased, thus overstating the effect of educational background on respondent's educational attainment. The sizes of these biases for father's and mother's education are 25 percent and 37 percent, respectively. If the previous pattern were to be followed, one might expect the OLS estimates of the effects of ability and academic preparation on educational attainment to be an underestimate of the true effect, due to the overestimation of the dependence of educational attainment on parental education. Such is not the case. The OLS estimate for ability is positively biased by about 20 percent, and academic preparation is positively biased by nearly 17 percent. Such fluctuation is to be expected in a multivariate model. The nature of measurement error causes OLS estimates not to be well behaved, so that the extent of bias becomes unpredictable in both magnitude and direction. Finally, the effect of college plans expressed in high school on educational attainment is underestimated by its OLS coefficient by about 23 percent.

CONCLUSION

This paper has examined the extent of bias inherent in ordinary least squares regression estimates when the presence of measurement error is ignored. While these results were based on but one population of high school students, and one structural model of educational attainment, the implications are much more widespread. It seems unwise to assume social variables are measured without error. This paper has demonstrated that the ordinary least squares estimates will be biased if measurement errors e incorrectly specified (e.g., assumed to be zero by choosing to ignore em). In some cases, the nature of bias can be predicted, but the more sual situation is that measurement error bias is unpredictable. Biases ay be offsetting, but are just as likely to be additive. Mason, et al. 1976) were wise to suggest that:

> Since the errors may be large or small, and their effects may be additive or offsetting, there is no way to access the biases in naive (uncorrected) models of achievement processes without first investigating the separate and joint effects of each type of measurement error (Mason, et al., 1976, p. 444).

Researchers would be well advised to heed such advice. This paper has found that biases may exceed 100 percent of the corrected estimates. While this degree of bias is serious, indeed, it pales against the extent of bias possible with correlated errors of measurement. In applicable situations, educational researchers should avail themselves of new analytical techniques which allow for the assessment of, and correction for, measurement error in models of educational processes.

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MISSING CELLS IN A TWO-WAY CLASSIFICATION

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bstract - A data set with one missing cell is investigated with a number of lausible hypotheses regarding the means. It is shown that the set of ypotheses likely to be of interest correspond to a result computationally identical to the unadjusted main effects solution.

The two-way fixed effects analysis of variance with disproportionate cell frequencies has been considered by many different researchers. The "full rank model" solution, described by Timm and Carlson (1975), has been purported to be a "best" solution. Overall, Spiegel and Cohen (1975) have also opted for this solution, though Overall and Spiegel (1969) earlier had shown a preference for the fitting constants solution. Cohen (1968) described a hierarchical model that has the advantage of being an additive solution. Jennings (1967) and Williams (1972) describe a solution that address probable hypotheses of interest. Jennings approached the problem in a classical regression formulation, whereas Williams showed that the same results could be computationally found in a simpler manner. Perhaps unfortunately, Williams termed the solution the unadjusted main effect solution.

Other researchers have used a combination of approaches rather than use exclusively a single solution. Among such researchers are Searle (1971) and Applebaum and Cramer (1974).

Focusing on the hypothesis tested has been the direction of Speed and Hocking (1976) and Searle, Speed and Henderson (1981). In the latter article, they show that, with missing cells, the usual hypotheses for rows and columns lose their meaning and that it is much more beneficial to concentrate on cells. This approach would seem to be in keeping with Jenning's (1967) earlier article.

Comparisons of the hypotheses tested in the full rank model solution of Timm and Carlson (1975) and the unadjusted main effects solution was shown in Williams (1977). In a companion to the present paper Williams (this issue) showed that when the data are proportional but not equal in cell frequencies that the hypotheses tested could vary from those a researcher wishes to test for the Timm and Carlson full rank model solution. The direction of the present paper is to examine the hypotheses when missing cells occur.

The data are taken from Williams (1974, p. 77) except that the three data points in the last cell (engineering females) are omitted.

ACT Scores

Sex	Arts and Sciences	Education	Engineering
Male	20	21	21
	18	17	22
	18	19	16
	16	14	18
	21	12	23
	22	26	
	24	28	
	28	21	
	29	14	
	16	15	
	18		
	13		
	15		
	18		
	17		
Female	19	23	
	17	29	
	17	21	
	16	17	
	18	15	
	27	13	
	14		
	15		
	16		

Note that if the three data points had been included for the engineering females, the data would be proportional, and the analysis is given in Williams (this issue).

To analyse the data, an analysis using contrast coding is used to effect what might be termed a "quasi-analysis of variance solution" using the full rank model approach of Timm and Carlson and the unadjusted main effect solution of Williams. In addition to the Y (criterion) variable, four other variables are defined:

$X_1^* = 1$ if male, -1 if female;

$X_2^* = 1$ if in the College of Arts and Sciences, 0 if in the College of Education, -1 if in the College of Engineering;

$X_3^* = 0$ if in the College of Arts and Sciences, 1 if in the College of Education, -1 if in the College of Engineering; and

$X_4^* = X_1^* \cdot X_2^*$.

Six models can be defined:

$Y = b_0 + b_1X_1^* + e_1$, (1)

$Y = b_0 + b_2X_2^* + b_3X_3^* + e_2$, (2)

$Y = b_0 + b_1X_1^* + b_2X_2^* + b_3X_3^* + e_3$, (3)

$Y = b_0 + b_1X_1^* + b_2X_2^* + b_3X_3^* + b_4X_4^* + e_4$, (4)

$Y = b_0 + b_1X_1^* + b_4X_4^* + e_5$, (5) and

$Y = b_0 + b_2X_2^* + b_3X_3^* + b_4X_4^* + e_6$. (6)

In equations 1 through 6, the b's are regression coefficients specific to an equation (b_0 will likely be different for the different equations; so also the b's are specific to an equation); the e's are error terms associated with each equation.

Table 1 shows the sums of squares generated by these models.

While the results for each main effect are different depending on whether the measurement is made in the presence of the other main effect or the main effect and the interaction, this outcome would be expected from our knowledge of the disproportionate case.

It is instructive to set up binary coded predictors and then state and test likely hypotheses of interest. Five cell variables can be defined:

Table 1

Two-Way Solution for the Missing Cell Data

Source of Variation	df	SS	MS	F
Sex	1	$SS_1= 7.51$	7.57	.35
Sex (Independent of **College**)	1	$SS_3-SS_2=10.68-5.64=5.64=5.14$	5.14	.24
Sex (Independent of **College and Interaction**)	1	$SS_4-SS_5=28.74-25.24=3.50$	3.50	.16
College	2	$SS_2=5.64$	2.84	.13
College (Independent of Sex)	2	$SS_3-SS_1=10.68-7.51=3.17$	1.59	.07
College (Independent of Sex and Interaction)	2	$SS_4-SS_6=28.74-8.71=20.03$	10.02	.46
Interaction	1	$SS_4-SS_5=28.74-10.68=18.06$	18.06	.83
Within	40	$SS_{DEV_4}=873.16$	21.83	

X_1 = 1 if in cell 1 (Male, Arts & Science), 0 otherwise;

X_2 = 1 if in cell 2 (Male, Education), 0 otherwise;

X_3 = 1 if in cell 3 (Male, Engineering), 0 otherwise;

X_4 = 1 if in cell 4 (Female, Arts and Science), 0 otherwise; and

X_5 = 1 if in cell 5 (Female, Education), 0 otherwise.

Then a full model can be defined:

$$Y = b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_7. \quad (7)$$

An alternative full model utilizing the unit vector is

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + e_7. \quad (8)$$

Hypotheses for Rows

Now, several different hypotheses that might be of interest for the row effect can be investigated. Four such hypotheses will be treated:

H_1: $b_1 + b_2 = b_4 + b_5$, a hypothesis for regression coefficients that corresponds to $\frac{\overline{Y}_1 + \overline{Y}_2}{2} = \frac{\overline{Y}_4 + \overline{Y}_5}{2}$. Note that H_1 fails to address altogether membership in cell 3; it also tests a hypothesis among the means that does not take into account the varying cell frequencies.

H_2: $\frac{b_1 + b_2 + b_3}{3} = \frac{b_4 + b_5}{2}$. While H_2 takes cell 3 into account, it does not address the varying cell frequencies.

H_3: $\frac{15b_1 + 10b_2}{25} = \frac{9b_4 + 6b_5}{15}$. H_3 takes into account the unequal cell frequencies, it does not take into account cell 3.

H_4: $\frac{15b_1 + 10b_2 + 5b_3}{30} = \frac{9b_4 + 6b_5}{15}$. H_4 takes into account both the unequal sized groups and cell 3.

t H_1, the restriction shown in H_1 is imposed on the full model;

$_1$: $b_1 + b_2 = b_4 + b_5$

r $b_1 = b_4 + b_5 - b_2$.

$= (b_4 + b_5 - b_2) X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + e_8$

r

$= b_2(X_2 - X_1) + b_4(X_4 + X_1) + b_5(X_5 + X_1) + b_3 X_3 + e_8$

$= X_2 - X_1$;

$= X_4 + X_1$; and

$= X_5 + X_1$,

$= b_2 D_1 + b_4 D_2 + b_5 D_3 + b_3 X_3 + e_8$. (9)

the model shown for equation 9,

$= 26.92$; $SS_4 - SS_9 = 28.74 - 26.92 = 1.82$;

$\frac{1.82}{21.83} = .08$, a value that does not correspond to any given in Table 1.

interest is in using R^2s rather than SS, the equation

$\frac{(R^2_{FM} - R^2_{RM})/df_1}{(1 - R^2_{FM})/df_2}$ where R^2_{FM} refers to the R^2 term for the full model

$_{RM}$ refers to the R^2 term for the restricted model.

$= \frac{(.03187 - .02985)/1}{(1 - .03187)/40} = .08$, as before.

H_2: $\frac{b_1 + b_2 + b_3}{3} = \frac{b_4 + b_5}{2}$;

$b_1 = 3/2b_4 + 3/2b_5 - b_2 - b_3$. Imposing this restriction of the full

ields

$(3/2b_4 + 3/2b_5 - b_2 - b_3)X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + e_9$.

$b_2(X_2 - X_1) + b_3(X_3 - X_1) + b_4(X_4 + 3/2X_1) + b_5(X_5 + 3/2X_1) + e_9$.

Let $D_1 = X_2 - X_1$;

$D_4 = X_3 - X_1$;

$D_5 = X_4 + 3/2\ X_1$; and

$D_6 = X_5 + 3/2\ X_1$.

Then $Y = b_2D_2 + b_3D_4 + b_4D_5 + b_5D_6 + e_9$. (10)

Using the model shown for equation 10, $SS_{10} = 23.72$; $SS_4 - SS_{10} = 28.74 - 23.72 =$

$F = \frac{5.02}{21.83} = .23$. Note that H_2 does not yield any solution for sex shown in Table 1.

Using H_3: $\frac{15b_1 + 10b_2 \qquad 9b_4 + 6b_5}{25 \qquad\qquad 15}$, or $b_1 = b_4 + 2/3b_5 - 2/3\ b_2$, an impositio

is made on the full model:

$Y = (b_4 + 2/3\ b_5 - 2/3b_2)X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_{10}$.

$Y = b_2(X_2 - 2/3X_1) + b_3X_3 + b_4(X_4 + X_1) + b_5(X_5 + 2/3X_1) + e_{10}$.

Let $D_7 = X_2 - 2/3X_1$;

$D_2 = X_4 + X_1$; and

$D_8 = X_5 + 2/3X_1$.

Then $Y = b_2D_7 + b_3X_3 + b_4D_2 + b_5D_8 + e_{10}$. (11)

Using the model shown for equation 11,

$SS_{11} = 23.70$, $SS_4 - SS_{11} = 28.74 - 23.70 = 5.04$.

$F = \frac{5.04}{21.83} = .23$, a value that does not correspond to any outcome for the sex effect shown in Table 1.

Consider H_4: $\frac{15b_1 + 10b_2 + 5b_3}{30} = \frac{9b_4 + 6b_5}{15}$, or

$$b_1 = 6/5\ b_4 + 4/5b_5 - 2/3b_2 - 1/3b_3.$$

Imposing this restriction on the full model yields

$Y = (6/5b_4 + 4/5b_5 - 2/3b_2 - 1/3b_3)X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_{11}.$

$Y = b_2(X_2 - 2/3X_1) + b_3(X_3 - 1/3X_1) + b_4(X_4 + 6/5X_1) + b_5(X_5 + 4/5X_1) + e_{11}.$

Let $D_9 = X_2 - 2/3X_1$;

$D_{10} = X_3 - 1/3X_1$;

$D_{11} = X_4 + 6/5X_1$; and

$D_{12} = X_5 + 4/5X_1$.

Then $Y = b_2D_9 + b_3D_{10} + b_4D_{11} + b_5D_{12} + e_{11}$. (12)

Using equation 12,

$SS_{12} = 21.23$; $SS_4 - SS_{12} = 28.74 - 21.23 = 7.51$;

$F = \frac{7.51}{21.83} = .35$. It can be noted that the result for H_4 is identical with the use of equation 1, which is the unadjusted sex effect.

Hypotheses for Columns

Four different hypotheses can be given for the column effect also:

H_5: $b_1 + b_4 = b_2 + b_5$. Note that H_5, like H_1, disregards cell 3 and does not take into account the unequal sized cell frequencies.

H_6: $\frac{b_1 + b_4}{2} = \frac{b_2 + b_5}{2} = b_3$. H_6, like H_2, does not take into account the unequal sized cell frequencies.

H_7: $\frac{15b_1 + 9b_4}{24} = \frac{10b_2 + 6b_5}{16}$. H_7, like H_3, takes into account the unequal sized cell frequencies, but disregards cell 3.

H_8: $\frac{15b_1 + 9b_4}{24} = \frac{10b_2 + 6b_5}{16} = b_3$. H_8, like H_4, takes into account the unequal sized cell frequencies and cell 3.

To test H_5: $b_1 + b_4 = b_2 + b_5$, or $b_1 = b_2 + b_5 - b_4$.

Imposing this restriction on the full model yields

$Y = .(b_2 + b_5 - b_4)X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_{12}.$

$Y = b_2(X_2 + X_1) + b_3X_3 + b_4(X_4 - X_1) + b_5(X_5 + X_1) + e_{12}.$

Let $D_{13} = X_2 + X_1;$

$D_{14} = X_4 - X_1;$ and

$D_3 = X_5 + x_1.$

Then $Y = b_2D_{13} + b_3X_3 + b_4D_{14} + b_5D_3 + e_{12}.$ (13)

Using equation 13, $SS_{13}= 25.68$; $SS_4 - SS_{13} = 28.74 - 25.68 = 3.06$.

$F = \frac{3.06}{21.83} = .14$, a value that does not correspond to any of the outcomes for the college effect in Table 1.

Regarding H_6: $\frac{b_1 + b_4}{2} = \frac{b_2 + b_5}{2} = b_3$, two different restrictions are implied.

Solving for b_1 and b_3, $b_1 = b_2 + b_5 - b_4$; $b_3 = \frac{b_2 + b_5}{2}$.

Imposing these restrictions on the full model,

$Y = (b_2 + b_5 - b_4)X_1 + b_2X_2 + \frac{(b_2 + b_5)X_3}{2} + b_4X_4 + b_5X_5 + e_{13}.$

$Y = b_2(X_2 + X_1 + 1/2X_3) + b_4(X_4 - X_1) + b_5(X_5 + X_1 + 1/2X_3) + e_{13}.$

Let $D_{15} = X_2 + x_1 + 1/2X_3;$

$D_{14} = X_4 - X_1;$ and

$D_{16} = X_5 + X_1 + 1/2X_3.$

Then $Y = b_2D_{15} + b_4D_{14} + b_5D_{16} + e_{13}.$ (14)

Using equation 14, $SS_{14} = 19.68$; $SS_4 - SS_{14} = 28.74 - 9.06 = 19.68$.

$F = \frac{9.06/2}{21.83} = .21$, a value that does not correspond to any outcome for the college effect in Table 1.

H_7 is given as $\frac{15b_1 + 9b_4}{24} = \frac{10b_2 + 6b_5}{16}$.

Solving for b_1, $b_1 = b_2 + 3/5b_5 - 3/5b_4$.

posing this restriction on the full model yields,

$$Y = (b_2 + 3/5b_5 - 3/5b_4)X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_{14}.$$

$$Y = b_2(X_2 + X_1) + b_3X_3 + b_4(X_4 - 3/5X_1) + b_5(X_5 + 3/5X_1) + e_{14}.$$

Let $D_{13} = X_2 + X_1$;

$D_{17} = X_4 - 3/5X_1$; and

$D_{18} = X_5 + 3/5X_1$.

ien $Y = b_2D_{13} + b_3X_3 + b_4D_{17} + b_5D_{18} + e_{14}$. (15)

ing equation 15, $SS_{15} = 28.24$; $SS_4 - SS_{15} = 28.74 - 28.24 = .50$.

$F = \frac{.50}{21.83} = .02$, a value that does not correspond to any outcome shown

r the college effect in Table 1.

Finally, regarding H_8: $\frac{15b_1 + 9b_4}{24} = \frac{10b_2 + 6b_5}{16} = b_3$, two restrictions

hown in terms of b_1 and b_3) are implied:

$b_1 = b_2 + 3/5b_5 - 3/5b_4$ and

$b_3 = 5/8b_2 + 3/8b_5$.

mposing these restrictions on the full model,

$$Y = (b_2 + 3/5b_5 - 3/5b_4)X_1 + b_2X_2 + (5/8b_2 + 3/8b_5)X_3 + b_4X_4 + b_5X_5 + e_{15}.$$

$$Y = b_2(X_2 + X_1 + 5/8X_3) + b_4(X_4 - 3/5X_1) + b_5(X_5 + 3/5X_1 + 3/8X_3) + e_{15}.$$

Let $D_{19} = X_2 + X_1 + 5/8X_3$;

$D_{17} = X_4 - 3/5X_1$; and

$D_{20} = X_5 + 3/5X_1 + 3/8X_3$.

Then $Y = b_2D_{19} + b_4D_{17} + b_5D_{20} + e_{15}$. (16)

Using equation 16, $SS_{16} = 23.10$; $SS_4 - SS_{16} = 28.74 - 23.10 = 5.64$.

$F = \frac{5.64/2}{21.83} = .13$, the same result, in a computational sense, of the unadjusted main effect for colleges.

Hypothesis for Interaction

In testing the hypothesis for interaction, it can be noted that cell 3 does not enter into the interaction. Thus, the likely hypothesis of interest in terms of the means is $\overline{Y}_1 - \overline{Y}_4 = \overline{Y}_2 - \overline{Y}_5$.

In terms of the regression coefficients, the null hypothesis would be tested by $b_1 - b_4 = b_2 - b_5$ (this hypothesis will be called H_9).

Then, in terms of b_1, $b_1 = b_2 - b_5 + b_4$. Imposing this restriction on the full model yields,

$$Y = (b_2 - b_5 + b_4)X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + e_{16}.$$

$$Y = b_2(X_2 + X_1) + b_3X_3 + b_4(X_4 + X_1) + b_5(X_5 - X_1) + e_{16}.$$

Let $D_{13} = X_2 + X_1$;

$D_2 = X_4 + X_1$; and

$D_{21} = X_5 - X_1$.

Then $Y = b_2D_{13} + b_3X_3 + b_4D_2 + b_5D_{21} + e_{16}$. (17)

Using equation 17, $SS_{17} = 10.68$; $SS_4 - SS_{17} = 28.74 - 10.68 = 18.06$, thus $F = \frac{18.06}{21.83} = .83$, the result given for interaction in Table 1. The results of using H_1 thru H_9 with both SS and R^2s are shown in Table 2.

Discussion

A considerable amount of effort has been expended by many different researchers in investigating the two-way analysis of variance with disproportionate cell frequencies. In regard to any two-way layout of data, four situations regarding the cell frequencies can be put in order of their stress on the analysis: 1) equal numbers in each cell; 2) unequal but proportional

Table 2

Hypotheses for Two-Way Analysis of Variance with a Missing Cell

Hypothesis	SS_R	$SS_F - SS_R$	R^2	$R_F^2 - R_R^2$
H_1: $b_1 + b_2 = b_4 + b_5$	26.92	1.82	.02985	.00202
H_2: $\frac{b_1 + b_2 + b_3}{3} = \frac{b_4 + b_5}{2}$	23.72	5.02	.02629	.00558
H_3: $\frac{15b_1 + 10b_2}{25} = \frac{9b_4 + 6b_5}{15}$	23.70	5.04	.02628	.00559
H_4: $\frac{15b_1 + 10b_2 + 5b_3}{30} = \frac{9b_4 + 6b_5}{15}$	21.23	7.51	.02354	.00833
H_5: $b_1 + b_4 = b_2 + b_5$	25.68	3.06	.02848	.00339
H_6: $\frac{b_1 + b_4}{2} = \frac{b_2 + b_5}{2} = b_3$	19.68	9.06	.02182	.01005
H_7: $\frac{15b_1 + 9b_4}{24} = \frac{10b_2 + 6b_5}{16}$	28.24	.50	.03131	.00056
H_8: $\frac{15b_1 + 9b_4}{24} = \frac{10b_2 + 6b_5}{16} = b_3$	23.10	5.64	.02562	.00625
H_9: $b_1 - b_4 = b_2 - b_5$	10.68	18.06	.01184	.02003

numbers in each cell; 3) disproportionate numbers in each cell; and 4) at least one missing cell.

Addressing the four situations, the solution described by Jennings (1967) and shown to be computationally equivalent to the unadjusted main effects solution by Williams (1972) is robust in that it addresses likely hypotheses regarding the cell means in all four instances. The full rank model, described by Timm and Carlson (1975) can be criticized as addressing likely hypotheses of interest only for the equal cell frequency situation; the hypotheses tested in the proportionate case may very well deviate from those a researcher is likely to be most interested in. The hypotheses that are tested in the missing cells case do not appear to have any likely contrasts among the cells that address usual analysis of variance questions. It is of course possible that the hypotheses tested by the full rank model are of interest to the researcher. However, as a general data-analytic tool, the full rank model as described by Timm and Carlson would seem to lack the robustness needed to suggest itself to the statistically unsophisticated user.

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PREDICTION OF MISSING DATA USING REGRESSION MODELS: A PROGRAMMED APPROACH FOR LARGE SPSS SYSTEM FILES

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Abstract

The purpose of this article is to present computer programming capable of automatically predicting missing data in an SPSS system file using multiple regression techniques. Two versions are presented. The first is an interactive approach designed to run via VSPC under OS/VS MVS. The second is designed to be used in a card batch environment. In addition, an equation which predicts the amount of CPU time required is included.

Incomplete data are often a major concern in behavioral research. SPSS, SASS, and BIOMED, three of the most commonly available statistical packages, all have provisions for handling missing data. These subroutines generally take the form of some type of case deletion. This approach, however, decreases the N size and therefore the power of the analysis. In addition, if regression methods are used, a decrease in the N size will increase the upward bias of the R (Newman, 1972) and decrease the stability of the weights (Newman, 1973).

Two other alternatives for handling missing data are sometimes used. One method is the substitution of the mean value calculated from the complete cases. However, this technique tends to decrease the variance and may therefore be inappropriate for analysis of variance procedures. The other approach is to predict the missing values through the use of a regression equation whose weights are calculated from the complete cases. This latter approach tends to bias any subsequent analysis to a lesser extent than the insertion of means.

While neither SPSS, SASS, or BIOMED makes direct provision for the insertion of means for missing values, it is possible by utilizing various calculation and data management subroutines within the packages. By comparison, the calculation and insertion of missing data based on the regression approach, utilizing subroutines existing within the packages, is so complicated that this procedure is impractical even when dealing with small amounts of missing data.

The following programming was designed to provide a solution to this problem. Two versions of the program are presented. The first was designed to run interactively via VSPC under OS/VS MVS where VS FORTRAN, the RUML subroutine of the IMSL (International Mathematical and Statistical Libraries, Inc., 1980), and SPSS Release 8.1 (McGraw-Hill, 1979) routines have been implemented. The second version was designed for use with a card deck in a batch mode and only requires the implementation of SPSS, the IMSL, and FORTRAN. The final section of this paper presents a method for predicting the amount of CPU time that will be required to predict and insert the missing data.

Interactive Programming

The interactive programming consists of two routines designed to run consecutively: INTRO and INTRO1. They will call on SPSS system file, sort the complete from the incomplete cases, and further sort the incomplete cases into two categories: $\leq 10\%$ and $> 10\%$ missing data. Next, the program will scan the first case in the $\leq 10\%$ missing data file and locate the first missing variable, build a regression equation predicting this variable utilizing only those predictors available from that particular case, and calculate the weights utilizing the complete case file. These weights, along with the values for the predictors from the first case, will be used to predict the missing value for the variable. An a priori decision was made to round the predicted value up or down as appropriate if the first three digits dropped are $> \pm.455$ of the last significant digit kept. If the first three digits dropped are in the range of $< \pm.455$ of the last significant digit kept, the value of the last significant digit is randomly rounded up or down. This value is then inserted into the data matrix and the program then continues its scan of the first case in the $\leq 10\%$ missing data file for other missing variables. If it should find another missing value the process is repeated. In no instance, however, are previously predicted variables entered into the prediction process of subsequent variables for that particular case.

Once the scan and prediction of missing variables for the first case is completed, the program repeats the process for the second and each subsequent case in the ≤ 10% missing data file. After the last case has been completed, the program combines the complete case file with newly completed version of the ≤ 10% missing data file. This new file is in BCD format and can be used as a raw data file for input into SPSS. Its name is USER FINISH CASES.

This new system file will contain complete data on all cases which initially were complete or had ≤ 10% missing data. Those cases which had > 10% missing data will have been discarded. The choice of the 10% cutoff is based on the fact that > 10% missing data for any one case will not unduly bias the prediction of the missing data (Cohen & Cohen, 1975).

The following sections document the interactive versions of INTRO and INTRO1.

INTRO

```
      DIMENSION H(72),NAME(80),IFORM(80),NAMSYS(8),IP(80)
     +,NAMSUB(8)
      INTEGER YES,GO
      DATA YES,NO,GO/'Y','N','GO  '/
      ITEST=0
      CALL OPSYS('COMMAND','ALL',IRTN,IMESS,'FILE JOBOUT')
      CALL OPSYS('ALLOC','JOBOUT',9)
      CALL CLEAR
      WRITE(6,200)
  200 FORMAT(6X,'ENTRY INTO THE DATA MANAGEMENT SOFTWARE.'/
     +1X,'PURPOSE   : DATA EDITING AND CASE FILE ADDITIONS. '/
     +1X,'PROCEDURE:                                          '/
     +1X,'             1. PLACE SPSS FILE ON DISK FILE.     '/
     +1X,'             2. SCAN CASES FOR COMPLETE CASES.     '/
     +1X,'             3. SPLIT CASES AND DELETE )10% MISSING VALUES. '/
     +1X,'             4. USE COMPLETE CASES TO REPLACE MISSING VALUES.'/
     +1X,'             5. REWRITE SPSS FILE WITH CORRECT CASES.   '//)
      CALL CLEAR
      WRITE(6,201)
  201 FORMAT(1X,'WELL, WELL RON;  HOW THE HELL ARE YOU')
      READ(5,100) IANS
  100 FORMAT(1A4)
      CALL REPLY(IANS)
    5 WRITE(6,202)
  202 FORMAT(1X,'ARE YOU READY TO DUMP THE SPSS DATA FILE???')
      CALL ANSWER(IRN)
      IF(IRN.EQ.0) GO TO 10
      IF(ITEST.EQ.99) GO TO 99
      WRITE(6.203)
```

```
  203 FORMAT(6X,'THEN WHY DID YOU START THIS PROGRAM ?????'/
     +1X,'I AM GOING TO ASSUME THAT YOU CAN NOT TYPE AND YOU NEED'/
     +1X,'ANOTHER CHANCE TO ANSWER THE QUESTION.'/)
      GO TO 5
   10 WRITE(6,204)
  204 FORMAT(1X,'VERY WELL THEN I AM READY TO TRANSFER THE DATA.')
      WRITE(6,205)
  205 FORMAT(1X,'I NEED SOME INFORMATION;')
      WRITE(6,206)
      READ(5,*) NVARS
  206 FORMAT(1X,'FIRST, THE NUMBER OF VARIABLES IN THE FILE ')
      WRITE(6,207)
      READ(5,101) (NAME(II),II=1,55)
  101 FORMAT(80A1)
  207 FORMAT(1X,'NEXT, THE NAMES  OF THE VARIABLES IN THE FILE:')
      WRITE(6,208)
      READ(5,101) (IFORM(II),II=1,55)
  208 FORMAT(1X,'NEXT, THE FORMAT TO READ THE VARIABLES IN THE FILE:')
      WRITE(6,209)
      READ(5,101) (NAMSYS(II),II=1,8)
      WRITE(6,500)
      READ(5,101) (NAMSUB(II),II=1,8)
  209 FORMAT(1X,'LAST, THE NAME OF THE SPSS SYSTEM FILE TO READ ')
  500 FORMAT(1X,'AND THE NAME OF THE SUBFILE (??):')
      WRITE(9,210) (NAMSYS(II),II=1,8),(NAMSYS(II),II=1,8)
```

```
210 FORMAT(
   +'//XXXXXXXX  JOB  04423,STALCUP,CLASS=A'
   +/'/*JOBPARM SKIP=YES,TIME=2
   +/'// EXEC SPSS,DSN9=''&&OUTCASE'',UNIT9=SYSDA,'
   +/'//  STATUS9=NEW,DISP9=PASS,TRK9=400,'
   +/'//  DCB9=''(RECFM=FB,LRECL=400,BLKSIZE=4000)'',DSN3=',8A1
   +''GET FILE         ',8A1)
    WRITE(9,501) (NAMSUB(II),II=1,8)
501 FORMAT(
   +'RUN SUBFILES    ',8A1)
    WRITE(9,211) (IFORM(II),II=1,55),(NAME(II),II=1,55)
211 FORMAT(
   +'WRITE CASES       ',55A1,
   +/'                 ',55A1)
    WRITE(9,212) (IFORM(II),II=1,55)
212 FORMAT(
   +'FINISH'
   +/'// EXEC FORT                                 '
   +/'      DIMENSION IDATA(200),DATA(200)'
   +/'  200 FORMAT',55A1)
    WRITE(9,213) NVARS
213 FORMAT(
   +'      IREC=0
   +/'      INVARS=',I5)
    WRITE(9,214)
214 FORMAT(
   +'    5 CONTINUE                                '
   +/'      READ(11,200,END=99) (DATA(I),I=1,INVARS)'
   +/'      DO 10 I=1,INVARS                      '
   +/'      IDATA(I)=DATA(I)                         '
   +/'   10 CONTINUE '
   +/'      WRITE(12) (IDATA(I),I=1,INVARS) '
   +/'      IREC=IREC+1                              '
   +/'      GO TO 5                                  '
```

```
    +/'    99 WRITE(6,300) IREC                          '
    +/'   300 FORMAT(''1''///                               '
    +/'       +25X,''   P R O C E S S I N G   E N D E D   W I T H   ''/  '
    +/'       +25X,''                                               ''/  '
    +/'       +25X,''   N O R M A L   R E T U R N   C O D E         ''/  '
    +/'       +25X,''                                               ''/  '
    +/'       +25X,''               # RECORDS============)'',I5)  '
    +/'        STOP                              ')
     WRITE(9,215) NVARS
 215 FORMAT(
    +'        END                                  '      .
    +/'// EXEC GOFORT                              '
    +/'//FT11F001 DD DSN=&&OUTCASE,DISP=(OLD,DELETE)'
    +/'//FT12F001 DD DSN=USER.CASES(OUT),DISP=SHR                            '
    +/'//*            UNIT=SYSDA,                  '
    +/'//*            DCB=(RECFM=VBS,LRECL=800,BLKSIZE=804),'
    +/'//*            SPACE=(TRK,(200,5),RLSE),VOL=SER=ACAD01'
    +/'// EXEC FORT'
    +/'        DIMENSION IDATA(200)'
    +/'        DATA INC1,INC2,INC3,INC4/-9,-8,-7,-6/'
    +/'        IREC=0                                                         '
    +/'        IREC1=0                                                       '
    +/'        IREC2=0                                                       '
    +/'        IFLAG=0                                                       '
    +/'        NREC=',I5
    +/'     5 READ(10,END=99) (IDATA(I),I=1,NREC)')
     WRITE(7,216)
 216 FORMAT(
    +'        IREC=IREC+1                                                   '
    +/'        DO 10 I=1,NREC                                                '
    +/'        IF(IDATA(I).EQ.INC1) IFLAG=IFLAG+1                  '
    +/'        IF(IDATA(I).EQ.INC2) IFLAG=IFLAG+1                  '
    +/'C       IF(IDATA(I).EQ.INC3) IFLAG=IFLAG+1                  '
    +/'C       IF(IDATA(I).EQ.INC4) IFLAG=IFLAG+1                  '
    +/'    10 CONTINUE                                                       '
```

```
+/'        IF(IFLAG.EQ.0) GO TO 20
+/'        IF(IFLAG.LE.7) GO TO 25
+/'        IFLAG=0
+/'        GO TO 5
+/'    20 WRITE(12) (IDATA(I),I=1,NREC)'
+/'        IREC1=IREC1+1
 WRITE(9,217)
217 FORMAT(
+'        IFLAG=0
+/'        GO TO 5
+/'    25 WRITE(11) (IDATA(I),I=1,NREC)
+/'        IREC2=IREC2+1
+/'        IFLAG=0
+/'        GO TO 5
+/'    99 CONTINUE
+/'        IREC3=IREC-(IREC1+IREC2)
+/'        WRITE(6,202) IREC,IREC1,IREC2,IREC3
+/'   202 FORMAT(1X,I6,'' RECORDS PROCESSED''
+/'       +/1X,I6,'' RECORDS COMPLETE''/'
+/'       +1X,I6,'' RECORDS INCOMPLETE''/ '
+/'       +/1X,I6,'' RECORDS DISCARDED'') ')
 WRITE(9,218)
218 FORMAT(
+'        STOP
+/'        END
+/'// EXEC GOFORT'
+/'//FT10F001 DD DSN=USER.CASES(OUT).DISP=SHR'
```

```
     +'//FT11F001 DD DSN=USER.INC.CASES,DISP=SHR
     +'//*          UNIT=SYSDA,
     +'//*          DCB=(RECFM=VBS,LRECL=800,BLKSIZE=804),'
     +'//*          SPACE=(TRK,(100,5),RLSE),VOL=SER=ACAD01'
     +'//FT12F001 DD DSN=USER.COM.CASES,DISP=SHR '
     +'//*          UNIT=SYSDA,
     +'//*          DCB=(RECFM=VBS,LRECL=800,BLKSIZE=804),'
     +'//*          SPACE=(TRK.(100,5),RLSE),VOL=SER=ACAD01')
   99 CONTINUE
      CALL CLEAR
      CALL OPSYS('COMMAND','ALL',IRTN,IMESS,'PF 12 ''SUB JOBOUT''  C E')
      CALL OPSYS('COMMAND','ALL',IRTN,IMESS,'PF 11 ''STATUS    ''  C E')
      CALL OPSYS('COMMAND','ALL',IRTN,IMESS,'PF 10 ''RUN INTRO1''  C E')
      WRITE(6,300)
  300 FORMAT(1X,'ALL RIGHT RON, NOW YOU NEED TO SUBMIT THE JOB: ',
     +/11X,'PRESS PF KEY #12.'
     +/11X,'TO CHECK THE STATUS OF THE JOB'
     +/11X,'PRESS PF KEY #11.'
     +/11X,'AND TO START THE NEXT PROGRAM'
     +/11X,'PRESS PF KEY #10.'//)
      STOP
      END
      SUBROUTINE CLEAR
      WRITE(6,200)
  200 FORMAT(1X,'PLEASE CLEAR YOUR SCREEN AND PRESS ENTER ... ..')
      READ(5,100)
  100 FORMAT(1A1)
      RETURN
```

```
      END
      SUBROUTINE REPLY(IANS)
      INTEGER*4 YES,OK,FINE.GOOD,BL,BAD
      DATA YES,NO,OK,FINE,GOOD,BL,BAD/'Y','N','OK  ',
     1'FINE','GOOD','    ','BAD '/
      IF(IANS.EQ.FINE) GO TO 20
      IF(IANS.EQ.GOOD) GO TO 30
      IF(IANS.EQ.BAD) GO TO 40
      IF(IANS.EQ.OK) GO TO 50
      WRITE(6,500)
      GO TO 99
   20 WRITE(6,100)
      GO TO 99
   30 WRITE(6,200)
      GO TO 99
   40 WRITE(6,300)
      GO TO 99
   50 WRITE(6,400)
   99 CONTINUE
  100 FORMAT(1X,'JUST FINE, NOW WHAT KIND OF ANSWER IS THAT ????')
  200 FORMAT(1X,'WELL, WE WILL SEE ABOUT CHANGING THAT.')
  300 FORMAT(1X,'SORRY TO HEAR THAT, PLEASE DON'T TAKE IT OUT ON ME')
  400 FORMAT(1X,'WELL DON'T COMMIT YOURSELF TO A POSITIVE ANSWER. ')
  500 FORMAT(1X,'NEXT TIME TRY HARDER, THINK OF A BETTER ANSWER.')
      RETURN
      END
      SUBROUTINE ANSWER(IRN)
      INTEGER YES
      DATA YES,NO/'Y','N'/
      WRITE(6,200)
  200 FORMAT(1X,'(Y/N):')
   10 READ(5,100) IANS
  100 FORMAT(A1)
      IF(IANS.EQ.YES) GO TO 20
      IF(IANS.EQ.NO) GO TO 30
      WRITE(6,201)
```

```
  201 FORMAT(1X,'PLEASE ANSWER Y OR N........')
      GO TO 10
   20 IRN=0
      GO TO 99
   30 IRN=1
   99 CONTINUE
      RETURN
      END
```

INTRO1

```
      DIMENSION N(72),NAME(80),IFORM(80),NAMSYS(8),IP(80)
      INTEGER YES,GO
      DATA YES,NO,GO/'Y','N','GO  '/
      WRITE(6,200)
  200 FORMAT(1X,'READY?    IF SO TYPE THE WORD GO.')
  100 FORMAT(1A4)
   50 READ(5,100) IANS
      IF(IANS.EQ.GO) GO TO 55
      GO TO 50
   55 CONTINUE
      CALL OPSYS('COMMAND','ALL',IRTN,IMESS,'FILE JOBOUT')
      CALL OPSYS('ALLOC','2078 REPLACE',10)
      CALL OPSYS('ALLOC','JOBOUT',9)
      REWIND 9
      DO 60 J=1,4
  110 FORMAT(80A1)
      READ(10,110) (IP(I),I=1,80)
      WRITE(9,110) (IP(I),I=1,80)
```

```
   60 CONTINUE
      WRITE(6,201)
  201 FORMAT(1X,'ENTER THE NUMBER OF VARIABLES: ')
      READ(5,*) NVARS
      WRITE(9,303) NVARS
  303 FORMAT(1X,'       NVARS=',I5)
   65 CONTINUE
      READ(10,110,END=88) (IF(I),I=1,80)
      WRITE(9,110) (IF(I),I=1,80)
      GO TO 65
   88 CALL CLEAR
      WRITE(6,304)
  304 FORMAT(//1X,'NOW THE JOB FOR THE MLR IS READY.'
     +/11X,'PRESS PF KEY # 12 TO SUBMIT THE JOB'
     +/11X,'PRESS PF KEY # 11 TO CHECK THE STATUS OF THE JOB')
      WRITE(6,305)
  305 FORMAT(/1X,'CONTROL RETURNED TO YOU, OVER AND OUT.........')
      STOP
      END
      SUBROUTINE CLEAR
      WRITE(6,200)
  200 FORMAT(1X,'PLEASE CLEAR YOUR SCREEN AND PRESS ENTER.......')
      READ(5,100)
  100 FORMAT(1A1)
      RETURN
      END
      SUBROUTINE REPLY(IANS)
      INTEGER*4 YES,OK,FINE,GOOD,BL,BAD
      DATA YES,NO,OK,FINE,GOOD,BL,BAD/'Y','N','OK  ',
     +'FINE','GOOD','    ','BAD '/
      IF(IANS.EQ.FINE) GO TO 20
      IF(IANS.EQ.GOOD) GO TO 30
      IF(IANS.EQ.BAD) GO TO 40
      IF(IANS.EQ.OK) GO TO 50
      WRITE(6-500)
```

```
      GO TO 99
   20 WRITE(6,100)
      GO TO 99
   30 WRITE(6,200)
      GO TO 99
   40 WRITE(6,300)
      GO TO 99
   50 WRITE(6,400)
   99 CONTINUE
  100 FORMAT(1X,'JUST FINE, NOW WHAT KIND OF ANSWER IS THAT ????')
  200 FORMAT(1X,'WELL, WE WILL SEE ABOUT CHANGING THAT.')
  300 FORMAT(1X,'SORRY TO HEAR THAT, PLEASE DON'T TAKE IT OUT ON ME')
  400 FORMAT(1X,'WELL DON'T COMMIT YOURSELF TO A POSITIVE ANSWER, ')
  500 FORMAT(1X,'NEXT TIME TRY HARDER, THINK OF A BETTER ANSWER.')
      RETURN
      END
      SUBROUTINE ANSWER(IRN)
      INTEGER YES
      DATA YES,NO/'Y','N'/
      WRITE(6,200)
  200 FORMAT(1X,'(Y/N):')
   10 READ(5,100) IANS
  100 FORMAT(1A1)
```

```
      IF(IANS.EQ.YES) GO TO 20
      IF(IANS.EQ.NO) GO TO 30
      WRITE(6,201)
  201 FORMAT(1X,'PLEASE ANSWER Y OR N.......')
      GO TO 10
   20 IRN=0
      GO TO 99
   30 IRN=1
   99 CONTINUE
      RETURN
      END
```

Batch Version

Both INTRO and INTRO1 are documented below in a form suitable for batch entry via cards. The cards which must be changed in each instance are noted.

```
/*JOBPARM SKIP=YES,TIME=2
// EXEC SPSS,DSN9='  OUTCASE',UNIT9=SYSDA,
// STATUS9=NEW,DISP9=PASS,TRK9=600,
// DCB9='(RECFM=FB,LRECL=400,BLKSIZE=4000)',DSN3=ABOD
GET FILE        ABOD
RUN SUBFILES    S10
WRITE CASES     (67F3.0)
                SCHOOL,ITEM01,ITEM66,AGE,GRADE
FINISH
// EXEC FORT
      DIMENSION IDATA(200),DATA(200)
  200 FORMAT(67F3.0)
      IREC=0
      INVARS= 67
    5 CONTINUE
      READ(11,200,END=99) (DATA(I),I=1,INVARS)
      DO 10 I=1,INVARS
      IDATA(I)=DATA(I)
   10 CONTINUE
      WRITE(12) (IDATA(I),I=1,INVARS)
      IREC=IREC+1
      GO TO 5
   99 WRITE(6,300) IREC
  300 FORMAT('1'///
     +25X,'   P R O C E S S I N G   E N D E D   W I T H   '/
     +25X,'                                               '/
     +25X,'   N O R M A L   R E T U R N   C O D E         '/
     +25X,'                                               '/
     +25X,'             RECORDS===========>',I5)
      STOP
      END
```

Computer Center

Initial SPSS file

Format of variables to be read from INITIAL SPSS file

Number of variables to be read from INITIAL SPSS file

```
// EXEC GOFORT
//FT11F001 DD DSN=  OUTCASE,DISP=(OLD,DELETE)
//FT12F001 DD DSN=USER.CASES(OUT),DISP=SHR
//*           UNIT=SYSDA,
//*           DCB=(RECFM=VBS,LRECL=800,BLKSIZE=804),
//*           SPACE=(TRK,(200,5),RLSE),VOL=SER=ACAD01
// EXEC FORT
      DIMENSION IDATA(200)
      DATA INC1,INC2,INC3,INC4/-9,-8,-7,-6/
      IREC=0
      IREC1=0
      IREC2=0
      IFLAG=0
      NREC=   67
    5 READ(10,END=99) (IDATA(I),I=1,NREC)
      IREC=IREC+1
      DO 10 I=1,NREC
      IF(IDATA(I).EQ.INC1) IFLAG=IFLAG+1
      IF(IDATA(I).EQ.INC2) IFLAG=IFLAG+1
C     IF(IDATA(I).EQ.INC3) IFLAG=IFLAG+1
C     IF(IDATA(I).EQ.INC4) IFLAG=IFLAG+1
   10 CONTINUE
      IF(IFLAG.EQ.0) GO TO 20
      IF(IFLAG.LE.7) GO TO 25
```

Computer Center VCL

```
      IFLAG=0
      GO TO 5
   20 WRITE(12) (IDATA(I),I=1,NREC)
      IREC1=IREC1+1
      IFLAG=0
      GO TO 5
   25 WRITE(11) (IDATA(I),I=1,NREC)
      IREC2=IREC2+1
      IFLAG=0
      GO TO 5
   99 CONTINUE
      IREC3=IREC-(IREC1+IREC2)
      WRITE(6,202) IREC,IREC1,IREC2,IREC3
  202 FORMAT(1X,I6,' RECORDS PROCESSED'
     +/1X,I6,' RECORDS COMPLETE'/
     +1X,I6,' RECORDS INCOMPLETE'/
     +/1X,I6,' RECORDS DISCARDED')
      STOP
      END
```

```
// EXEC GOFORT
//FT10F001 DD DSN=USER.CASES(OUT),DISP=SHR
//FT11F001 DD DSN=USER.INC.CASES,DISP=SHR          Computer Center VCC
//*          UNIT=SYSDA,
//*          DCB=(RECFM=VBS,LRECL=800,BLKSIZE=804),
//*          SPACE=(TRK,(100,5),RLSE),VOL=SER=ACAD01
//FT12F001 DD DSN=USER.COM.CASES,DISP=SHR
//*          UNIT=SYSDA,
//*          DCB=(RECFM=VBS,LRECL=800,BLKSIZE=804),
//*          SPACE=(TRK,(100,5),RLSE),VOL=SER=ACAD01

//XXXXXXXX  JOB  04423,STALCUP,CLASS=C
/*JOBPARM SKIP=YES,TIME=15
// EXEC FORT
      COMMON /COM1/ INCOM(200)
       NVARS= 67 ———————————————— Number of variables
      I=0
      NNNN=NVARS-1
   10 READ(10,END=99)   ISC,(INCOM(J),J=1,NNNN)
      WRITE(6,201)  ISC,(INCOM(J),J=1,NNNN)
      CALL REP(NVARS,ISC)
      CALL WRITE(NVARS,ISC)
      I=I+1
      GO TO 10
   99 CONTINUE
      WRITE(6,200)  I
  200 FORMAT('1','  J O B   S U M M A R Y',
     +//11X,'JOB FINISHED;  COMPLETION CODE=0'
     +/11X,'NUMBER OF CASES=',I5)
  201 FORMAT(1X,'INCOMPLETE CASE LISTED BELOW:',
     +50(/1X,30I4))
      STOP
      END
```

```
C...........................................................
      SUBROUTINE REP(NREC,ISC)
      DIMENSION XX(1350,200),TEMP(200),

     +N(200,7),VARS(200),IREP(200),NRR(6),
     +INDEX(200)
      COMMON /COM1/INDATA(200)
      COMMON /COM2/ISUB(1350,200)
      COMMON /COM3/IFILE(1350,200)
      COMMON /COM4/INFILE(200)
      ICOMP=1
      IT=ISC
      NNREC=NREC-1
      REWIND 11
   10 READ(11,END=99)   ISC,(INFILE(I),I=1,NNREC)
C     WRITE(6,777)   ISC,(INFILE(I),I=1,NNREC)
  777 FORMAT(1X,30I3/)
      IF(IT.EQ.ISC)  GO TO 20
      GO TO 10
   20 CONTINUE
      J=0
      L=0
C.................        J = COUNTER FOR INCOMPLETE VARS
C.................        L = COUNTER FOR COMPLETE VARS
C.................     IREP = INDEX OF LOCATION FOR COMPLETE VARS
C.................    INDEX = INDEX OF LOCATION FOR INCOMPLETE VARS
C.................    IFILE = VALUES OF THE DEPENDENT VARS (COMPLETE)
C.................     ISUB = VALUES OF THE INDEPENDENT VARS (INCOMPLETE)
```

```
      DO 30 I=1,NNREC
      IF(INDATA(I).EQ.-9.OR.INDATA(I).EQ.-8) GO TO 40
      GO TO 50
   40 J=J+1
      INDEX(J)=I
      GO TO 30
   50 CONTINUE
      L=L+1
      IREP(L)=I
   30 CONTINUE
      M=0
   35 M=M+1
      DO 60 K=1,J
      IFILE(M,K)=INFILE(INDEX(K))
   60 CONTINUE
      DO 70 K=1,L
      ISUB(M,K)=INFILE(IREP(K))
   70 CONTINUE
      READ(11,END=88)   ISC,(INFILE(LL),LL=1,NNREC)
      IF(IT.EQ.ISC) GO TO 35
      LL=L+J
      L1=L+1
      M1=L+1
      MTM=M1*M1
C     WRITE(6,888)
      CALL SUB(J,L,M,NVARS,IREP,INDEX,XY,8,LL,L1,M1)
      RETURN
   88 CONTINUE
C     WRITE(6,666)
```

```
  555 FORMAT(1X,'*REC=',I5)
  666 FORMAT(1X,'CALL AT END')
  888 FORMAT(1X,'CALL AT MID')
      LL=L+J
      L1=L+1
      M1=L+1
      NTM=M1*M1
      CALL SUB(J,L,N,NVARS,IREP,INDEX,XY,B,LL,L1,M1)
   99 CONTINUE
      RETURN
      END
C.....................................................................
      SUBROUTINE SUB(J,L,N,NVARS,IREP,INDEX,XY,B,LL,L1,M1)
      DIMENSION XY(N,M1),TEMP(200),
     *XYBAR(200),A(5400),ANOVA(14),B(M1,7),VARB(2100),IREP(200),
     *RRR(6),INDEX(200)
      COMMON /COM1/INDATA(200)
      COMMON /COM2/ISUB(1350,200)
      COMMON /COM3/IFILE(1350,200)
      COMMON /COM4/INFILE(200)
C.................     J = COUNTER FOR INCOMPLETE VARS
C.................     L = COUNTER FOR COMPLETE VARS
C.................     N = COUNTER FOR NUMBER OF CASES
C.................  IREP = INDEX OF LOCATION FOR COMPLETE VARS
C................. INDEX = INDEX OF LOCATION FOR INCOMPLETE VARS
C................. IFILE = VALUES OF THE DEPENDENT VARS (COMPLETE)
C.................  ISUB = VALUES OF THE INDEPENDENT VARS (INCOMPLETE)
C     WRITE(6,666) N,L,J
```

```
  666 FORMAT(1X,3I10/20(20I5/))
      DO 10 LL=1,N
      DO 10 KK=1,L
      XY(LL,KK)=ISUM(LL,KK)
   10 CONTINUE
C     WRITE(6,666) N,L,J,(IFILE(LL,1),LL=1,N)
      ICON=0
    5 CONTINUE
      ICON=ICON+1
      IF(ICON.GT.J) GO TO 999
      KK=L+1
      DO 20 LL=1,N
      XY(LL,KK)=IFILE(LL,ICON)
   20 CONTINUE
C     WRITE(6,666) KK,ICON,LL
      NN=N
      MM=L
      II=N
      NBR(1) = MM + 1
      NBR(2) = NN
      NBR(3) = NN
      NBR(4) = 1
```

```
      NBR(5) = 1
      NBR(6) = 1
      ALFA = 0.10
      IB = N1
C     WRITE(6,555)
  555 FORMAT(1X,'JUST BEFORE THE CALL',I10)
      CALL BECOVM (XY,IX,NBR,TEMP,XYBAR,A,IER)

      CALL RLMUL (A,XYBAR,NN,MM,ALFA,ANOVA,B,IB,VARB,IER)
C     WRITE(6,666) B(1,1),B(NN+1,1)
      SUM=B(NN+1,1)
      DO 30 II=1,NN
      SUM=SUM+B(II,1)*INDATA(IREP(II))
   30 CONTINUE
      CALL ROUND(SUM,IS)
      INDATA(INDEX(ICOD))=IS
      GO TO 5
  999 CONTINUE
      RETURN
      END
C...........................................................
      SUBROUTINE ROUND(X,I)
      I=X
      XX=X*100
      II=I*100
      Y=XX-II
      IF(Y.GE.45.0.AND.Y.LE.55.0) GO TO 10
      IF(Y.LT.45.0) RETURN
      IF(Y.GT.55.0) I=I+1
      RETURN
```

```
  200 FORMAT(1X,'FINISHED CASE LISTED BELOW:',
     +50(/1X,30I4))
      RETURN
      END
// EXEC GOFORT,GOSIZE=3500K
//FT10F001 DD DSN=USER.INC.CASES,DISP=SHR
//FT11F001 DD DSN=USER.COM.CASES,DISP=SHR
//FT16F001 DD DSN=USER.HAL01,DISP=SHR
// EXEC FORT
      DIMENSION N(80)
      K=0
    1 READ(12,END=88) (N(J),J=1,65)        Number of variables
      WRITE(16,100) (N(I),I=1,65)
      K=K+1
      GO TO 1
   88 CONTINUE
    5 READ(13,100,END=99) (N(I),I=1,65)      Number of variables
      WRITE(16,100) (N(I),I=1,65)
      K=K+1
      GO TO 5
   99 CONTINUE
      WRITE(6,201) K
  100 FORMAT(80I3)
  200 FORMAT(/1X,3(1X,30I3/))
  201 FORMAT(///1X,'  OF CASES ADDED=',I6)
      STOP
      END
// EXEC GOFORT
//FT12F001   DD DSN=USER.COM.CASES,DISP=SHR
//FT13F001   DD DSN=USER.HAL01,DISP=SHR
//FT16F001   DD DSN=USER.FINISH.CASES,DISP=(MOD,KEEP)
//
```

Prediction of Time Required

The programming requires fairly large amounts of CPU time when dealing with large files. The total time required is a function of the number of variables, the number of cases, the number of cases with missing data, and the interaction between the number of missing cases and the total number of cases.

To date, run time data have been gathered on 23 files which ranged in size from an $N = 122$ to $N = 1{,}081$. The number of missing data cases ranged from $N = 12$ to $N = 191$. All the files had 67 variables. Required CPU times ranged from 2.145 minutes to 203.072 minutes.

This information was used to build to test a regression equation which could be used to predict the CPU time required. The $R^2 = .9960$ had a $p < .00000$, but the equation is limited by the fact that all the test files had 67 variables. Even considering this limitation, the following regression equation may be useful in predicting CPU time.

$$\mathrm{CPUT} = (.22248840)u + (.00779903)\mathrm{NC} + (.08595389)\mathrm{NM} + (.00100721)\mathrm{NC} * \mathrm{NM} + E$$

Where:

CPUT = CPU time required
U = Unit vector (+)
NC = Number of cases
NM = Number of cases with _ 10% missing data
E = Error vector (ignored when predicting CPUT)

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